Deterministic radio propagation modeling and ray tracing

1) Introduction to deterministic propagation modelling

2) Geometrical Theory of Propagation I - The ray concept – Reflection and transmission

3) Geometrical Theory of Propagation II - Diffraction, multipath

4) Ray Tracing I

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6) Deterministic channel modelling I

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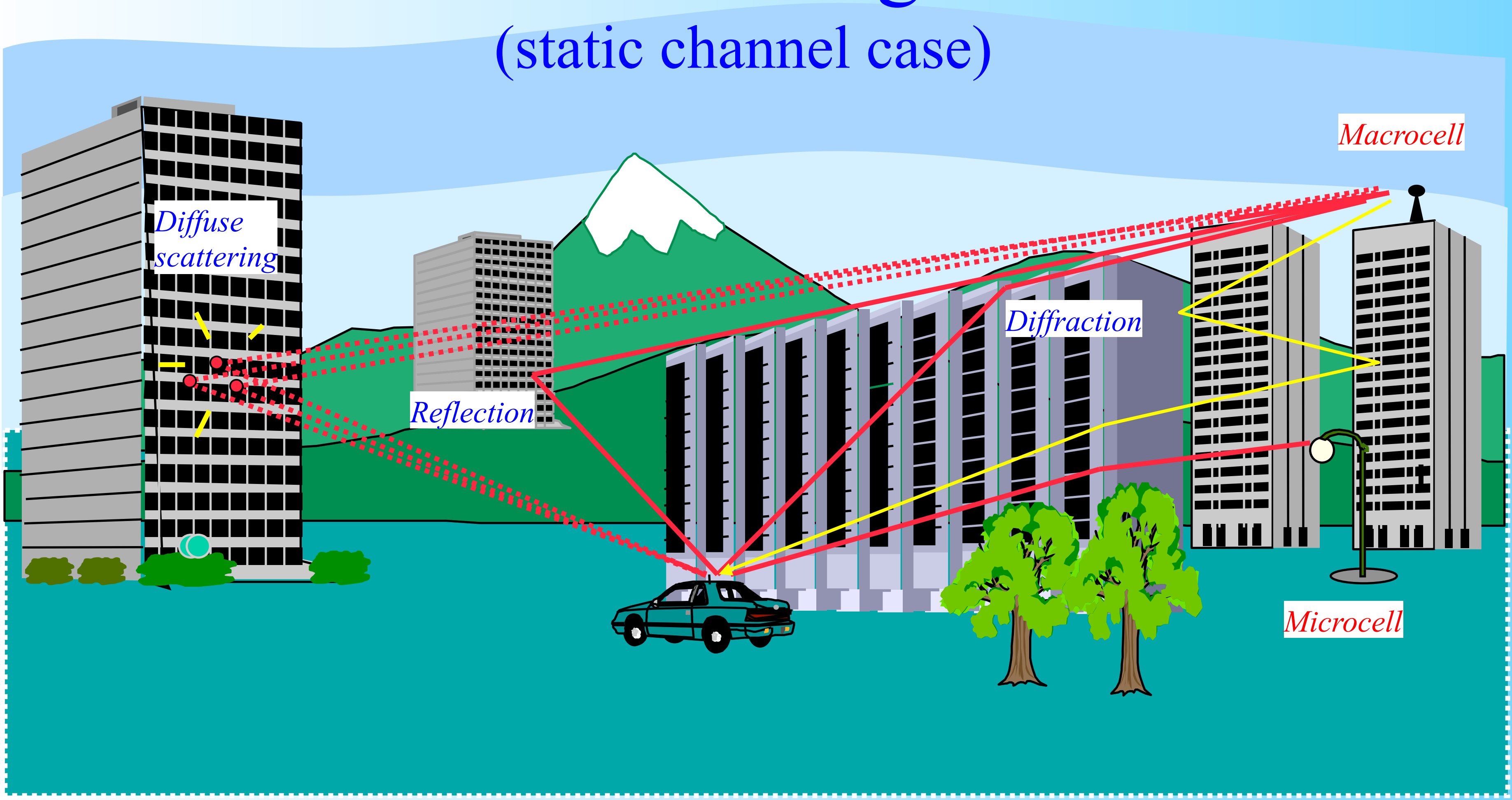
8) Project - discussion

Deterministic multipath channel modelling

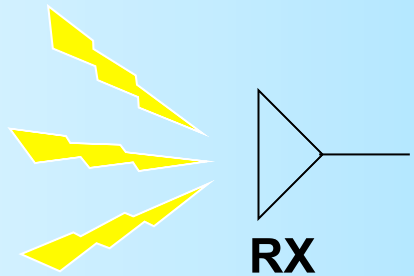
(static channel case)



Received signal with 1 path

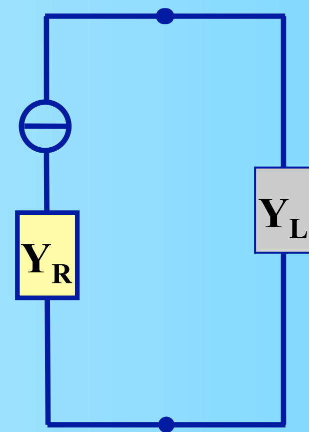
i-th ray $\vec{E}_T^i$

$\chi_i = \left(\theta_R^i, \phi_R^i\right)$



$\equiv$

$I_R^i\left(\vec{E}_T^i\right)$



Complex number representing the received signal (current) :

$$I_R^i = -j\lambda\sqrt{\frac{\Re e\left(Y_R\right) g_R\left(\theta_R^i, \phi_R^i\right)}{\pi\eta}}\left\{\hat{p}_R\left(\theta_R^i, \phi_R^i\right)\cdot \vec{E}_T^i\right\} = \left|I_R^i\right| e^{j\arg\left(I_R^i\right)} = \rho_i e^{j\vartheta_i}$$

(of course it is a funcion of the current I_T at the transmitter end)

In particular we have:

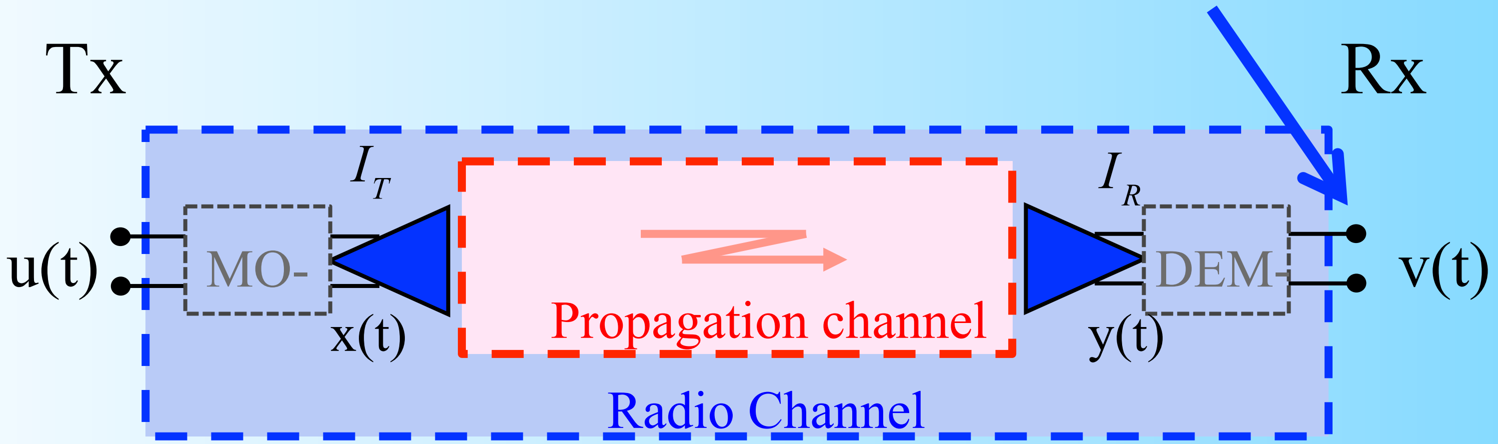
ρ_i	amplitude	s_i, t_i	lenth, delay
θ_i	phase	χ_i	dicertion of arrival
f_i	*Doppler freq.*	ψ_i	dicertion of departure

Received signal with N_r paths (1/2)

$$I_R = \sum_{i=1}^{N_r} I_R^i = \sum_{i=1}^{N_r} \rho_i e^{j\vartheta_i} \quad (1)$$

- In the narrowband case the new signal at the Rx is still a sinusoid, but with amplitude and phase given by the coherent sum (1). Time does not appear.

- In the wideband case, i.e. when a transmitted signal is modulated on the carrier we have to include the MO-DEM and consider the time domain.



Received signal with N_r paths (2/2)

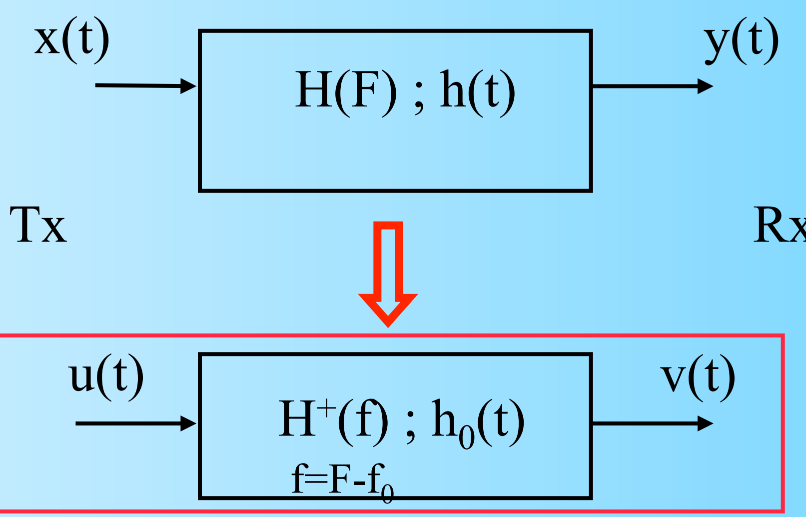
- Not only the carrier has a new amplitude and phase now, but different propagation delays of different paths create echoes of the modulating signal at the Rx!

- The **baseband- or lowpass-equivalent radio channel** must be considered now:

$$x(t) = A(t)\cos\left[2\pi f_o t + \alpha(t) - \varphi_o\right] = $$
$$= \mathrm{Re}\left\{u(t)e^{j2\pi f_0 t}\right\}$$

u(t) signal's complex envelope: it contains the modulation law

$$u(t) = A(t)e^{j\alpha(t)}e^{-j\varphi_o}$$



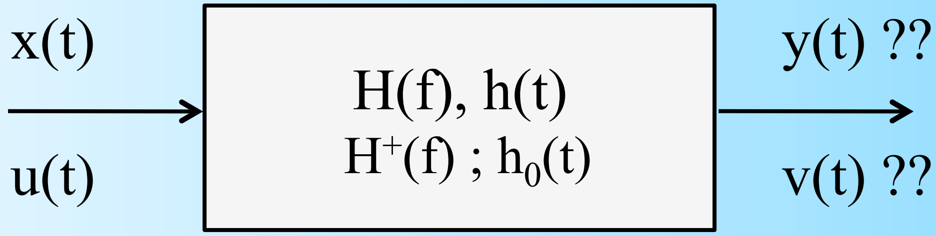
lowpass-equivalent radio channel

Input-Output Multipath Channel Functions

- The presence of multipath can be formally described by the some proper "*I/O Channel Functions*" that can be associated with the radio channel

- **hp1** - *discrete channel*: N_r rays/paths

  hp2- *static channel*: channel properties don't vary in time ➔ terminals don't move (in practice, fluctuations in time can be neglected during transmission)



- The I/O channel functions establish a correspondence between the input and the output signals, i.e. formulates the effects of the environment on the propagating signal

Channel Lowpass Impulse Response (1/2)

- According to hp 1-2, the i-th path ($i=1,.., N_r$) introduces:
  - amplitude loss (ρ_i) due to the attenuation produced by propagation and by the interactions between the wave and the environment along the path;
  - time shift (t_i) due to propagation delay;
  - phase shift (θ_i) due to the phase change along the path;

$$y_i(t) = \rho_i \cdot A(t - t_i) \cdot \cos\left(2\pi f_0 (t - t_i) + \alpha(t - t_i) - \varphi_0 + \theta_i\right)$$

$$y(t) = \sum_{i=1}^{N_r} y_i(t) = \Re e\left(\sum_{i=1}^{N_r} \rho_i \cdot A(t - t_i) \cdot e^{j2\pi f_0 t} \cdot e^{-j2\pi f_0 t_i} \cdot e^{j\alpha(t - t_i)} \cdot e^{-j\phi_0} e^{j\theta_i} \right)$$

$$y(t) = \Re e\left(\underbrace{\sum_{i=1}^{N_r} \underbrace{\rho_i \cdot A(t - t_i) \cdot e^{j\alpha(t - t_i)} \cdot e^{-j\phi_0}}_{u(t-t_i)} \cdot e^{-j2\pi f_0 t_i} \cdot e^{j\theta_i}}_{v(t)} \cdot e^{j2\pi f_0 t} \right)$$

Channel Lowpass Impulse Response (2/2)

- v(t) represents the complex envelope of the received signal:

$$v(t) = \sum_{i=1}^{N_r} \rho_i \cdot u(t - t_i) \cdot e^{j(\theta_i - 2\pi f_0 \cdot t_i)}$$

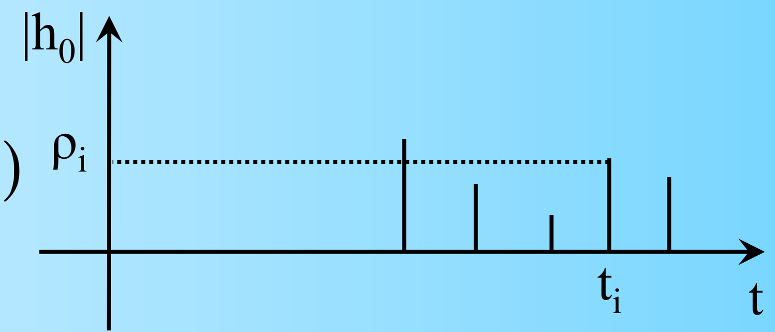
- $u(t - t_i) = \int_{-\infty}^{\infty} \delta(\xi - t_i) \cdot u(t - \xi) d\xi$ (well known property of the δ-distribution)

$$v(t) = \sum_{i=1}^{N_r} \rho_i \int_{-\infty}^{\infty} \delta(\xi - t_i) \cdot u(t - \xi) d\xi \cdot e^{j(\theta_i - 2\pi f_0 \cdot t_i)} = \int_{-\infty}^{\infty} \sum_{i=1}^{N_r} \rho_i \, \delta(\xi - t_i) e^{j(\theta_i - 2\pi f_0 \cdot t_i)} u(t - \xi) d\xi$$

$$h_0(t) \equiv \sum_{i=1}^{N_r} \rho_i \delta(t - t_i) e^{j(\theta_i - 2\pi f_0 \cdot t_i)}$$

Channel lowpass impulse response

$$v(t) = \int_{-\infty}^{\infty} h_0(\xi) \cdot u(t - \xi) d\xi = \int_{-\infty}^{\infty} h_0(t - \xi) \cdot u(\xi) d\xi = h_0(t) \otimes u(t)$$



Channel low-pass and band-pass transfer functions

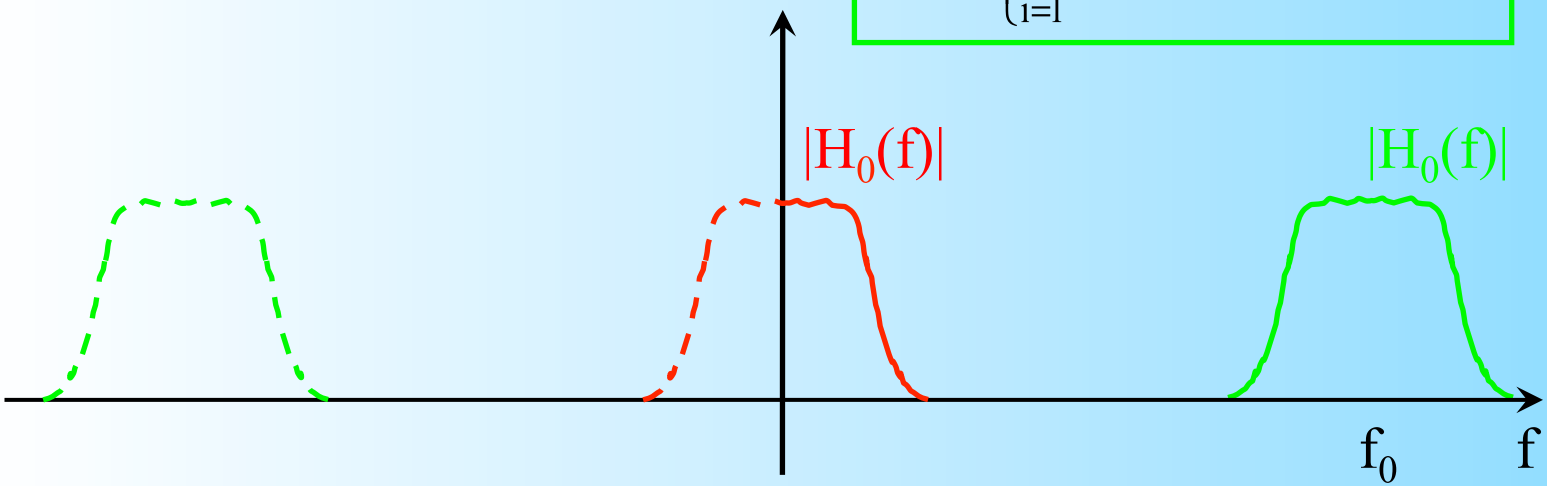
- The Fourier-transform of $h_0(t)$ represents the *channel* *<u>low-pass</u>* *transfer function* $H_0(f)$

$$H_0(f)=\Im\left[h_0(t)\right]=\sum_{i=1}^{N_r}\int_{-\infty}^{\infty}\rho_i\,\delta(t-t_i)\,e^{j(\theta_i-2\pi f_0\cdot t_i)}\cdot e^{-j2\pi f\cdot t}dt=\sum_{i=1}^{N_r}\rho_i\;e^{-j\left(2\pi(f+f_0)\cdot t_i-\theta_i\right)}$$

- $H_0(f)$ is related to the *channel transfer function* $H(f)$ through the following, general relation:

$$H(f)=\begin{cases}H_0(f-f_0) & f\geq 0\\ H_0^*(-f-f_0) & f<0\end{cases}$$

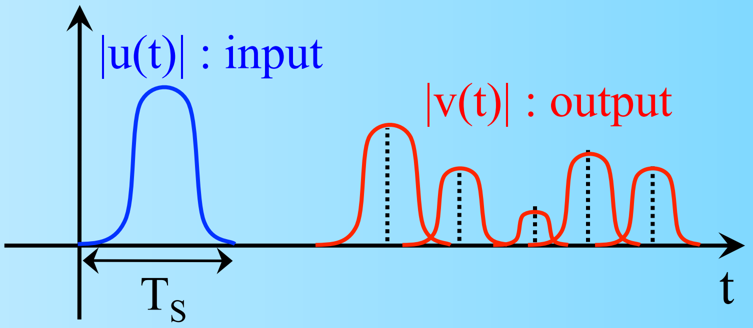
$$H(f)=\begin{cases}\sum_{i=1}^{N_r}\rho_i\;e^{-j(2\pi f\cdot t_i-\theta_i)} & f\geq 0\\ \sum_{i=1}^{N_r}\rho_i\;e^{-j(2\pi f\cdot t_i+\theta_i)} & f<0\end{cases}$$



Channel Time Dispersion (time domain)

- With reference to the signals complex envelope:

$$v(t)=\sum_{i=1}^{N_r}\rho_i\cdot u(t-t_i)\cdot e^{j(\theta_i-2\pi f_0\cdot t_i)}$$

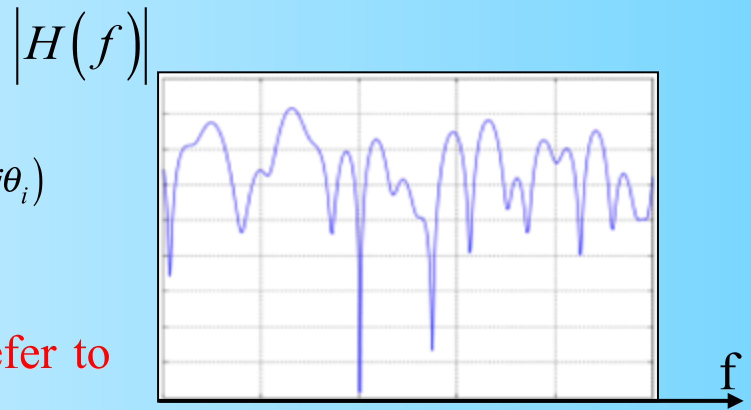


- Because of the multipath and different propagation delays, the radio channel is affected by **time dispersion** at the Rx.
- In digital communication systems, symbols may overlap at the receiver, thus producing the so called **intersymbol interference - ISI (**avoided only if $T_S >> \Delta t = t_{i,max} - t_{i,min}$)

Channel frequency selectivity (frequency domain)

Equivalent low-pass channel transfer function

$$H(f) = F\{h(t)\} = \sum_{i=1}^{N} \rho_i \, e^{-j2\pi f t_i} \, e^{(-j2\pi f_0 t_i + j\theta_i)}$$



Note: we neglect now the footer "0" we always refer to the low-pass functions

- Because of the multipath and different propagation delays, the radio channel frequency response is non-flat at the Rx → **distortion** for wideband signals or **frequency-selective fading**.
- If the signal is narrowband then we have **frequency-flat fading**

Example: 2 paths

The time origin is arbitrary, therefore we can choose $t_1 = \theta_1 = 0$. Then we can normalze w.r.t. the amplitude of the first path:

$$H(F) = 1 + \frac{\rho_2}{\rho_1} e^{-j\left\{2\pi\left(f+f_0\right)t_2 - \vartheta_2\right\}} = 1 + \rho e^{-j\left\{2\pi F \Delta t \ - \vartheta \ \right\}}$$

Thus the frequency response module is:

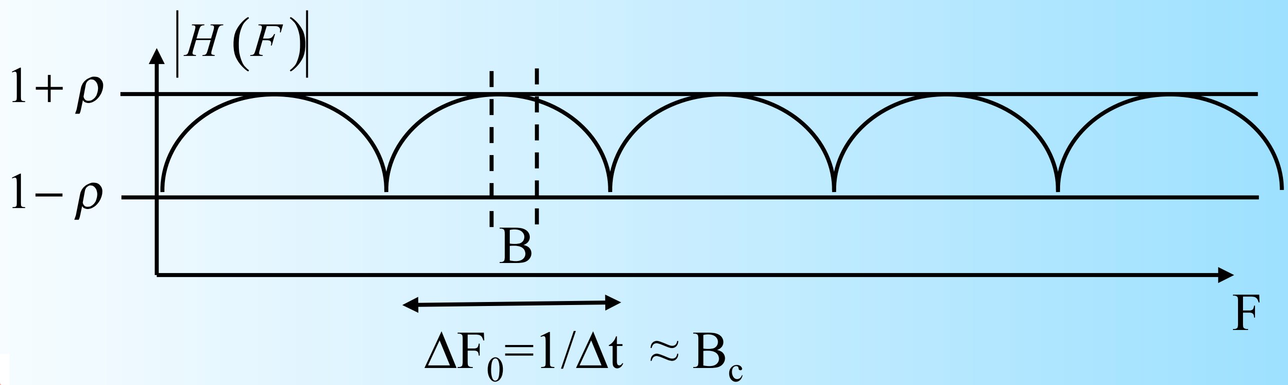
$$\left|H\left(F\right)\right| = \sqrt{\left[1+\rho\cos(2\pi F\Delta t - \theta)\right]^2 + \rho^2 \sin^2(2\pi F \Delta t - \theta)} = \sqrt{1+\rho^2+2\rho\cos(2\pi F\Delta t - \theta)}$$

Notches of |H(F)|:

$2\pi\ F_{0k}\Delta t - \theta = (2k+1)\,\pi$

Distance between two notches:

$2\pi\,(F_{0\,k+1} - F_{0\,k})\,\Delta t = 2\pi \longrightarrow \Delta F_0 = (F_{0\,k+1} - F_{0\,k}) = 1/\Delta t$



Flat fading condition:

$$B \ll B_c \approx \frac{1}{\Delta t}$$

Wideband channel parmeters (1/2)

Real-world h(t) (e.g: measured) is a time- continuous function. The following functions can therefore be defined:

$$p(t) = \frac{|h(t)|^2}{\int |h(t)|^2 \, dt} \quad \text{[W/s];} \qquad \text{it's normalized: } \int p(t)\,dt = 1$$

It’s the **deterministic *power-delay profile***

If an estimate of the power-delay profile for a given environment is needed, then by averaging N samples of $p(t)$ for different Tx-Rx positions over the environment we can get :

$$q(t) \approx \frac{1}{N} \sum_{i=1}^{N} p_i(t)$$

mean *power-delay profile*

Wideband channel parmeters (2/2)

Channel time-dispersion can be estimated through the following parameters:

RMS delay spread (DS)

$$DS = \sqrt{\int p(t)\left(t - T_{M0}\right)^2 dt} \quad T_{M0} = \int p(t)\, t\, dt \quad \text{(deterministic DS)}$$

$$DS = \sqrt{\int q(t)\left(t - T_{M0}\right)^2 dt} \quad T_{M0} = \int q(t)\, t\, dt \quad \text{(average DS)}$$

DS is simply the standard deviations of *p* or *q* interpreted as a pdf.

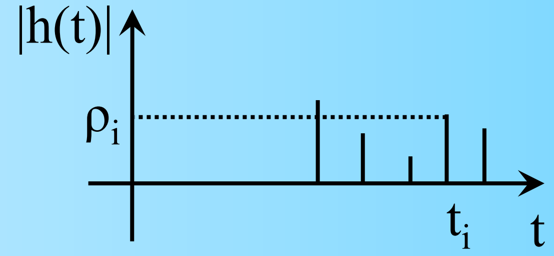
Moreover the following frequency-coherence parameter can be derived

Coherence bandwidth B_c

$$B_c \simeq \frac{1}{DS}$$

Discrete case (2/4)

$$h(t)=\sum_{i=1}^{N_r}\rho_i\,\delta(t-t_i)\,e^{j(-2\pi f_0\cdot t_i+\theta_i)}$$



Since h(t) is a discrete function which is defined only for t={t_i}, and therefore the impulses do not overlap, we have:

$$\left|h(t)\right|=\left|\sum_{i=1}^{N_r}\rho_i\,\delta(t-t_i)\,e^{j(-2\pi f_0\cdot t_i+\theta_i)}\right|=\sum_{i=1}^{N_r}\left|\rho_i\,\delta(t-t_i)\,e^{j(-2\pi f_0\cdot t_i+\theta_i)}\right|=\sum_{i=1}^{N_r}\left|\rho_i\right|\delta(t-t_i)$$

then

$$\left|h(t)\right|^2=\left(\sum_{i=1}^{N_r}\rho_i^2\cdot\delta^2(t-t_i)\right)\left(\sum_{j=1}^{N_r}\rho_j^2\cdot\delta^2(t-t_j)\right)=\sum_{i=j=1}^{N_r}\rho_i^2\cdot\delta(t-t_i)$$

were the last equal sign is due to the fact that the double products at the left hand side are non-zero only when i=j. Also, for simplicity we have assumed that $\delta^2(t-t_i)=\delta(t-t_i)$

Ideal wideband channel parameters (discrete case)

Using the time-discrete channel impulse response, derived for example from a ray tracing program or with an ideal, infinite bandwidth system, the deterministic wideband channel parameters can be defined as follows:

Power delay profile

$$p(t) = \frac{\sum_{i=1}^{N_r} \rho_i^2 \, \delta(t - t_i)}{\sum_{i=1}^{N_r} \rho_i^2}$$

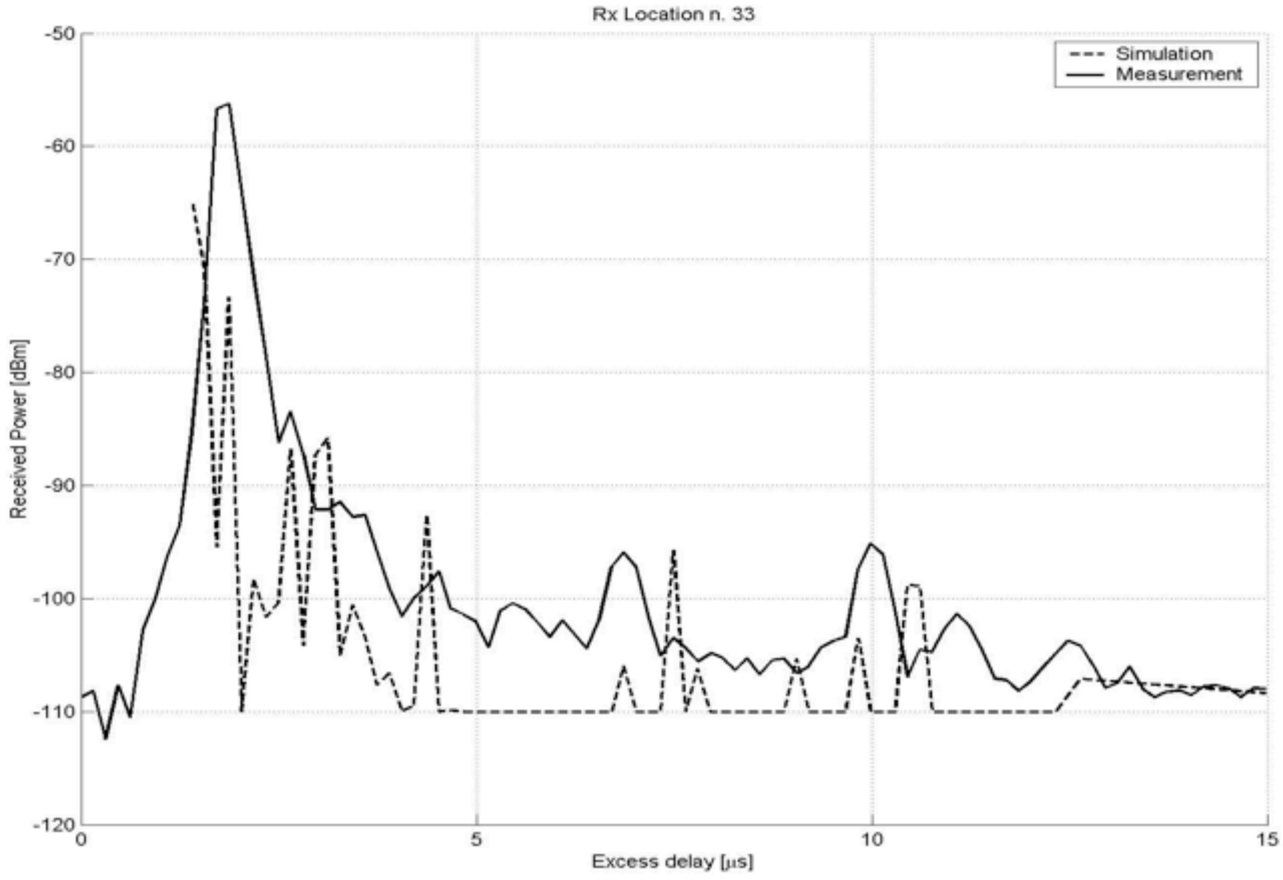
Delay Spread

$$TM_0 = \sum_{i=1}^{N_r} t_i \cdot p_i$$

$$DS = \sqrt{\sum_{i=1}^{N_r} (t_i - TM_0)^2 \cdot p_i}$$

where: $p_i = \frac{\rho_i^2}{P_{TOT}} = \frac{\rho_i^2}{\sum_{k=1}^{N_r} \rho_k^2}$

Example of deterministic power-delay profile

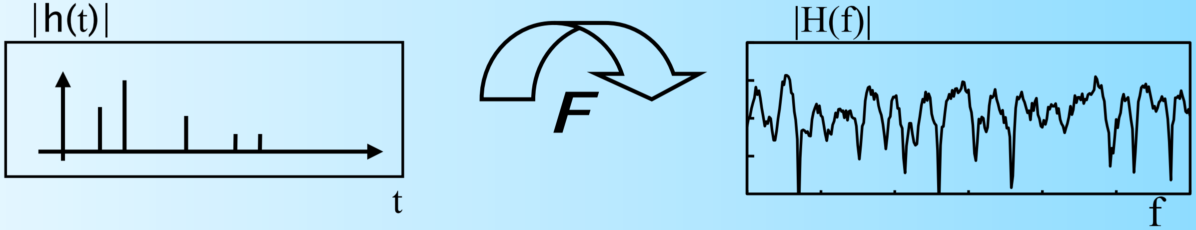


Fourier-related domains

The channel transfer functions have the form:

$$M(x)=\sum_{i=1}^{N}\delta(x-x_i)\,f(\ldots)\quad\overset{x\rightarrow F^{\pm1}\rightarrow y}{\Longrightarrow}\quad N(y)=\sum_{i=1}^{N}e^{\mp j2\pi y x_i}\,f(\ldots)$$

Es:



Since the Fourier transform of a delayed δ is an exponential, we always have such a relation between Fourier-related domains.

Vice-versa if the functional dependance is exponential, then the Fourier transform gives a δ-dependance in the transformed domain

$$\delta\text{-dependance}\quad\overset{F}{\Longleftrightarrow}\quad e\text{-dependance}$$

Extension to the space domain (1/4)

Each ray has one and only one angle of arrival. Therefore we can extrapolate the **angle-dependent impulse response** (ex: azimuth only)

$$h(t,\phi) = \sum_i \rho_i \delta\left[t - t_i\right]\delta\left[\phi - \phi_i\right] e^{j\left\{-2\pi f_0 t_i + \vartheta_i\right\}}$$

Also the **angle-dependent transfer function** can be defined:

$$H(f,\phi) = \sum_i \rho_i \delta\left[\phi - \phi_i\right] e^{j\left\{-2\pi\left(f + f_0\right)t_i + \vartheta_i\right\}}$$

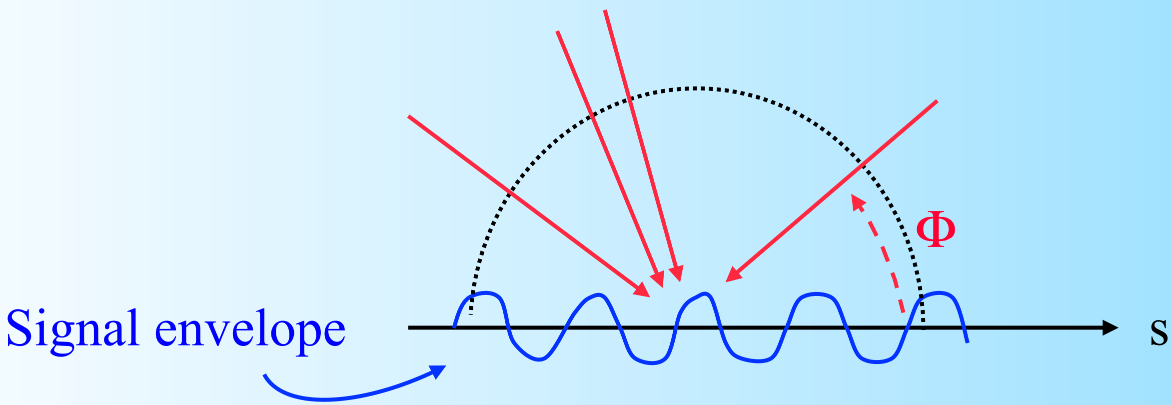
Similarly the **elevation** could be considered.
Also, the angle of departure could be considered in a similar way

Extension to the space domain (2/4)

What is the *F*-related domain of *Φ*?

It is space. We get an exponential dependence in the space domain (Fourier Optics, not covered here):

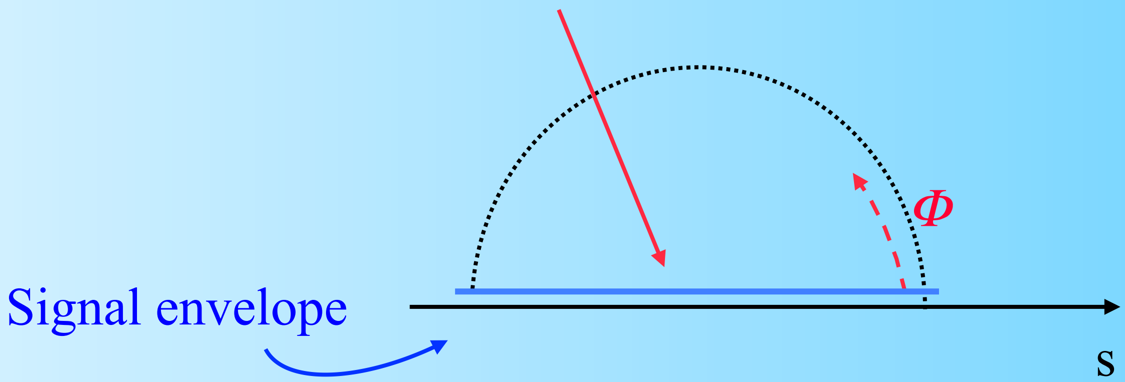
$$\Rightarrow \quad h(t,s) = \sum_i \rho_i \delta\left[t - t_i\right] e^{-j2\pi\phi_i s} e^{j\left\{-2\pi f_0 t_i + \vartheta_i\right\}}$$



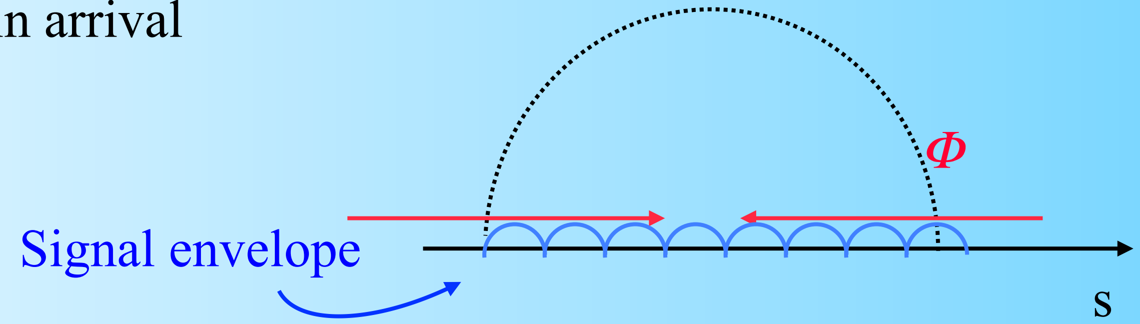
Φ can also be called spatial- frequency

Extension to the space domain (3/4)

Ex. 1: one wave in arrival



Ex 2: two opposite waves in arrival



The extreme case is the Rayleigh case: a very large number of waves uniformly distributed in $\Phi \rightarrow$ Rayleigh fading along s.

Extension to the space domain (4/4)

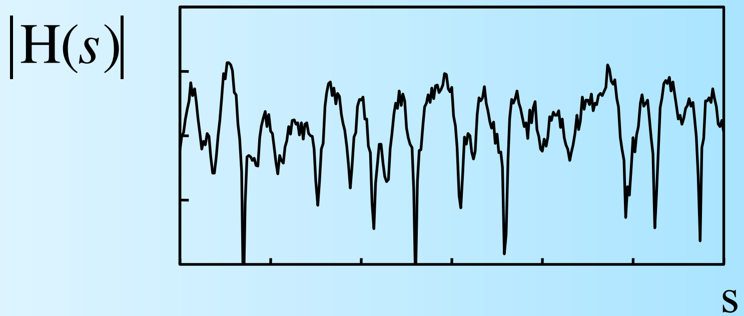
Let's consider now the angle-dependent low-pass channel transfer function:

$$H(f,\phi) = \sum_i \rho_i \delta\left[\phi - \phi_i\right] e^{j\left\{-2\pi\left(f+f_0\right)t_i + \vartheta_i\right\}}$$

It can be F-transformed in the angle domain to obtain :

$$H(f,s) = \sum_i \rho_i e^{-j2\pi\phi_i s} e^{j\left\{-2\pi\left(f+f_0\right)t_i + \vartheta_i\right\}}$$

Therefore H(s) is of the e-kind in space. We have therefore **space-selective multipath fading** or **fast fading**



Power-angle profile

The **power-azimuth profile** can be defined:

$$p_\phi(\phi) = \frac{\left|H(\phi)\right|^2}{\int \left|H(\phi)\right|^2 d\phi} \; ; \qquad H(\phi) = H(f=0,\phi)$$

In the discrete case the power-azimuth profile has the simple form:

$$p_\phi(\phi) = \frac{\sum_{i=1}^{N} \rho_i^2 \, \delta(\phi - \phi_i)}{\sum_{i=1}^{N} \rho_i^2} = \sum_{i=1}^{N} p_i \, \delta(\phi - \phi_i)$$

Through the power-angle profile the Angle-Spread can be defined

Angle-Spread

Mean angle (azimuth) of arrival:

$$\bar{\phi} = \int_0^{2\pi} \phi \, p_\phi(\phi) \, d\phi$$

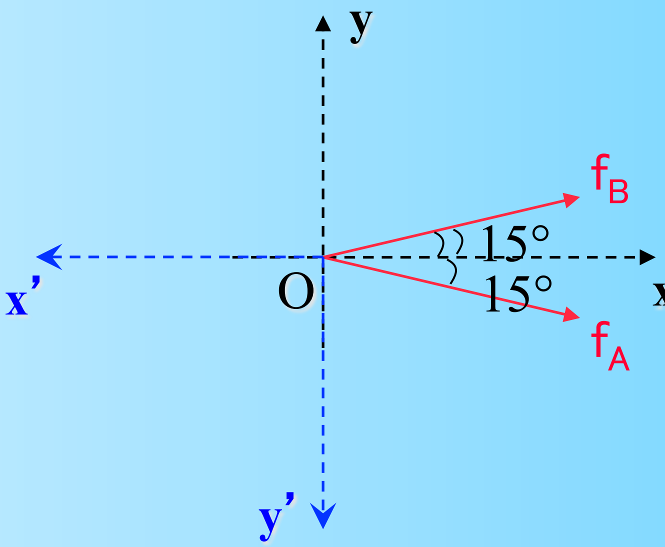
RMS Azimuth Spread:

$$AS = \sqrt{\int_0^{2\pi} \left(\phi - \bar{\phi}\right)^2 p_\phi(\phi) \, d\phi}$$

In the discrete case we have:

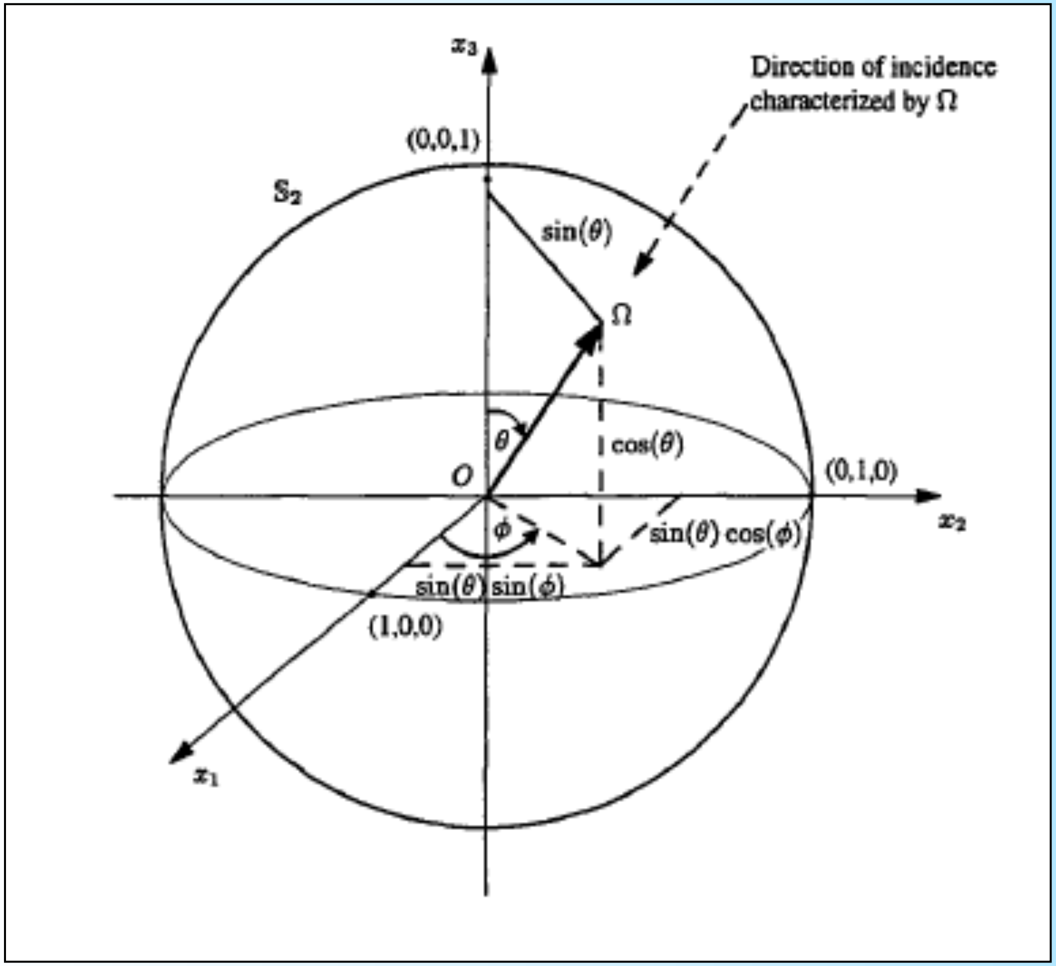
$$AS = \sqrt{\sum_{i=1}^{N} p_i \cdot \left(\phi_i - \bar{\phi}\right)^2}$$

Angle spread problem:



The reference system yielding to the minimum AS should always be adopted. In this case x’, y’.

3D Angle-spread



Each direction can be represented by a unit vector $\vec{\Omega} = \vec{\Omega}(\theta, \phi)$. The initial point of $\vec{\Omega}$ is anchored at the reference location O, while its tip is located on a sphere of unit radius centered on O (see figure)

$$\vec{\Omega} = \vec{\Omega}(\theta,\phi) = \left[\cos(\phi)\sin(\theta), \sin(\phi)\sin(\theta), \cos(\theta)\right]^T$$

3D Angle-spread (II)

Mean Direction Of Arrival (DOA):

$$\left\langle \vec{\Omega} \right\rangle = \int_{4\pi} \vec{\Omega}\, p_{\Omega}\left(\vec{\Omega}\right) d\Omega$$

$p_{\Omega}\left(\vec{\Omega}\right)$ 3D power-angle profile

3D angle spread[*]:

$$AS^{3D} = \sigma_{\vec{\Omega}} = \sqrt{\int_{4\pi} \left| \vec{\Omega} - \left\langle \vec{\Omega} \right\rangle \right|^2 p_{\Omega}\left(\vec{\Omega}\right) d\Omega} = \sqrt{\left\langle \left| \vec{\Omega} \right|^2 \right\rangle - \left| \left\langle \vec{\Omega} \right\rangle \right|^2} = \sqrt{1 - \left| \left\langle \vec{\Omega} \right\rangle \right|^2}$$

(the last equality results from: $\left| \vec{\Omega} \right| = 1$)

In the discrete case the definitions above become:

$$\left\langle \vec{\Omega} \right\rangle = \sum_{k=1}^{N} p_k \vec{\Omega}_k$$

$$\sigma_{\vec{\Omega}} = \sqrt{\sum_{k=1}^{N} \left| \vec{\Omega} - \left\langle \vec{\Omega} \right\rangle \right|^2 p_k} = \sqrt{1 - \left| \left\langle \vec{\Omega} \right\rangle \right|^2}$$

3D Angle-spread (III)

- $\sigma_{\vec{\Omega}}$ does not depend on the choice of the reference system in the RX location

- $\sigma_{\vec{\Omega}}$ provides a 3D description of the angle dispersion of the channel.

  - Notice that, in general, results: $\sigma_{\vec{\Omega}} \in [0 \ , \ 1]$

    Therefore it has the meaning of percentage of the whole solid angle

A completely similar formulation holds for the angle of departure