Chapter 6
Deterministic channel parameter estimation

Spectral-based method

- Periodogram and correlogram
- Bartlett beamformer [Bartlett1948]
- Capon beamformer [Capon1969]
- MUSIC [Schmidt-86],
- ESPIRT
- SAGE, RiMAX

Signal Model

Received signal vector:

$$\boldsymbol{Y}(t) = \sum_{\ell=1}^{L} \boldsymbol{s}(t;\boldsymbol{\theta}_\ell) + \sqrt{\frac{N_0}{2}}\boldsymbol{W}(t),$$

where

- $\boldsymbol{Y}(t) \in \mathbb{C}^{M_2}$: output of the Rx array.
- $\boldsymbol{W}(t) \in \mathbb{C}^{M_2}$: circularly symmetric spatially and temporally white Gaussian noise with spectral height N_0.
- $\boldsymbol{s}(t;\boldsymbol{\theta}_\ell) \in \mathbb{C}^{M_2}$: signal contributed by the ℓth path at the output of the Rx array.

Signal Model

The signal contribution of individual specular path:

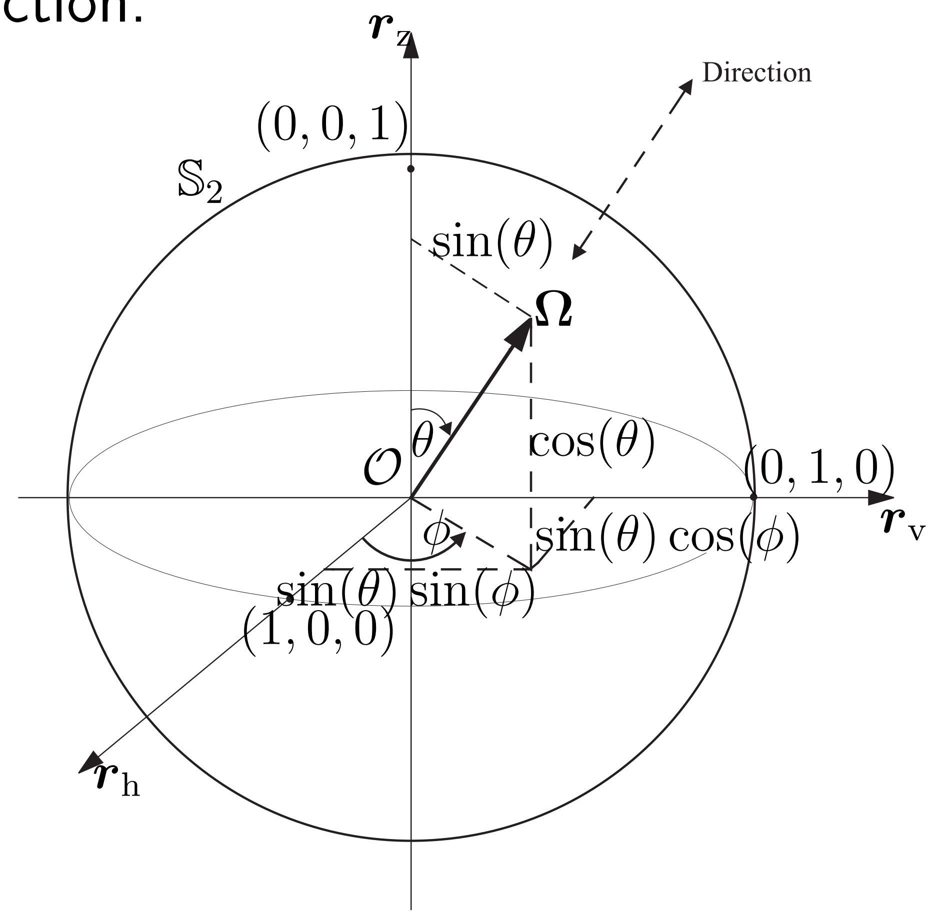
$$\begin{aligned}\boldsymbol{s}(t;\boldsymbol{\theta}_\ell) \doteq & [s_1(t;\boldsymbol{\theta}_\ell),\ldots,s_{M_2}(t;\boldsymbol{\theta}_\ell)]^{\mathrm{T}} \\ = & \exp\{j2\pi\nu_\ell t\}\boldsymbol{C}_2(\boldsymbol{\Omega}_{2,\ell})\boldsymbol{A}_\ell\boldsymbol{C}_1(\boldsymbol{\Omega}_{1,\ell})^T\boldsymbol{u}(t-\tau_\ell).\end{aligned}$$

with

- $\boldsymbol{\theta}_\ell \doteq [\boldsymbol{\Omega}_{1,\ell},\boldsymbol{\Omega}_{2,\ell},\tau_\ell,\nu_\ell,\boldsymbol{A}_\ell]$: parameter vector of the ℓth path;
- $\boldsymbol{C}_k(\boldsymbol{\Omega})\doteq[\boldsymbol{c}_{k,1}(\boldsymbol{\Omega}),\boldsymbol{c}_{k,2}(\boldsymbol{\Omega})] \in \mathbb{C}^{M_k\times 2},\ k{=}1,2$: response of Array k in direction $\boldsymbol{\Omega}$;
- $\boldsymbol{A}_\ell \doteq \begin{bmatrix}\alpha_{\ell,1,1} & \alpha_{\ell,1,2}\\ \alpha_{\ell,2,1} & \alpha_{\ell,2,2}\end{bmatrix} \in \mathbb{C}^{2\times 2}$: polarization matrix ;
- $\boldsymbol{u}(t) \doteq [u_1(t),...,u_{M_1}(t)]^{\mathrm{T}} \in \mathbb{C}^{M_1}$: input signal vector.

Signal Model

Definition of a direction:



$$\boldsymbol{\Omega} = \boldsymbol{e}(\phi,\theta) \doteq [\cos(\phi)\sin(\theta), \sin(\phi)\sin(\theta), \cos(\theta)]^\mathrm{T} \in \mathbb{S}_2$$

The SAGE Algorithm

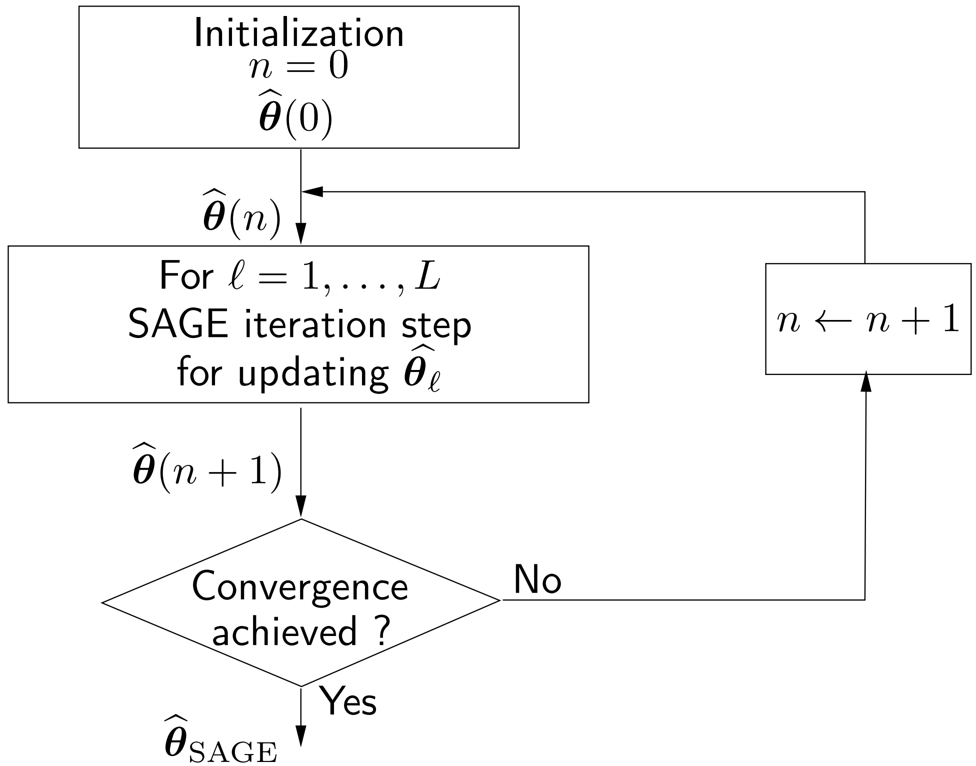
- Parameter vector:

$$\boldsymbol{\theta} \doteq [\boldsymbol{\theta}_1, ..., \boldsymbol{\theta}_L].$$

- Incomplete data: $\boldsymbol{Y}(t)$
- Hidden data: $\boldsymbol{x}_\ell(t)$

$$\boldsymbol{x}_\ell(t) \doteq \boldsymbol{s}(t; \boldsymbol{\theta}_\ell) + \sqrt{\frac{N_0}{2}}\boldsymbol{W}(t), \quad \ell = 1, \ldots, L$$

(not the only choice)



The SAGE Algorithm

- Expectation (E-) step:

$$\begin{aligned}\hat{\boldsymbol{x}}_\ell(t) &= \mathrm{E}[\boldsymbol{x}_\ell(t)|\boldsymbol{y}(t), \hat{\boldsymbol{\theta}}(n)] \\ &= \boldsymbol{y}(t) - \sum_{\ell'=1,\ell'\neq\ell}^{L} \boldsymbol{s}(t; \hat{\boldsymbol{\theta}}_{\ell'}(n))\end{aligned}$$

where $\hat{\boldsymbol{\theta}}(n)$ is the current estimate of $\boldsymbol{\theta}$.

The SAGE Algorithm

- Objective function maximized in the M-step:

$$z(\bar{\boldsymbol{\theta}}_\ell; x_\ell) \doteq \boldsymbol{f}(\bar{\boldsymbol{\theta}}_\ell)^{\mathrm{H}} \boldsymbol{D}(\boldsymbol{\Omega}_{2,\ell}, \boldsymbol{\Omega}_{1,\ell})^{-1} \boldsymbol{f}(\bar{\boldsymbol{\theta}}_\ell)$$

where

- $\bar{\boldsymbol{\theta}}_\ell \doteq [\boldsymbol{\Omega}_{1,\ell}, \boldsymbol{\Omega}_{2,\ell}, \tau_\ell, \nu_\ell]$;
- $\boldsymbol{D}(\boldsymbol{\Omega}_{2,\ell}, \boldsymbol{\Omega}_{1,\ell}) \doteq \left[\boldsymbol{C}_2(\boldsymbol{\Omega}_{2,\ell})^{\mathrm{H}} \ \boldsymbol{C}_2(\boldsymbol{\Omega}_{2,\ell})\right] \otimes \left[\boldsymbol{C}_1(\boldsymbol{\Omega}_{1,\ell})^{\mathrm{H}} \ \boldsymbol{C}_1(\boldsymbol{\Omega}_{1,\ell})\right]$;

The SAGE Algorithm

- Objective function maximized in the M-step:

$$z(\bar{\boldsymbol{\theta}}_\ell; x_\ell) \doteq \boldsymbol{f}(\bar{\boldsymbol{\theta}}_\ell)^{\mathrm{H}} \boldsymbol{D}(\boldsymbol{\Omega}_{2,\ell}, \boldsymbol{\Omega}_{1,\ell})^{-1} \boldsymbol{f}(\bar{\boldsymbol{\theta}}_\ell)$$

where

- $\boldsymbol{f}(\bar{\boldsymbol{\theta}}_\ell) \doteq \begin{bmatrix} \boldsymbol{c}_{2,1}^{\mathrm{H}}(\boldsymbol{\Omega}_{2,\ell})\boldsymbol{X}_\ell(\tau_\ell, \nu_\ell)\boldsymbol{c}_{1,1}(\boldsymbol{\Omega}_{1,\ell})^* \\ \boldsymbol{c}_{2,1}^{\mathrm{H}}(\boldsymbol{\Omega}_{2,\ell})\boldsymbol{X}_\ell(\tau_\ell, \nu_\ell)\boldsymbol{c}_{1,2}(\boldsymbol{\Omega}_{1,\ell})^* \\ \boldsymbol{c}_{2,2}^{\mathrm{H}}(\boldsymbol{\Omega}_{2,\ell})\boldsymbol{X}_\ell(\tau_\ell, \nu_\ell)\boldsymbol{c}_{1,1}(\boldsymbol{\Omega}_{1,\ell})^* \\ \boldsymbol{c}_{2,2}^{\mathrm{H}}(\boldsymbol{\Omega}_{2,\ell})\boldsymbol{X}_\ell(\tau_\ell, \nu_\ell)\boldsymbol{c}_{1,2}(\boldsymbol{\Omega}_{1,\ell})^* \end{bmatrix}.$
- $\boldsymbol{X}_\ell(\tau_\ell, \nu_\ell)$ is a $M_2 \times M_1$ dim. matrix with entries

$$X_{\ell,m_2,m_1}(\tau_\ell, \nu_\ell) = \sum_{i=1}^{I} \exp(-j2\pi\nu_\ell t_{i,m_2,m_1}) \cdot \int_0^{T_{\mathrm{sc}}} u^*(t-\tau_\ell) x_\ell(t + t_{i,m_2,m_1}) \mathrm{d}t,$$

The SAGE Algorithm

- Conditions for $\boldsymbol{D}(\boldsymbol{\Omega}_2, \boldsymbol{\Omega}_1)$ to be non-singular:

$$\det\big(\boldsymbol{D}(\boldsymbol{\Omega}_2, \boldsymbol{\Omega}_1)\big) \neq 0,$$

  which holds, if and only if,

$$\boldsymbol{c}_{k,1}(\boldsymbol{\Omega}_k) \neq \gamma_k \cdot \boldsymbol{c}_{k,2}(\boldsymbol{\Omega}_k)$$

  for some complex number $\gamma_k, k = 1, 2$.

  - A necessary and sufficient condition for $\boldsymbol{D}(\boldsymbol{\Omega}_2, \boldsymbol{\Omega}_1)$ to be always invertible is that the vectors $\boldsymbol{c}_{k,1}(\boldsymbol{\Omega}_k)$ and $\boldsymbol{c}_{k,2}(\boldsymbol{\Omega}_k), k = 1, 2$ are linearly independent for any $\boldsymbol{\Omega}_2$ and $\boldsymbol{\Omega}_1$.
  - When $\boldsymbol{D}(\boldsymbol{\Omega}_2, \boldsymbol{\Omega}_1)$ is non-invertible, the four coefficients in the polarization matrix $\boldsymbol{A}_\ell$ cannot be estimated separately.

The SAGE Algorithm

- Maximization (M-) step:

$$\begin{aligned}
\hat{\tau}_\ell'' &= \arg\max_{\tau_\ell} z(\hat{\phi}_{1,\ell}', \hat{\theta}_{1,\ell}', \hat{\phi}_{2,\ell}', \hat{\theta}_{2,\ell}', \tau_\ell, \hat{\nu}_\ell'; \hat{x}_\ell) \\
\hat{\nu}_\ell'' &= \arg\max_{\nu_\ell} z(\hat{\phi}_{1,\ell}', \hat{\theta}_{1,\ell}', \hat{\phi}_{2,\ell}', \hat{\theta}_{2,\ell}', \hat{\tau}_\ell'', \nu_\ell; \hat{x}_\ell) \\
\hat{\theta}_{2,\ell}'' &= \arg\max_{\theta_{2,\ell}} z(\hat{\phi}_{1,\ell}', \hat{\theta}_{1,\ell}', \hat{\phi}_{2,\ell}', \theta_{2,\ell}, \hat{\tau}_\ell'', \hat{\nu}_\ell''; \hat{x}_\ell) \\
\hat{\phi}_{2,\ell}'' &= \arg\max_{\phi_{2,\ell}} z(\hat{\phi}_{1,\ell}', \hat{\theta}_{1,\ell}', \phi_{2,\ell}, \hat{\theta}_{2,\ell}'', \hat{\tau}_\ell'', \hat{\nu}_\ell''; \hat{x}_\ell) \\
\hat{\theta}_{1,\ell}'' &= \arg\max_{\theta_{1,\ell}} z(\hat{\phi}_{1,\ell}', \theta_{1,\ell}, \hat{\phi}_{2,\ell}'', \hat{\theta}_{2,\ell}'', \hat{\tau}_\ell'', \hat{\nu}_\ell''; \hat{x}_\ell) \\
\hat{\phi}_{1,\ell}'' &= \arg\max_{\phi_{1,\ell}} z(\phi_{1,\ell}, \hat{\theta}_{1,\ell}'', \hat{\phi}_{2,\ell}'', \hat{\theta}_{2,\ell}'', \hat{\tau}_\ell'', \hat{\nu}_\ell''; \hat{x}_\ell). \\
\boldsymbol{\alpha}_\ell'' &= (IPT_{\mathrm{sc}})^{-1} \boldsymbol{D}(\boldsymbol{\Omega}_{2,\ell}'', \boldsymbol{\Omega}_{1,\ell}'')^{-1} \boldsymbol{f}(\bar{\boldsymbol{\theta}}_\ell'')
\end{aligned}$$

$$\boldsymbol{\alpha}_\ell \doteq \mathrm{vec}(\boldsymbol{A}_\ell)$$

Initialization

- Successive Interference Cancellation:

$$y^{(\ell)}(t) = y(t) - \sum_{\ell'=1}^{\ell-1} s(t; \hat{\boldsymbol{\theta}}_{\ell'}(0))$$

- Non-Coherent Maximum Likelihood (NC-ML) estimator for initializing $\hat{\tau}_\ell$, $\hat{\nu}_\ell$, and $\hat{\boldsymbol{\Omega}}_{2,\ell}$.

- Coherent Maximum Likelihood (C-ML) estimator for initializing $\hat{\boldsymbol{\Omega}}_{1,\ell}$ and $\hat{\boldsymbol{\alpha}}_\ell$.

Initialization

- NC-ML estimate of delay τ_ℓ

$$\hat{\tau}_\ell(0) = \arg\max_{\tau_\ell}\left\{\sum_{i=1}^{I}\sum_{m_2=1}^{M_2}\sum_{m_1=1}^{M_1}\left|\int_0^{T_{\rm sc}} y^{(\ell)}(t+t_{i,m_2,m_1})u^*(t-\tau_\ell)\mathrm{d}t\right|^2\right\}.$$

- NC-ML estimate of Doppler frequency ν_ℓ:

$$\hat{\nu}_\ell(0) = \arg\max_{\nu_\ell}\left\{\sum_{m_2=1}^{M_2}\sum_{m_1=1}^{M_1}\left|\sum_{i=1}^{I}\exp(-j2\pi\nu_\ell t_{i,m_2,m_1})\right.\right.$$
$$\left.\left.\cdot\int_0^{T_{\rm sc}} y^{(\ell)}(t+t_{i,m_2,m_1})u^*\big(t-\hat{\tau}_\ell(0)\big)\mathrm{d}t\right|^2\right\}.$$

Initialization

- NC-ML estimate of direction of arrival $\mathbf{\Omega}_{2,\ell}$:

$$\hat{\mathbf{\Omega}}_{2,\ell}(0) = \arg\max_{\mathbf{\Omega}_{2,\ell}} \Bigg\{ \sum_{m_1=1}^{M_1} \Bigg[\big|\tilde{\boldsymbol{c}}_{2,1}^{\mathrm{H}}(\mathbf{\Omega}_{2,\ell})\boldsymbol{y}_{m_1}^{(\ell)}\big|^2 + \big|\tilde{\boldsymbol{c}}_{2,2}^{\mathrm{H}}(\mathbf{\Omega}_{2,\ell})\boldsymbol{y}_{m_1}^{(\ell)}\big|^2$$

$$-2\mathcal{R}\{\boldsymbol{y}_{m_1}^{(\ell)H}\tilde{\boldsymbol{c}}_{2,2}(\mathbf{\Omega}_{2,\ell}\tilde{\boldsymbol{c}}_{2,1}^{\mathrm{H}}(\mathbf{\Omega}_{2,\ell})\boldsymbol{y}_{m_1}^{(\ell)H}\tilde{\boldsymbol{c}}_{2,2}^{\mathrm{H}}(\mathbf{\Omega}_{2,\ell})\tilde{\boldsymbol{c}}_{2,1}(\mathbf{\Omega}_{2,\ell})\} \Bigg] \Bigg\}.$$

- C-ML Estimate of direction of departure $\mathbf{\Omega}_{1,\ell}$:

$$\hat{\mathbf{\Omega}}_{1,\ell}(0) = \arg\max_{\mathbf{\Omega}_{1,\ell}} \Big\{ z\Big(\hat{\mathbf{\Omega}}_{2,\ell}(0), \mathbf{\Omega}_{1,\ell}, \hat{\tau}_\ell(0), \hat{\nu}_\ell(0); \hat{y}_\ell\Big) \Big\}$$

- C-ML Estimate of the complex polarization vector $\boldsymbol{\alpha}_\ell$

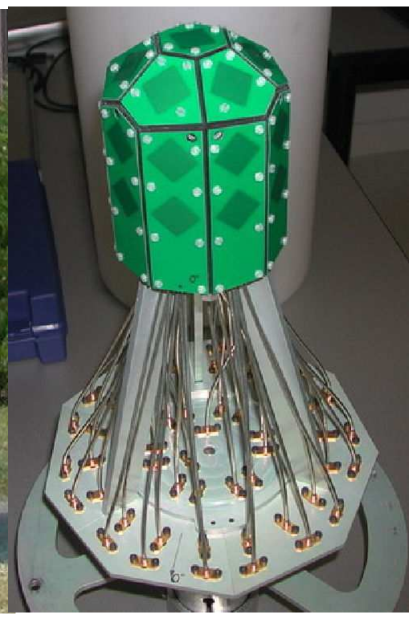
$$\hat{\boldsymbol{\alpha}}_\ell(0) = (IPT_{\mathrm{sc}})^{-1}\boldsymbol{D}\big(\hat{\mathbf{\Omega}}_{2,\ell}(0), \hat{\mathbf{\Omega}}_{1,\ell}(0)\big)^{-1}\boldsymbol{f}\big(\hat{\bar{\boldsymbol{\theta}}}_\ell(0)\big)$$

Experimental Investigations

Characteristics of the measurement setup:

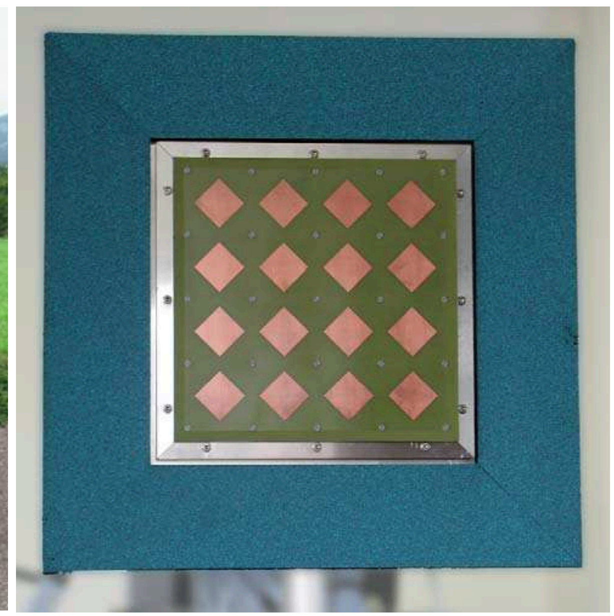
- MIMO channel sounder: Propsound
- Tx Array: 3x8 omni-directional dual-polarized array (M_1=54),
- Rx Array: 4x4 planar dual-polarized array (M_2=32),
- 2.45 GHz Carrier frequency and 100 MHz bandwidth

Tx



Tx Array

Rx

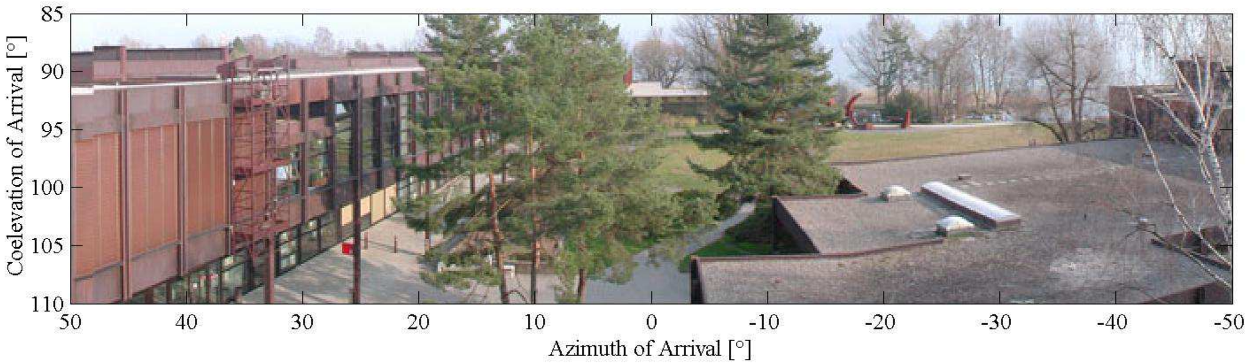


Rx Array

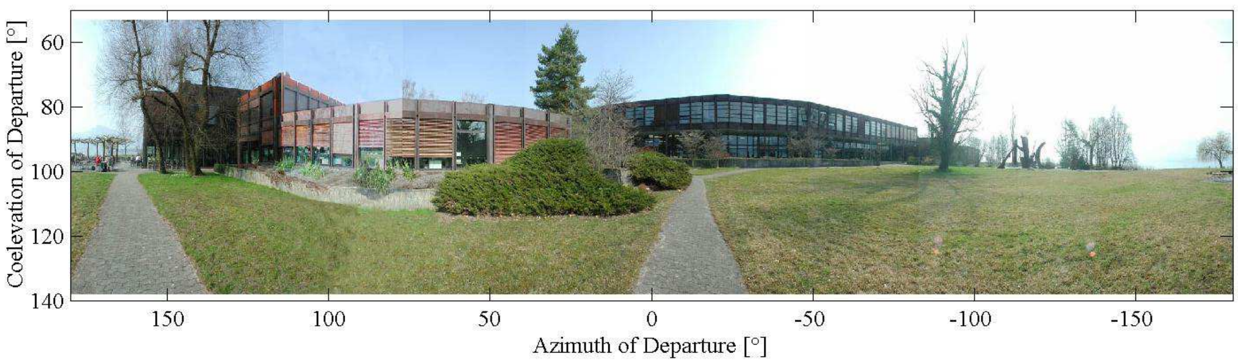
Experimental Investigations

Investigated propagation environment:

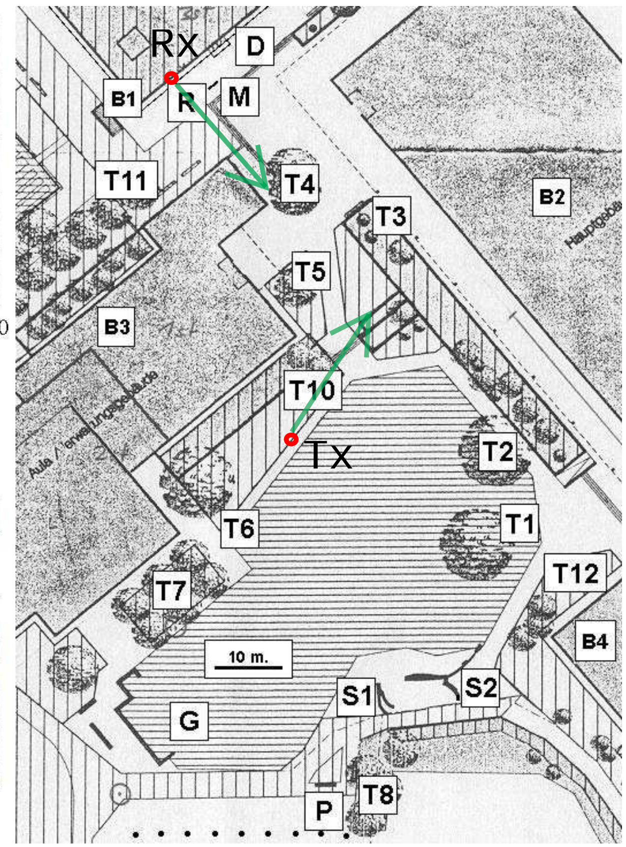
Surrounding of the Rx



Surrounding of the Tx

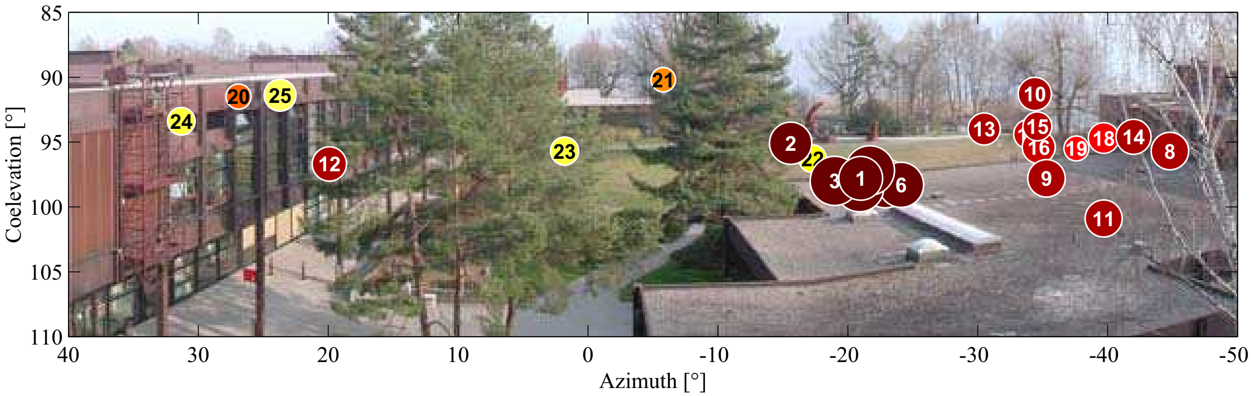


Map of the environment

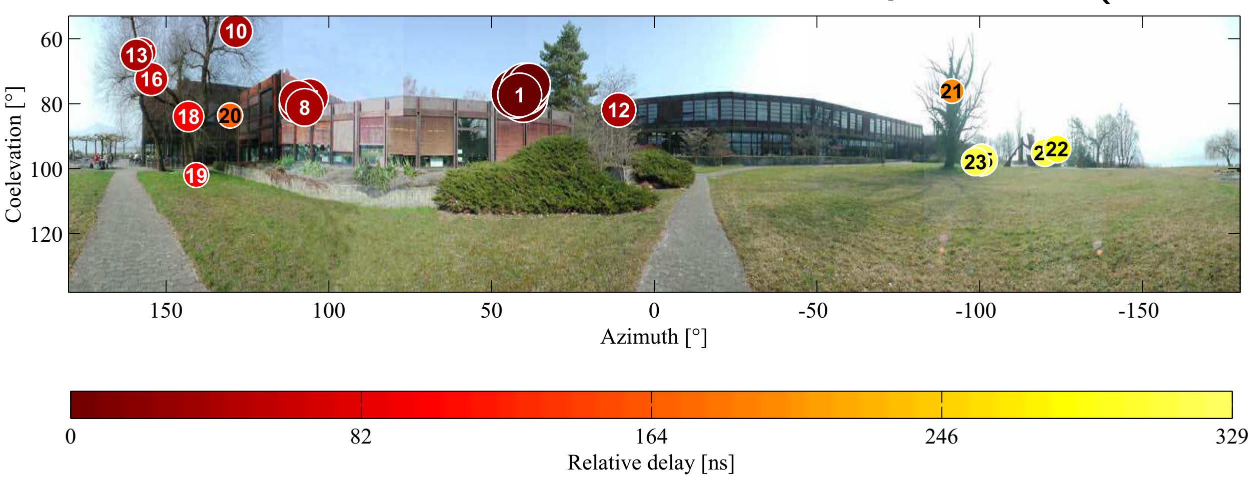


Experimental Investigations (NLOS)

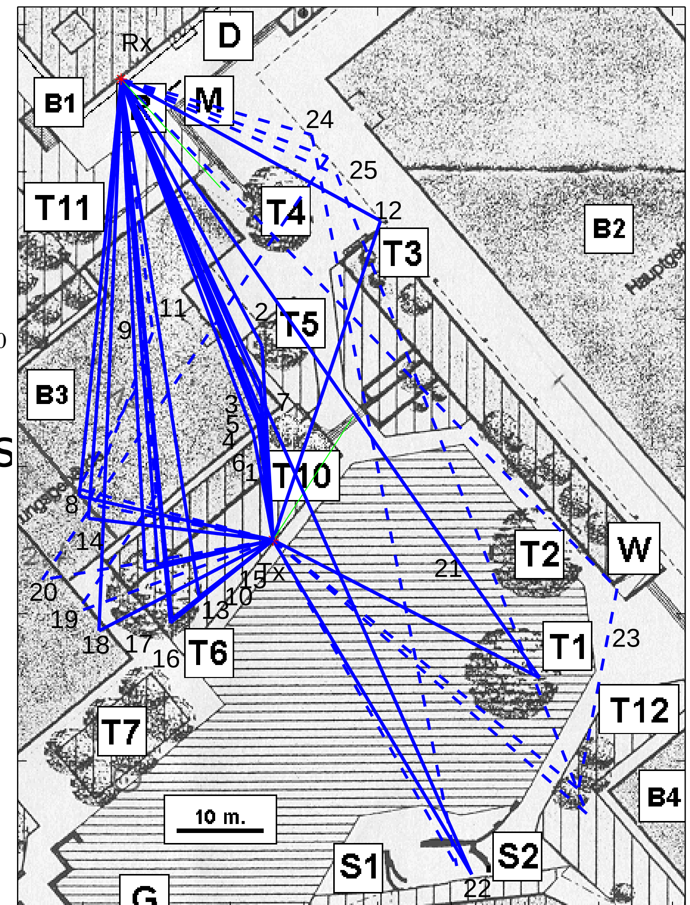
Estimated Directions of Arrival (DoAs)



Estimated Directions of Departure (DoDs

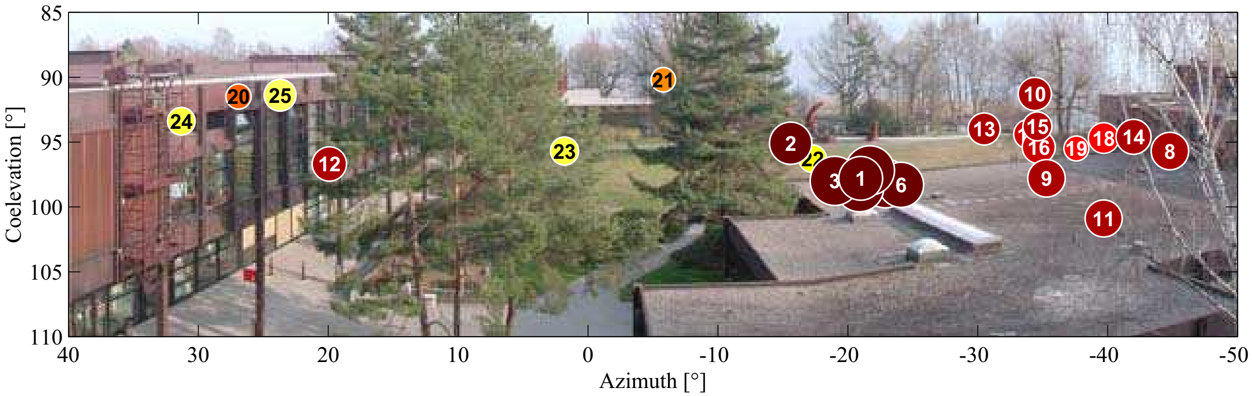


Reconstructed Paths

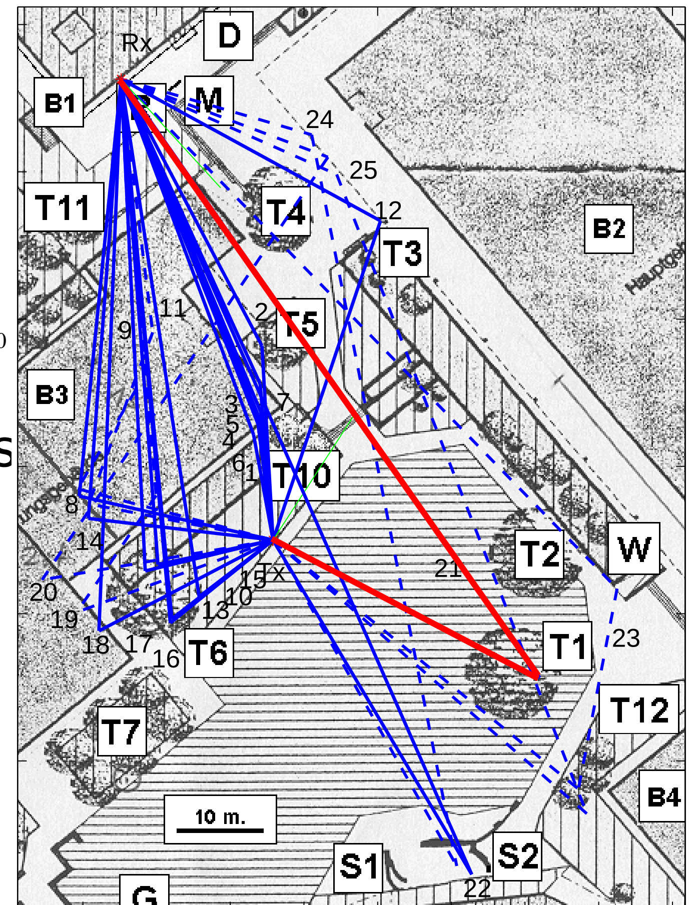


Experimental Investigations (NLOS)

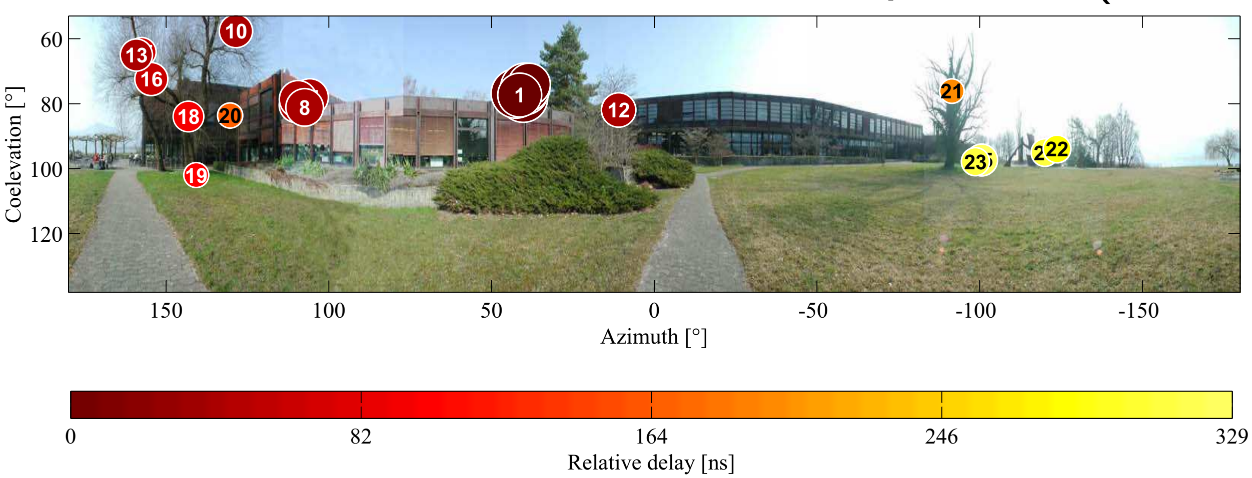
Estimated Directions of Arrival (DoAs)



Reconstructed Path 21

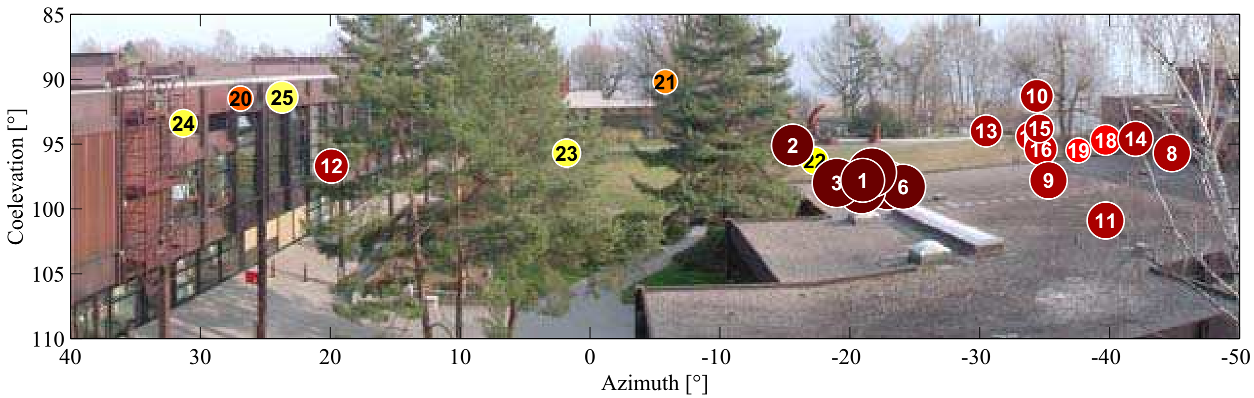


Estimated Directions of Departure (DoDs)

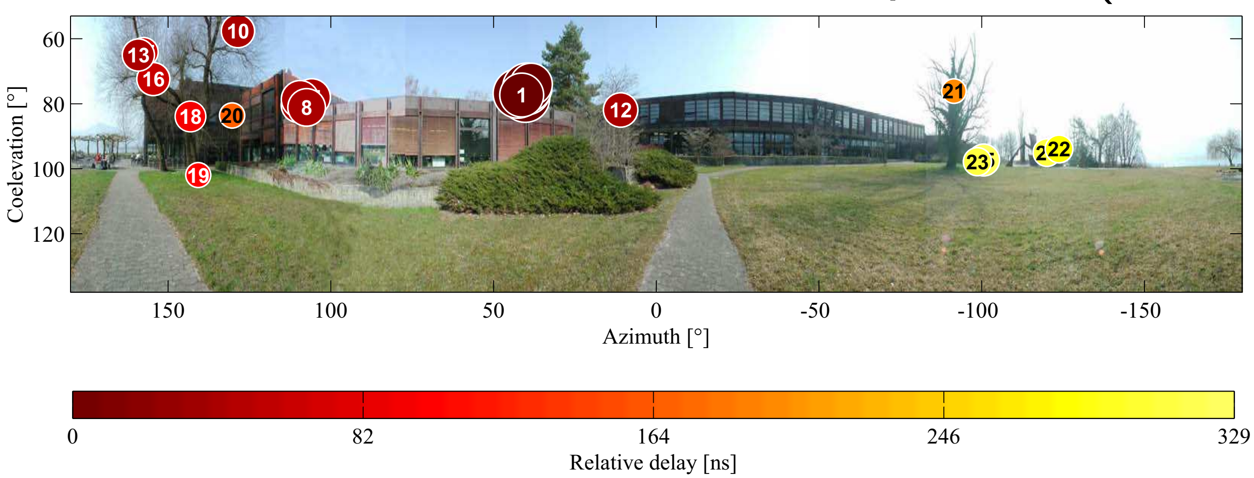


Experimental Investigations (NLOS)

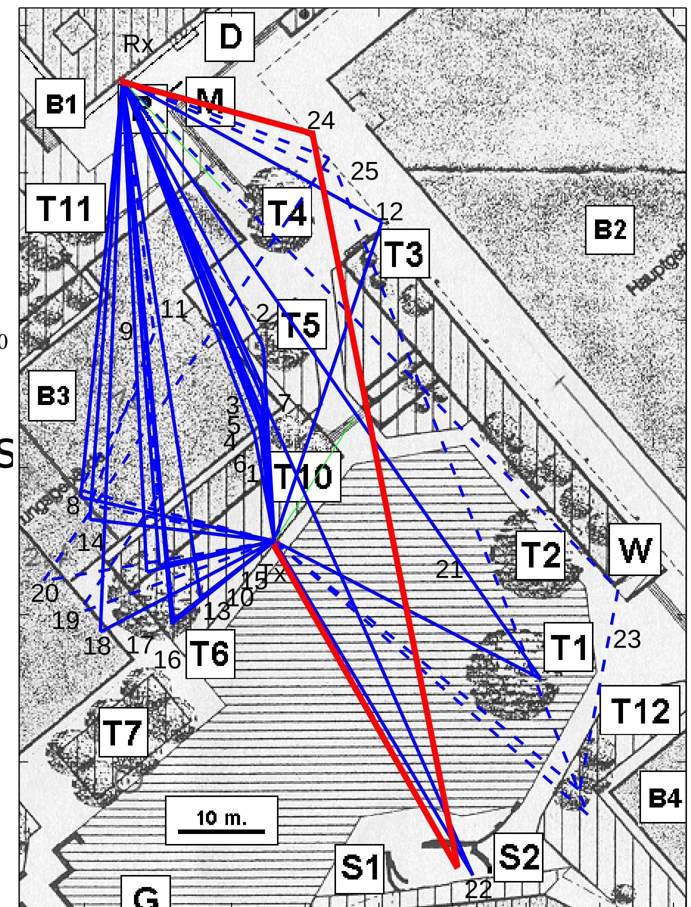
Estimated Directions of Arrival (DoAs)



Estimated Directions of Departure (DoDs

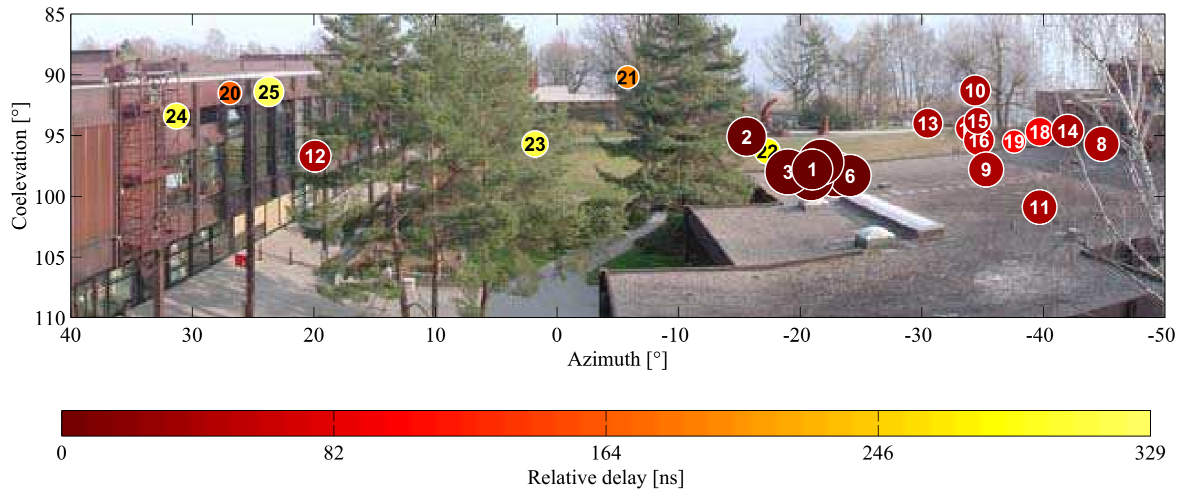


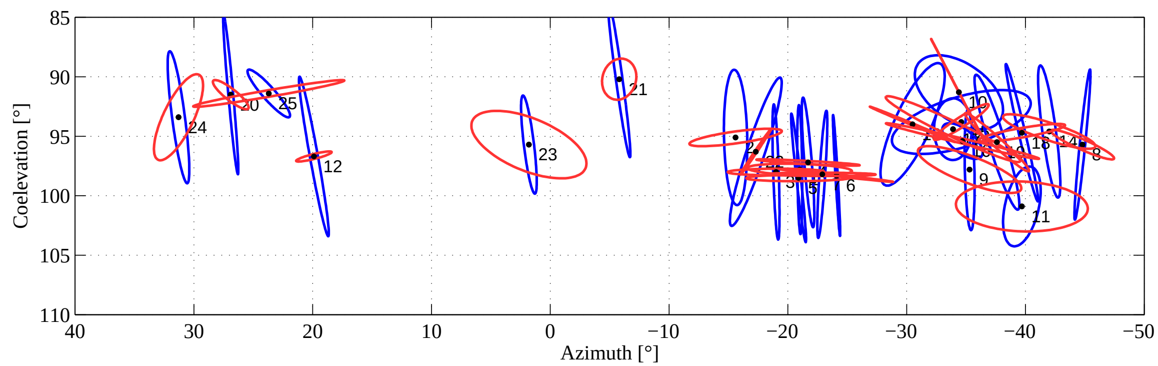
Reconstructed Path 24



Experimental Investigations (NLOS)

Estimated polarization:





- Blue ellipses : polarization ellipses calculated using $\left[\hat{a}_{d,1,1} \quad \hat{a}_{d,2,1}\right]^{\mathrm{T}}$,
- Red ellipses : polarization ellipses calculated using $\left[\hat{a}_{d,1,2} \quad \hat{a}_{d,2,2}\right]^{\mathrm{T}}$.