Chapter 7
Statistical channel parameter estimation

Signal model

Narrow-band transmission, i.e. $B\sigma_\tau \ll 1$

$$\begin{aligned} \boldsymbol{Y}(t) &= \boldsymbol{H}(t)u(t) + \boldsymbol{W}(t) \in \mathbb{C}^M \\ &= \left[\int_{\mathbb{S}_2} \boldsymbol{c}(\boldsymbol{\Omega}) h(t;\boldsymbol{\Omega}) \mathrm{d}\boldsymbol{\Omega}\right] u(t) + \boldsymbol{W}(t). \end{aligned}$$

- $\boldsymbol{Y}(t)$: the output signals of the Rx array observed at time instance t.
- $u(t)$: scalar function denoting the complex envelope of the transmitted sounding signal at time t. It is known to the Rx and that $\int_0^T u(t)u(t)^* \mathrm{d}t = 1$, where $[\cdot]^*$ denotes complex conjugate and T represents the duration of observation interval
- $\boldsymbol{H}(t)$: the time-variant response of the SIMO system.
- $h(t;\boldsymbol{\Omega})$: (time-variant) DoA spread function of the propagation channel
- $\boldsymbol{c}(\boldsymbol{\Omega}) \doteq [c_1(\boldsymbol{\Omega}), c_2(\boldsymbol{\Omega}), \ldots, c_M(\boldsymbol{\Omega})]^{\mathrm{T}}$: the responses of the Rx array.

Signal model

In a scenario where the electromagnetic energy propagates from the Tx to the Rx via D paths,

- $h(t;\mathbf{\Omega}) = \sum_{d=1}^{D} h_d(t;\mathbf{\Omega})$
- $\boldsymbol{H}(t)$ fluctuates over the overall sounding period, but remains constant within individual observation intervals:

$$\boldsymbol{H}(t) \doteq \boldsymbol{H}_n, \quad t \in [t_n, t_n + T) \text{ and } n \in [1, \ldots, N].$$

- $h_d(t;\mathbf{\Omega}),\ d = 1, \ldots, D$ are constant within individual observation intervals: $h_d(t;\mathbf{\Omega}) = h_d(t_n;\mathbf{\Omega}) \doteq h_{d,n}(\mathbf{\Omega}), \quad t \in [t_n, t_n + T)$.
- Assumption: Uncorrelated complex (zero-mean) orthogonal stochastic measures

$$\mathrm{E}[h^*_{d,n}(\mathbf{\Omega}) h_{d',n'}(\mathbf{\Omega}')] = P_d(\mathbf{\Omega}) \delta_{nn'} \delta_{dd'} \delta(\mathbf{\Omega} - \mathbf{\Omega}').$$

Power spectrum model

- $P_d(\mathbf{\Omega}) \doteq \mathrm{E}[|h_{d,n}(\mathbf{\Omega})|^2]$ denotes the direction psdf of the dth path component.
- $P_d(\mathbf{\Omega}) = P_d \cdot f_d(\mathbf{\Omega})$ with P_d representing the average power of the dth path component and $f_d(\mathbf{\Omega})$ being a normalized direction psdf.
- $f_d(\mathbf{\Omega})$ coincides with the FB_5 pdf

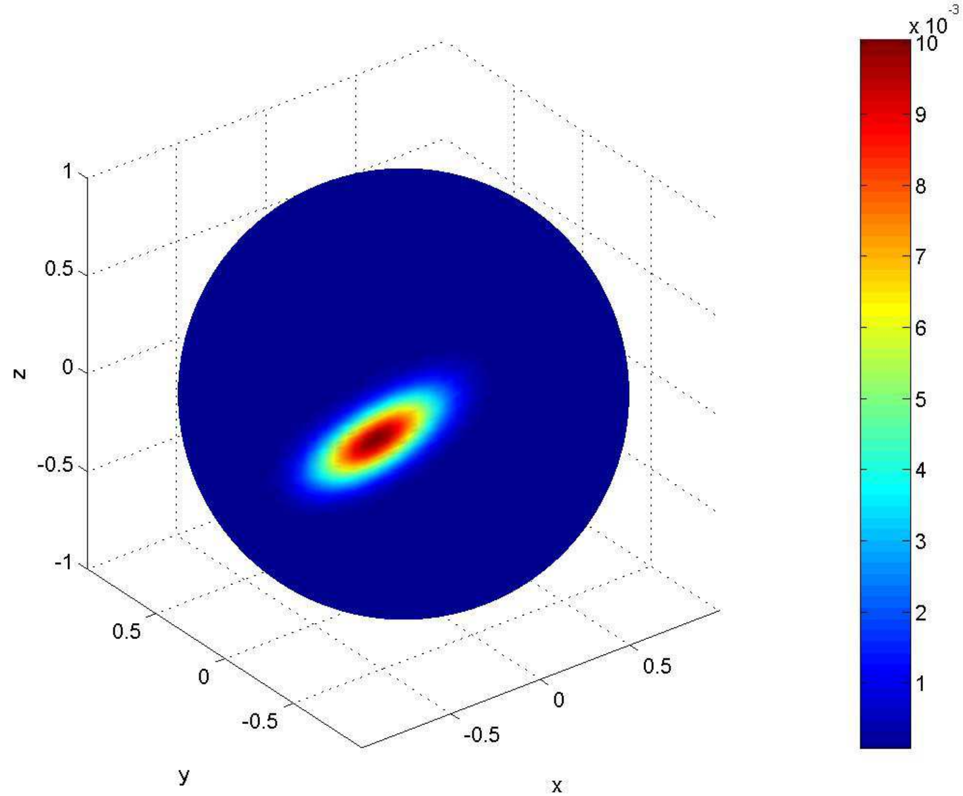
$$f_{\mathrm{FB}_5}(\mathbf{\Omega}) = C(\kappa,\eta)^{-1} \exp\{\kappa \boldsymbol{\gamma}_1^{\mathrm{T}} \mathbf{\Omega} + \kappa \cdot \eta[(\boldsymbol{\gamma}_2^{\mathrm{T}} \mathbf{\Omega})^2 - (\boldsymbol{\gamma}_3^{\mathrm{T}} \mathbf{\Omega})^2]\},$$

where $\kappa \geq 0$ represents the concentration parameter and $\eta \in [0, 1/2)$ is an ovalness factor, $C(\kappa, \eta)$ denotes a normalization constant depending on κ and η, $\boldsymbol{\gamma}_1$, $\boldsymbol{\gamma}_2$, and $\boldsymbol{\gamma}_3 \in \mathbb{R}^3$ are unit vectors, the matrix $\mathbf{\Gamma} \doteq [\boldsymbol{\gamma}_1, \boldsymbol{\gamma}_2, \boldsymbol{\gamma}_3]$ is uniquely determined by three angular parameters $\bar{\theta}$, $\bar{\phi}$ and α according to

$$\mathbf{\Gamma} = \begin{bmatrix} \sin(\bar{\theta})\cos(\bar{\phi}) & -\sin(\bar{\phi}) & \cos(\bar{\theta})\cos(\bar{\phi}) \\ \sin(\bar{\theta})\sin(\bar{\phi}) & \cos(\bar{\phi}) & \cos(\bar{\theta})\sin(\bar{\phi}) \\ \cos(\bar{\theta}) & 0 & -\sin(\bar{\theta}) \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\alpha) & -\sin(\alpha) \\ 0 & \sin(\alpha) & \cos(\alpha) \end{bmatrix}.$$

FB5 pdf

The FB$_5$ pdf with $\bar{\phi} = 135°, \bar{\theta} = 18°, \alpha = 144°, \kappa = 80$ and $\eta = 0.375$. The color bar to the right of the plot shows the magnitude expressed in linear scale.



SAGE algorithm

- all unknown model parameters

$$\boldsymbol{\theta} \doteq [P_1, P_2, \ldots, P_D, \tilde{\boldsymbol{\theta}}_1, \tilde{\boldsymbol{\theta}}_2, \ldots, \tilde{\boldsymbol{\theta}}_D]$$

with $\tilde{\boldsymbol{\theta}}_d \doteq [\bar{\phi}_d, \bar{\theta}_d, \kappa_d, \eta_d, \alpha_d]$.

- Subsets of parameters updated at the different iterations of the SAGE algorithm: the sets including the parameters characterizing individual path components, i.e. $\boldsymbol{\theta}_d \doteq [P_d, \tilde{\boldsymbol{\theta}}_d]$ with $d = [(i-1) \mod D] + 1$
- Admissible hidden data with $\boldsymbol{\theta}_d$ as

$$\begin{aligned} \boldsymbol{X}_d(t) &\doteq \boldsymbol{H}_d(t)u(t) + \boldsymbol{W}(t) \\ &= \left[\int_{\mathbb{S}_2} \boldsymbol{c}(\boldsymbol{\Omega}) h_d(t; \boldsymbol{\Omega}) \mathrm{d}\boldsymbol{\Omega}\right] u(t) + \boldsymbol{W}(t). \end{aligned} \tag{16}$$

where $\boldsymbol{H}_d(t) \doteq \boldsymbol{H}_{d,n} = \int_{\mathbb{S}_2} \boldsymbol{c}(\boldsymbol{\Omega}) h_{d,n}(\boldsymbol{\Omega}) \mathrm{d}\boldsymbol{\Omega}$.

SAGE algorithm

- The output of a correlator

$$\tilde{\boldsymbol{H}}_{d,n} \doteq \int_{t_n}^{t_n+T} \boldsymbol{x}_d(t)u(t)^*\mathrm{d}t, \quad n = 1, \ldots, N$$

when the input is the observation $\boldsymbol{X}_d(t) = \boldsymbol{x}_d(t)$ can be written as

$$\tilde{\boldsymbol{H}}_{d,n} = \boldsymbol{H}_{d,n} + \boldsymbol{N}_n,$$

- Invoking the central limit theorem, the elements of $\tilde{\boldsymbol{H}}_{d,n}$ are assumed to be Gaussian random variables.
- The vectors $\tilde{\boldsymbol{H}}_{d,1}, \ldots, \tilde{\boldsymbol{H}}_{d,N}$ form a sufficient statistic for the estimation of $\boldsymbol{\theta}_d$.

Expectation (E-) step of Iteration i

- Compute the expectation of the likelihood of $\boldsymbol{\theta}_d$ conditioned on the observation $\boldsymbol{Y}(t) = \boldsymbol{y}(t)$ and assuming that $\boldsymbol{\theta} = \widehat{\boldsymbol{\theta}}^{[i-1]}$:

$$Q(\boldsymbol{\theta}_d|\widehat{\boldsymbol{\theta}}^{[i-1]}) \doteq \mathrm{E}\big[\Lambda(\boldsymbol{\Omega}_d;\boldsymbol{x}_d)|\boldsymbol{Y}(t) = \boldsymbol{y}(t), \widehat{\boldsymbol{\theta}}^{[i-1]})\big]. \tag{17}$$

  where
  - $\widehat{\boldsymbol{\theta}}^{[i-1]}$ denotes the parameter estimates obtained in the $(i-1)$th iteration and
  - $\Lambda(\boldsymbol{\Omega}_d;\boldsymbol{x}_d)$ represents the log-likelihood function of $\boldsymbol{\Omega}_d$ given an observation $\boldsymbol{X}_d(t) = \boldsymbol{x}_d(t)$

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$$Q(\boldsymbol{\theta}_d|\widehat{\boldsymbol{\theta}}^{[i-1]}) = -\ln|\boldsymbol{\Sigma}_{\tilde{\boldsymbol{H}}_d}(\boldsymbol{\theta}_d)| - \mathrm{tr}\big[(\boldsymbol{\Sigma}_{\tilde{\boldsymbol{H}}_d}(\boldsymbol{\theta}_d))^{-1} \cdot \widehat{\boldsymbol{\Sigma}}_{\tilde{\boldsymbol{H}}_d}(\widehat{\boldsymbol{\theta}}^{[i-1]})\big], \tag{18}$$

  where $\mathrm{tr}[\cdot]$ is the trace of the matrix given as an argument

Expectation (E-) step of Iteration i

- $Q(\boldsymbol{\theta}_d|\widehat{\boldsymbol{\theta}}^{[i-1]}) = -\ln|\boldsymbol{\Sigma}_{\tilde{\boldsymbol{H}}_d}(\boldsymbol{\theta}_d)| - \mathrm{tr}\big[(\boldsymbol{\Sigma}_{\tilde{\boldsymbol{H}}_d}(\boldsymbol{\theta}_d))^{-1} \cdot \widehat{\boldsymbol{\Sigma}}_{\tilde{\boldsymbol{H}}_d}(\widehat{\boldsymbol{\theta}}^{[i-1]})\big]$, where
  - $\boldsymbol{\Sigma}_{\tilde{\boldsymbol{H}}_d}(\boldsymbol{\theta}_d)$: the covariance matrix of $\tilde{\boldsymbol{H}}_{d,n}$: $\boldsymbol{\Sigma}_{\tilde{\boldsymbol{H}}_d}(\boldsymbol{\theta}_d) = P_d \int_{\mathbb{S}_2} \boldsymbol{c}(\boldsymbol{\Omega})\boldsymbol{c}(\boldsymbol{\Omega})^{\mathrm{H}} f_d(\boldsymbol{\Omega})\mathrm{d}\boldsymbol{\Omega} + \sigma_w^2 \boldsymbol{I}_M$
  - $\widehat{\boldsymbol{\Sigma}}_{\tilde{\boldsymbol{H}}_d}(\boldsymbol{\theta})$ is the conditional covariance matrix of $\tilde{\boldsymbol{H}}_{d,n}$ given the observation $\boldsymbol{y}(t)$ for $\boldsymbol{\theta}$:

$$\widehat{\boldsymbol{\Sigma}}_{\tilde{\boldsymbol{H}}_d}(\widehat{\boldsymbol{\theta}}^{[i]}) = \boldsymbol{\Sigma}_{\tilde{\boldsymbol{H}}_d}(\widehat{\boldsymbol{\theta}}_d^{[i]}) + \boldsymbol{\Sigma}_{\tilde{\boldsymbol{H}}_d}(\widehat{\boldsymbol{\theta}}_d^{[i]})\big[\boldsymbol{\Sigma}_{\tilde{\boldsymbol{H}}}(\widehat{\boldsymbol{\theta}}^{[i]})\big]^{-1}(\widehat{\boldsymbol{\Sigma}}_{\tilde{\boldsymbol{H}}} - \boldsymbol{\Sigma}_{\tilde{\boldsymbol{H}}_d}(\widehat{\boldsymbol{\theta}}_d^{[i]})) \cdot \big[\boldsymbol{\Sigma}_{\tilde{\boldsymbol{H}}}(\widehat{\boldsymbol{\theta}}^{[i]})\big]^{-1}\boldsymbol{\Sigma}_{\tilde{\boldsymbol{H}}_d}(\widehat{\boldsymbol{\theta}}_d^{[i]}),$$

  with
    - $\boldsymbol{\Sigma}_{\tilde{\boldsymbol{H}}}(\widehat{\boldsymbol{\theta}}^{[i]}) = \sum_{d=1}^{D} \boldsymbol{\Sigma}_{\tilde{\boldsymbol{H}}_d}(\widehat{\boldsymbol{\theta}}_d^{[i]}) + \sigma_w^2 \boldsymbol{I}_M$,
    - $\widehat{\boldsymbol{\Sigma}}_{\tilde{\boldsymbol{H}}} = \frac{1}{N}\sum_{n=1}^{N} \tilde{\boldsymbol{H}}_n \tilde{\boldsymbol{H}}_n^{\mathrm{H}}$ with $\tilde{\boldsymbol{H}}_n \doteq \int_{t_n}^{t_n+T} \boldsymbol{y}(t)u(t)^*\mathrm{d}t$, $n = 1, \ldots, N$.

Maximization (M-) step of Iteration i

- In the M-step, the estimate $\widehat{\boldsymbol{\theta}}_d^{[i]}$ is calculated as

$$\widehat{\boldsymbol{\theta}}_d^{[i]} = \arg\max_{\boldsymbol{\theta}_d} Q(\boldsymbol{\theta}_d|\widehat{\boldsymbol{\theta}}^{[i-1]}).$$

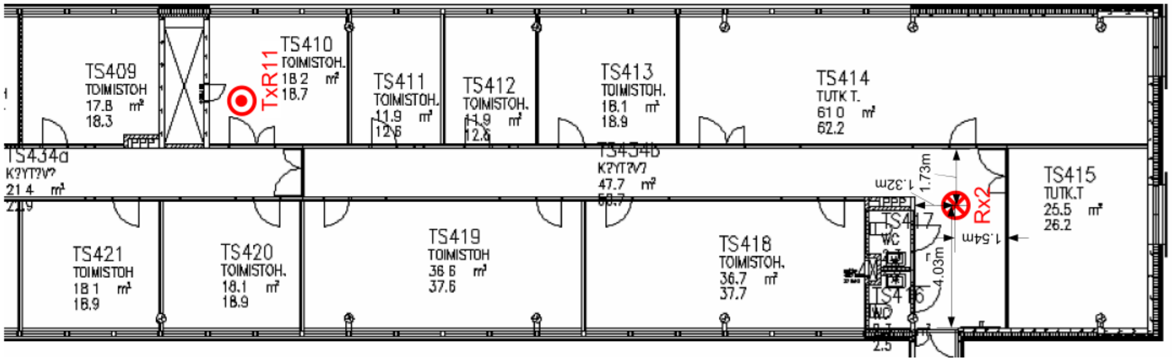
- By applying a coordinate-wise updating procedure, the required multiple-dimensional maximization can be reduced to multiple one-dimensional maximization problems.
- Notice that this coordinate-wise updating still remains within the SAGE framework with the admissible data given in (16).

Experimental Investigations

(a) Surroundings of the Tx.

(b) Surroundings of the Rx.

(c) Map of the premises.



Estimate of Azimuth-Elevation Power Spectrum

- Bartlett spectrum of the signal received at delay 160 ns
- Power spectrum estimate using the proposed characterization method
- Bartlett spectrum of the reconstructed signal