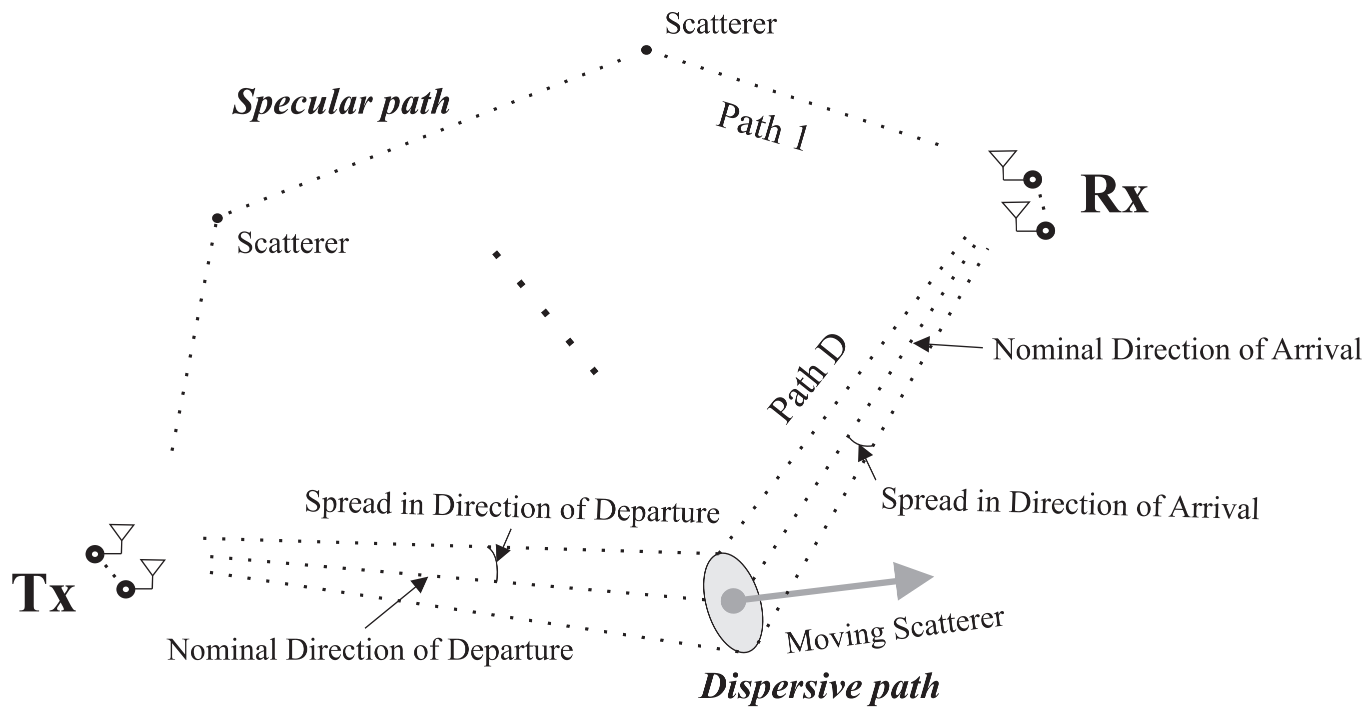
Lecture 5
Parametric estimation for the signal contributions of slightly distributed scatterers

Xuefeng Yin, Jukka-Pekka Nuutinen and Bernard H. Fleury

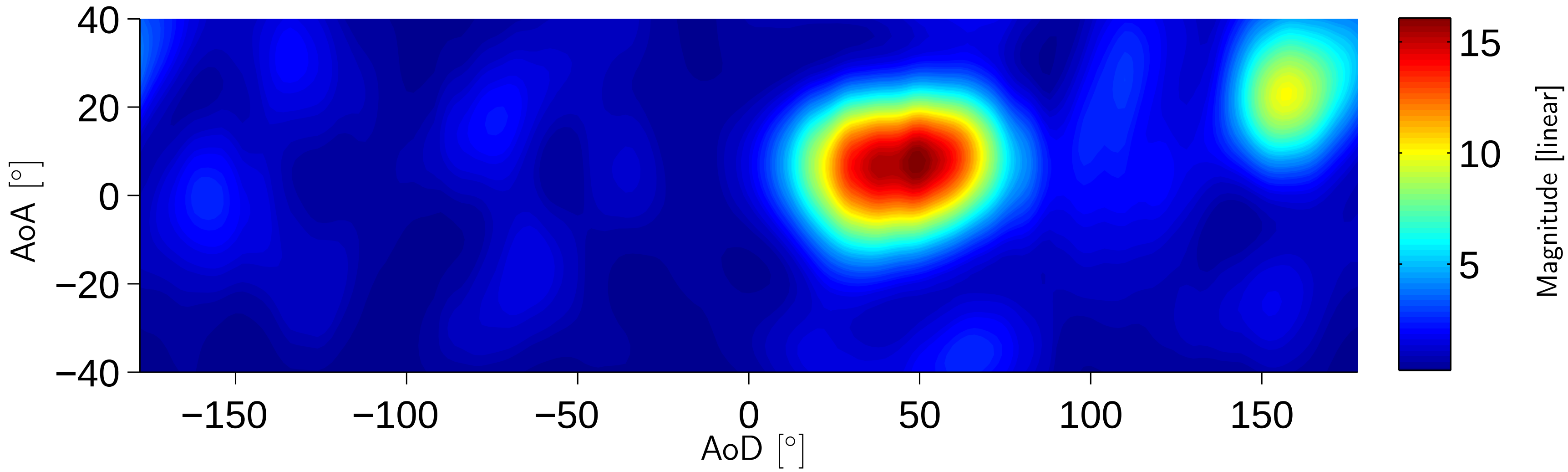
Multipath Propagation Environment



- Dispersion dimensions : delay, direction of departure (DoD), direction of arrival (DoA), polarizations and Doppler frequency
- Dispersion parameters of a propagation path:
  - center of gravity and spread, one pair per dimension
  - parameter characterizing dependence between the dimensions

An Example of Dispersion of Individual Path Components

An estimated power spectrum with respect to azimuth of departure (AoD) and azimuth of arrival (AoA) using measurement data



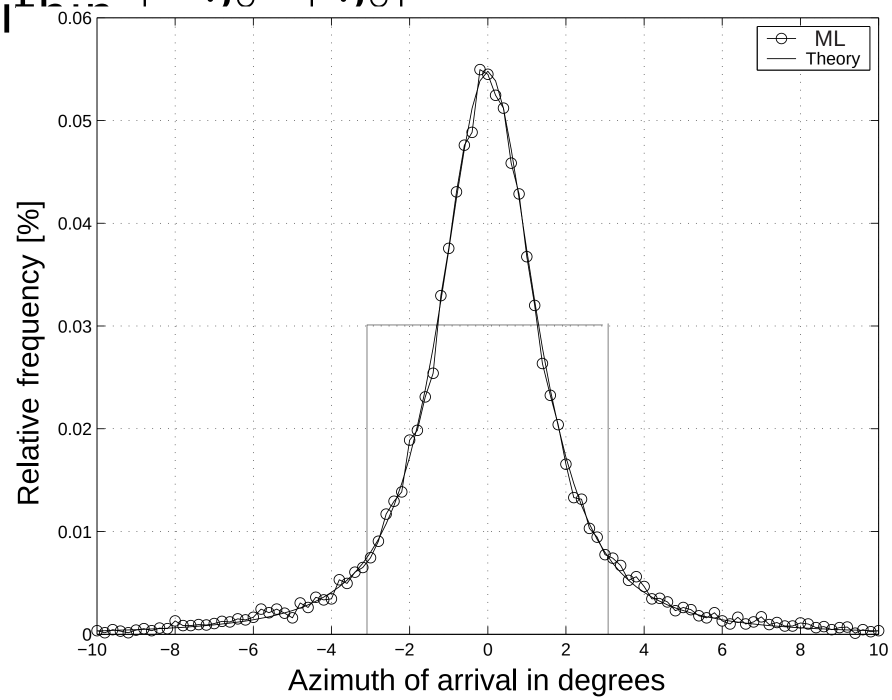
- Correlator outputs of the sounder at a specific delay are considered.
- Bartlett beamformer is used to estimate the power spectrum.

Motivation

In the scenario where individual path components are dispersive, Specular-Scatterer (SS) model-based parameter estimators return estimates with heavy-tail pdfs [Bengtsson & Völcker 2001].

Simulated pdf of the azimuth of arrival estimates using SS-ML estimator:
Simulation setting:

- A single dispersed-path-component scenario
- ϕ : uniform distributed within $[-3^\circ, +3^\circ]$
- $\bar{\phi} = 0^\circ$
- SNR $= 40$ dB



Effective Model for Individual Path Components

The received signal in a scenario with one dispersed path component can be modeled as

$$\boldsymbol{y}(t) = \boldsymbol{h}(t)s(t) + \boldsymbol{w}(t),$$

In the case where azimuth of arrival is considered,

- $\boldsymbol{h}(t)$: time-varying complex weight described by

$$\boldsymbol{h}(t) = \sum_{\ell=1}^{L} \alpha_\ell(t)\boldsymbol{c}(\bar{\phi} + \tilde{\phi}_\ell),$$

  with
  - $\alpha_\ell(t)$: complex weight of the ℓth individual sub-path;
  - $\bar{\phi}$: nominal azimuth of arrival of the path component;
  - $\tilde{\phi}_\ell$: azimuth deviation from $\bar{\phi}$ of the ℓth individual sub-path.
- $s(t)$: complex envelope of the transmitted signal.

Generalized Array Manifold (GAM) Model

First-order Taylor series approximation of $\boldsymbol{c}(\bar{\phi}+\tilde{\phi}_\ell)$ around $\bar{\phi}$:

$$\boldsymbol{c}(\bar{\phi}+\tilde{\phi}_\ell) \approx \boldsymbol{c}(\bar{\phi}) + \tilde{\phi}_\ell \boldsymbol{c}'(\bar{\phi}), \text{ when } \tilde{\phi}_\ell \text{ is small,}$$

Approximation of the received signal:

$$\begin{aligned}\boldsymbol{y}(t) &\approx \sum_{\ell=1}^{L} \alpha_\ell(t)[\boldsymbol{c}(\bar{\phi}) + \tilde{\phi}_\ell \boldsymbol{c}'(\bar{\phi})] + \boldsymbol{w}(t), \\ &= \alpha(t)\boldsymbol{c}(\bar{\phi}) + \beta(t)\boldsymbol{c}'(\bar{\phi}) + \boldsymbol{w}(t), \quad t = t_1, \ldots, t_N,\end{aligned}$$

where $\alpha(t) \doteq \sum_{\ell=1}^{L} \alpha_\ell(t)$, $\beta(t) \doteq \sum_{\ell=1}^{L} \alpha_\ell(t)\tilde{\phi}_\ell$.

Generalized Array Manifold (GAM) Model

Further assumption:

$\alpha(t)$ and $\beta(t)$ are zero-mean uncorrelated circularly-symmetric WSS Gaussian processes with autocorrelation functions

$$R_\alpha(\tau) = \sum_{\ell=1}^{L} R_{\alpha_\ell}(\tau) \text{ and } R_\beta(\tau) = \sigma_{\tilde{\phi}}^2 R_\alpha(\tau).$$

In particular,

$$\sigma_{\tilde{\phi}}^2 = \sigma_\beta^2/\sigma_\alpha^2,$$

where $\sigma_{\tilde{\phi}}$ is called azimuth spread, $\sigma_\beta^2 = R_\beta(0)$ and $\sigma_\alpha^2 = R_\alpha(0)$.

Simulation Study

Derived estimators using the GAM model:

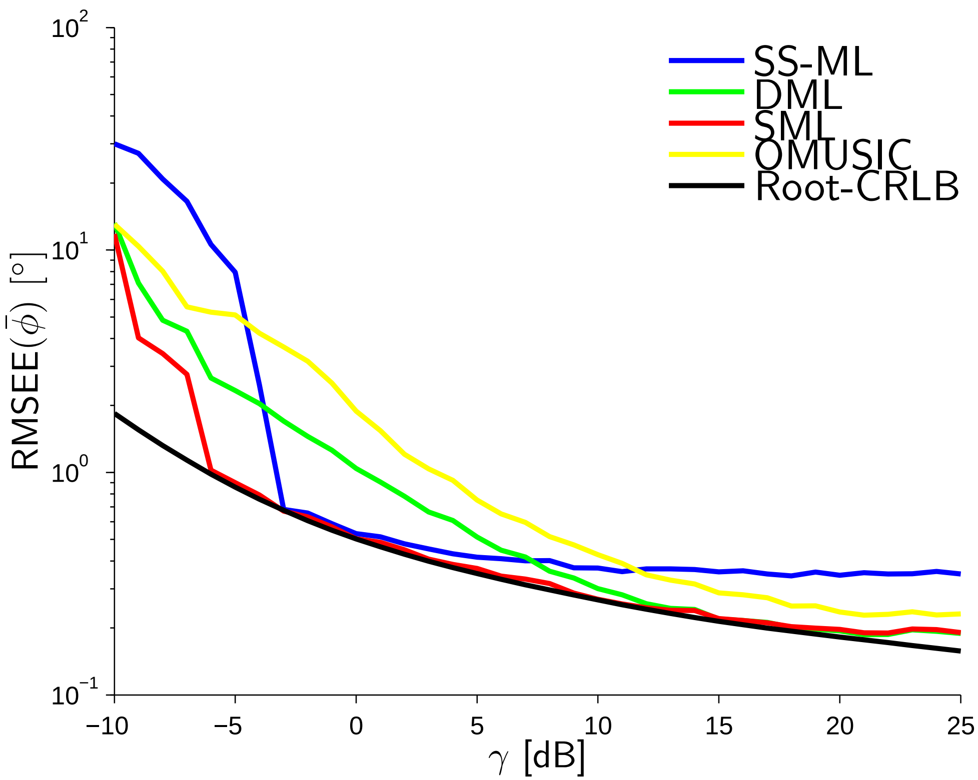
- stochastic and deterministic ML (SML and DML) estimators,
- orthonormal-MUSIC (OMUSIC) estimator: an extension of the standard MUSIC [Schmidt, 1986].

Root mean square estimation error (RMSEE) for the nominal azimuth of arrival:

Simulation settings:

- $D = 1$
- $L = 50$
- $\bar{\phi} = 110^o$
- $\sigma_{\tilde{\phi}} = 3^\circ$
- 50 realizations
- 8-element ULA
- 500 runs

(a)



When Direction of Arrival is Considered

Received signal in a single-SDS scenario:

$$\boldsymbol{y}(t)=\boldsymbol{h}(t)\cdot s(t)+\boldsymbol{w}(t)$$

with

$$\boldsymbol{h}(t)=\sum_{\ell=1}^{L}a_\ell(t)\boldsymbol{c}(\boldsymbol{\Omega}_\ell),$$

where

- $a_\ell(t)$: complex weight for the ℓth individual path;
- $\boldsymbol{\Omega}_\ell=\boldsymbol{e}(\phi_\ell,\theta_\ell)$ is a unit vector characterizing direction with azimuth ϕ_ℓ and elevation θ_ℓ.
- $\phi_\ell=\bar{\phi}+\tilde{\phi}_\ell$, and $\theta_\ell=\bar{\theta}+\tilde{\theta}_\ell$.
- $\bar{(\cdot)}$: nominal value of the argument;
- $\tilde{(\cdot)}$: deviation from the nominal value of the argument.

Generalized Array Manifold (GAM) model

First-order Taylor series expansion of $\boldsymbol{c}(\boldsymbol{\Omega}_\ell)$ around nominal DoA $\bar{\boldsymbol{\Omega}}$:

$$\boldsymbol{c}(\boldsymbol{\Omega}_\ell) \approx \boldsymbol{c}(\bar{\boldsymbol{\Omega}}) + \tilde{\phi}_\ell \boldsymbol{c}_\phi^{'}(\bar{\boldsymbol{\Omega}}) + \tilde{\theta}_\ell \boldsymbol{c}_\theta^{'}(\bar{\boldsymbol{\Omega}}),$$

where

$$\boldsymbol{c}_\phi^{'}(\boldsymbol{\Omega}) = \frac{1}{\sin(\theta)} \cdot \frac{\partial \boldsymbol{c}(\boldsymbol{\Omega})}{\partial \phi}, \quad \text{and} \quad \boldsymbol{c}_\theta^{'}(\boldsymbol{\Omega}) = \frac{\partial \boldsymbol{c}(\boldsymbol{\Omega})}{\partial \theta}.$$

Approximation of the received signal:

$$\begin{aligned} \boldsymbol{y}(t) &\approx \sum_{\ell=1}^{L} a_\ell(t) \left[\boldsymbol{c}(\bar{\boldsymbol{\Omega}}) + \tilde{\phi}_\ell \boldsymbol{c}_\phi^{'}(\bar{\boldsymbol{\Omega}}) + \tilde{\theta}_\ell \boldsymbol{c}_\theta^{'}(\bar{\boldsymbol{\Omega}}) \right] + \boldsymbol{w}(t), \\ &= \alpha(t)\boldsymbol{c}(\bar{\boldsymbol{\Omega}}) + \beta_\phi(t)\boldsymbol{c}_\phi^{'}(\bar{\boldsymbol{\Omega}}) + \beta_\theta(t)\boldsymbol{c}_\theta^{'}(\bar{\boldsymbol{\Omega}}) + \boldsymbol{w}(t), \quad t = t_1, \ldots, t_N, \end{aligned}$$

where

$$\alpha(t) \doteq \sum_{\ell=1}^{L} a_\ell(t), \quad \beta_\phi(t) \doteq \sum_{\ell=1}^{L} a_\ell(t)\tilde{\phi}_\ell, \quad \beta_\theta(t) \doteq \sum_{\ell=1}^{L} a_\ell(t)\tilde{\theta}_\ell.$$

Generalized Array Manifold model (Cont.)

Written in matrix notation:

$$\boldsymbol{y}(t) = \boldsymbol{F}(\bar{\phi})\boldsymbol{\xi}(t) + \boldsymbol{w}(t), \ \ t = t_1, \ldots, t_N,$$

where

$$\boldsymbol{F}(\bar{\boldsymbol{\Omega}}) = [\boldsymbol{c}(\bar{\boldsymbol{\Omega}}), \boldsymbol{c}_{\phi}^{'}(\bar{\boldsymbol{\Omega}}), \boldsymbol{c}_{\theta}^{'}(\bar{\boldsymbol{\Omega}})], \text{ and } \boldsymbol{\xi}(t) = [\alpha(t), \beta_{\phi}(t), \beta_{\theta}(t)]^{\mathrm{T}}.$$

Received signal in a multiple-SDS scenario:

$$\boldsymbol{y}(t) = \sum_{d=1}^{D} \boldsymbol{F}(\bar{\boldsymbol{\Omega}}_d)\boldsymbol{\xi}_d(t) + \boldsymbol{w}(t), \ \ t = t_1, \ldots, t_N,$$

where

$$\boldsymbol{F}(\bar{\boldsymbol{\Omega}}_d) = [\boldsymbol{c}(\bar{\boldsymbol{\Omega}}_d), \boldsymbol{c}_{\phi}^{'}(\bar{\boldsymbol{\Omega}}_d), \boldsymbol{c}_{\theta}^{'}(\bar{\boldsymbol{\Omega}}_d)], \ \boldsymbol{\xi}_d(t) = [\alpha_d(t), \beta_{\phi,d}(t), \beta_{\theta,d}(t)]^{\mathrm{T}}.$$

Generalized Array Manifold model (Cont.)

Realistic assumption:

$\alpha(t)$, $\beta_\phi(t)$ and $\beta_\theta(t)$ are complex circularly-symmetric zero-mean WSS processes with autocorrelation functions

$$R_\alpha(\tau)=\sum_{\ell=1}^{L} R_{a_\ell}(\tau),\; R_{\beta_\phi}(\tau)=\sigma^2_{\tilde{\phi}} R_\alpha(\tau), \;\; \text{and} \;\; R_{\beta_\theta}(\tau)=\sigma^2_{\tilde{\theta}} R_\alpha(\tau)$$

and cross-correlation functions

$$R_{\alpha\beta_\theta}(\tau)=R_{\alpha\beta_\phi}(\tau)=0,\; R_{\beta_\phi\beta_\theta}(\tau)=\sigma_{\tilde{\phi}}\sigma_{\tilde{\theta}}\rho_{\tilde{\phi}\tilde{\theta}} R_\alpha(\tau).$$

In particular,

$$\sigma^2_{\tilde{\phi}} = \frac{\sigma^2_{\beta_\phi}}{\sigma^2_\alpha}, \;\; \sigma^2_{\tilde{\theta}} = \frac{\sigma^2_{\beta_\theta}}{\sigma^2_\alpha}, \; \text{and} \;\; \rho_{\tilde{\phi}\tilde{\theta}} = \frac{R_{\beta_\phi\beta_\theta}(0)}{\sigma_{\tilde{\phi}}\sigma_{\tilde{\theta}}\sigma^2_\alpha},$$

where $\sigma^2_{(\cdot)} = R_{(\cdot)}(0)$.

The SAGE algorithm

Unknown parameter vector in a multiple-SDS scenario:

$$\boldsymbol{\theta} \doteq [\sigma_w^2, \bar{\phi}_d, \bar{\theta}_d, \alpha_d(t), \beta_{\phi,d}(t), \beta_{\theta,d}(t); \\ d = 1, \ldots, D, \quad t = t_1, \ldots, t_N].$$

A natural subset of unknown parameters:

$$\boldsymbol{\theta}_d \doteq [\sigma_w^2, \bar{\phi}_d, \bar{\theta}_d, \alpha_d(t), \beta_{\phi,d}(t), \beta_{\theta,d}(t), \quad t = t_1, \ldots, t_N].$$

Incomplete data (received signal):

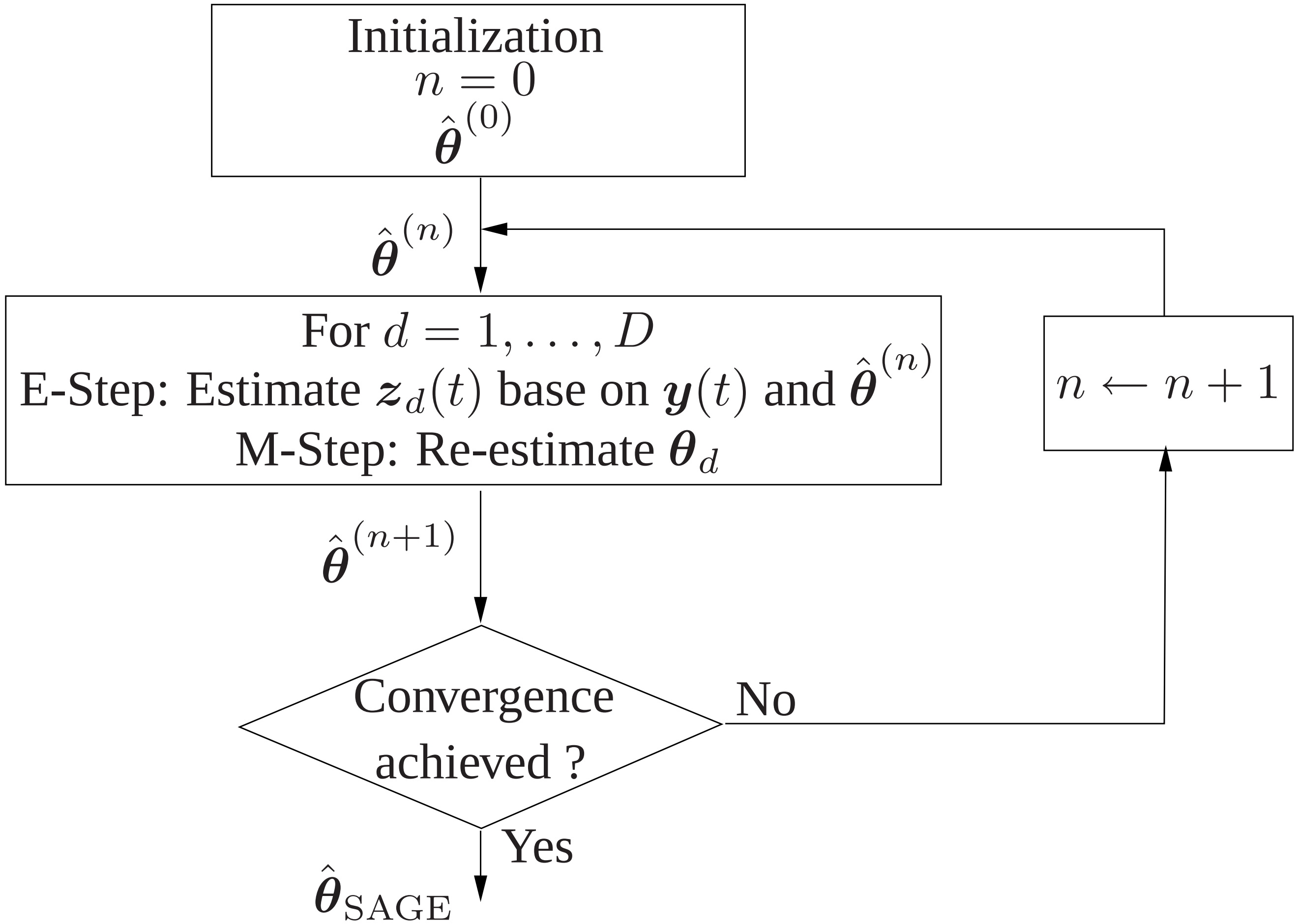
$$\boldsymbol{y}(t) = \sum_{d=1}^{D} \boldsymbol{F}(\bar{\boldsymbol{\Omega}}_d)\boldsymbol{\xi}_d(t) + \boldsymbol{w}(t), \quad t = t_1, \ldots, t_N.$$

Admissible hidden data:

$$\boldsymbol{z}_d(t) = \alpha_d(t)\boldsymbol{c}(\bar{\boldsymbol{\Omega}}_d) + \beta_{\phi,d}(t)\boldsymbol{c}_\phi'(\bar{\boldsymbol{\Omega}}_d) + \beta_{\theta,d}(t)\boldsymbol{c}_\theta'(\bar{\boldsymbol{\Omega}}_d) + \boldsymbol{w}(t).$$

The SAGE algorithm

Flow graph:



The SAGE algorithm

E-step:

$$\hat{\boldsymbol{z}}_d^{(n)}(t) \;=\; \mathrm{E}[\boldsymbol{z}_d(t)|\boldsymbol{y}(t), \hat{\boldsymbol{\theta}}^{(n)}].$$

Successive interference cancellation:

$$\hat{\boldsymbol{z}}_d^{(n)}(t) \;=\; \boldsymbol{y}(t) - \sum_{d'=1, d'\neq d}^{D} \boldsymbol{F}(\hat{\bar{\boldsymbol{\Omega}}}_{d'}^{(n)})\hat{\boldsymbol{\xi}}_{d'}^{(n)}(t),$$

where $(\hat{\cdot})^{(n)}$ is the estimate of the argument parameter in the nth iteration.

The SAGE algorithm

Objective function maximized in the M-step:

$$Z(\bar{\boldsymbol{\theta}}_d; \hat{\boldsymbol{z}}_d^{(n)}) \doteq \mathrm{tr}[\Pi_{\boldsymbol{F}\left(\bar{\boldsymbol{\Omega}}_d\right)} \hat{\boldsymbol{\Sigma}}_{\boldsymbol{z}_d^{(n-1)} \boldsymbol{z}_d^{(n-1)}}]$$

where

- $\mathrm{tr}(\cdot)$ denotes the trace operation.
- $\Pi_{\boldsymbol{F}(\bar{\boldsymbol{\Omega}}_d)} = \boldsymbol{F}(\bar{\boldsymbol{\Omega}}_d)\boldsymbol{F}(\bar{\boldsymbol{\Omega}}_d)^{\dagger}$.
- $\boldsymbol{F}(\bar{\boldsymbol{\Omega}}_d)^{\dagger} \doteq \left[\boldsymbol{F}(\bar{\boldsymbol{\Omega}}_d)^{\mathrm{H}}\boldsymbol{F}(\bar{\boldsymbol{\Omega}}_d)\right]^{-1}\boldsymbol{F}(\bar{\boldsymbol{\Omega}}_d)^{\mathrm{H}}$.
- $\hat{\boldsymbol{\Sigma}}_{\boldsymbol{z}_d^{(n-1)} \boldsymbol{z}_d^{(n-1)}} = \frac{1}{N}\sum_{t=t_1}^{t_N} \hat{\boldsymbol{z}}_d^{(n-1)}(t)\left(\hat{\boldsymbol{z}}_d^{(n-1)}(t)\right)^{\mathrm{H}}$.

The SAGE algorithm

Maximization (M-) step:

$$
\begin{aligned}
\hat{\bar{\phi}}_d^{(n)} &= \arg\max_{\bar{\phi}_d}\{\mathrm{tr}[\Pi_{\boldsymbol{F}\left(\boldsymbol{e}(\bar{\phi}_d,\hat{\bar{\theta}}_d^{(n-1)})\right)}\hat{\boldsymbol{\Sigma}}_{\boldsymbol{z}_d^{(n-1)}\boldsymbol{z}_d^{(n-1)}}]\},\\
\hat{\bar{\theta}}_d^{(n)} &= \arg\max_{\bar{\theta}_d}\{\mathrm{tr}[\Pi_{\boldsymbol{F}\left(\boldsymbol{e}(\hat{\bar{\phi}}_d^{(n)},\bar{\theta}_d)\right)}\hat{\boldsymbol{\Sigma}}_{\boldsymbol{z}_d^{(n-1)}\boldsymbol{z}_d^{(n-1)}}]\}.\\
\hat{\boldsymbol{\xi}}_d^{(n)}(t) &= \boldsymbol{F}(\hat{\bar{\boldsymbol{\Omega}}}_d^{(n)})^{\dagger}\hat{\boldsymbol{z}}_d^{(n-1)}(t),\ t = t_1,\ldots,t_N,\\
(\widehat{\sigma_w^2})^{(n)} &= \frac{1}{NM}\mathrm{tr}[\Pi^{\perp}_{\boldsymbol{F}(\hat{\bar{\boldsymbol{\Omega}}}_d^{(n)})}\hat{\boldsymbol{\Sigma}}_{\boldsymbol{z}_d^{(n-1)}\boldsymbol{z}_d^{(n-1)}}],
\end{aligned}
$$

where

- $\Pi^{\perp}_{\boldsymbol{F}(\hat{\bar{\boldsymbol{\Omega}}}_d^{(n)})} = \boldsymbol{I} - \Pi_{\boldsymbol{F}(\hat{\bar{\boldsymbol{\Omega}}}_d^{(n)})}$.

Direction Spread Estimation

GAM model-based estimators for azimuth-spread (AS), elevation-spread (ES) and azimuth-elevation correlation coefficient (AECC):

$$\hat{\sigma}_{\tilde{\phi}_d} = \sqrt{\frac{\widehat{\sigma^2_{\beta_{\phi,d}}}}{\widehat{\sigma^2_{\alpha_d}}}}, \quad \hat{\sigma}_{\tilde{\theta}_d} = \sqrt{\frac{\widehat{\sigma^2_{\beta_{\theta,d}}}}{\widehat{\sigma^2_{\alpha_d}}}}, \text{ and } \quad \hat{\rho}_{\tilde{\phi}\tilde{\theta}_d} \quad = \quad \frac{\hat{R}_{\beta_{\phi,d}\beta_{\theta,d}}(0)}{\hat{\sigma}_{\tilde{\phi}_d}\hat{\sigma}_{\tilde{\theta}_d}\widehat{\sigma^2_{\alpha_d}}}$$

where

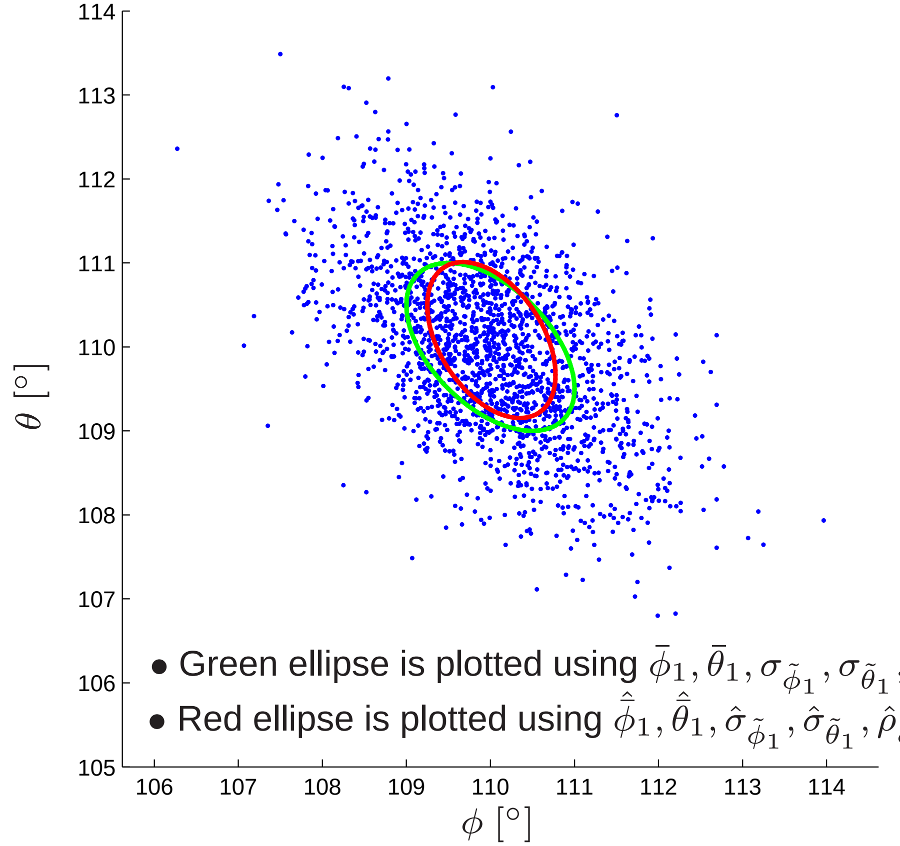
- $\hat{R}_{\beta_{\phi,d}\beta_{\theta,d}}(0) = \frac{1}{N}\sum_{t=t_1}^{t_N} \hat{\beta}_{\phi,d}(t)^* \hat{\beta}_{\theta,d}(t)$
- $\widehat{\sigma^2_\alpha} = \frac{1}{N}\sum_{t=t_1}^{t_N} |\hat{\alpha}(t) - <\hat{\alpha}(t)>|^2$, with $<\hat{\alpha}(t)> = \frac{1}{N}\sum\limits_{t=t_1}^{t_N} \hat{\alpha}(t)$.
- $\widehat{\sigma^2_{\beta_{\phi,d}}}$ and $\widehat{\sigma^2_{\beta_{\theta,d}}}$ are calculated similarly to $\widehat{\sigma^2_\alpha}$.

Simulation Studies

Estimation of the parameter $\bar{\phi}_1, \bar{\theta}_1, \sigma_{\tilde{\phi}_1}, \sigma_{\tilde{\theta}_1}, \rho_{\tilde{\phi}\tilde{\theta}_1}$

Example with simulation settings:

- single SDS
- $L = 100$
- $\bar{\phi}_1 = 110^o$
- $\bar{\theta}_1 = 110^o$
- $\sigma_{\tilde{\phi}_1} = 1^o$
- $\sigma_{\tilde{\theta}_1} = 1^o$
- $\rho_{\tilde{\phi}\tilde{\theta}_1} = -0.5$
- $N = 20$
- 50 dB output SNR
- 6×6 uniform planar array

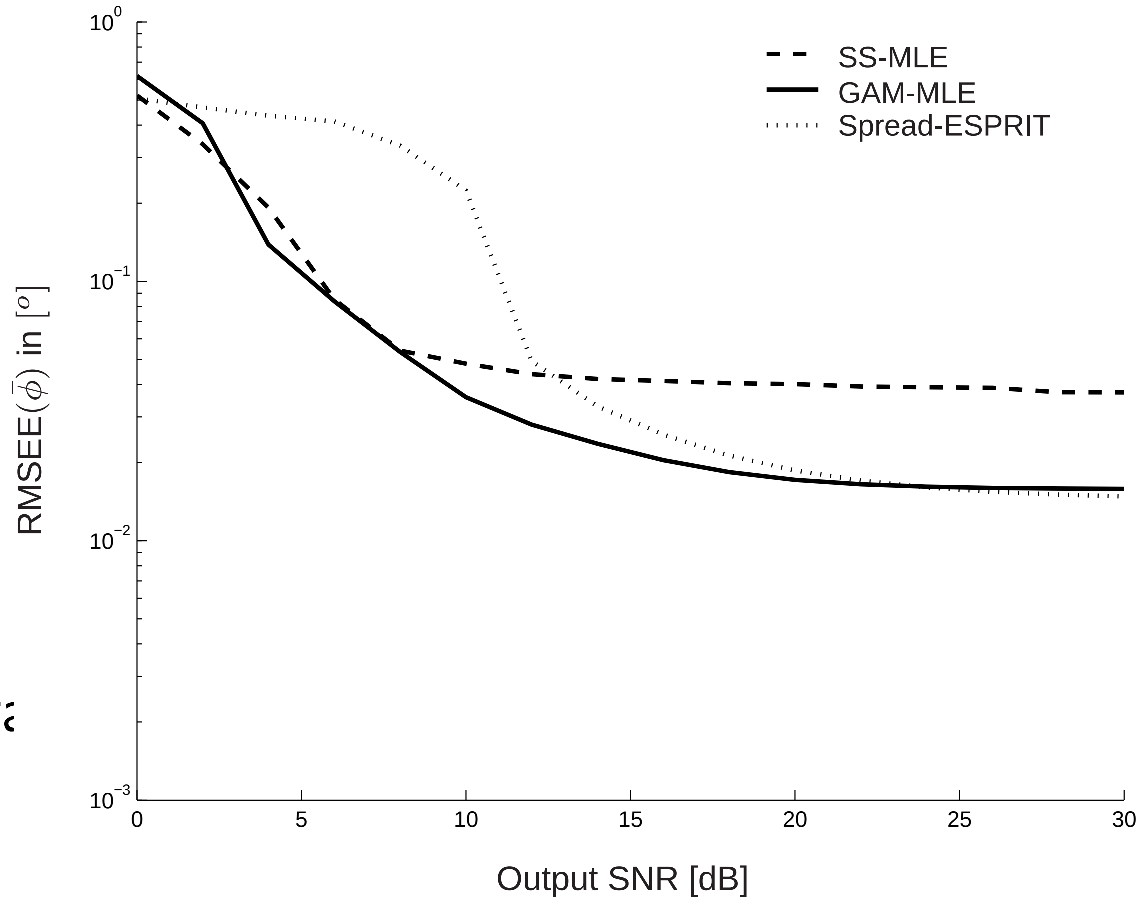


Simulation Studies

RMSEE for the nominal azimuth:

Simulation settings:

- single SDS
- $L = 100$
- $\bar{\phi}_1 = 110^o$
- $\bar{\theta}_1 = 110^o$
- $\sigma_{\tilde{\phi}_1} = 7^o$
- $\sigma_{\tilde{\theta}_1} = 7^o$
- $\rho_{\tilde{\phi}\tilde{\theta}_1} = 0.5$
- $N = 20$
- 6×6 uniform planar arra
- 100 simulation runs

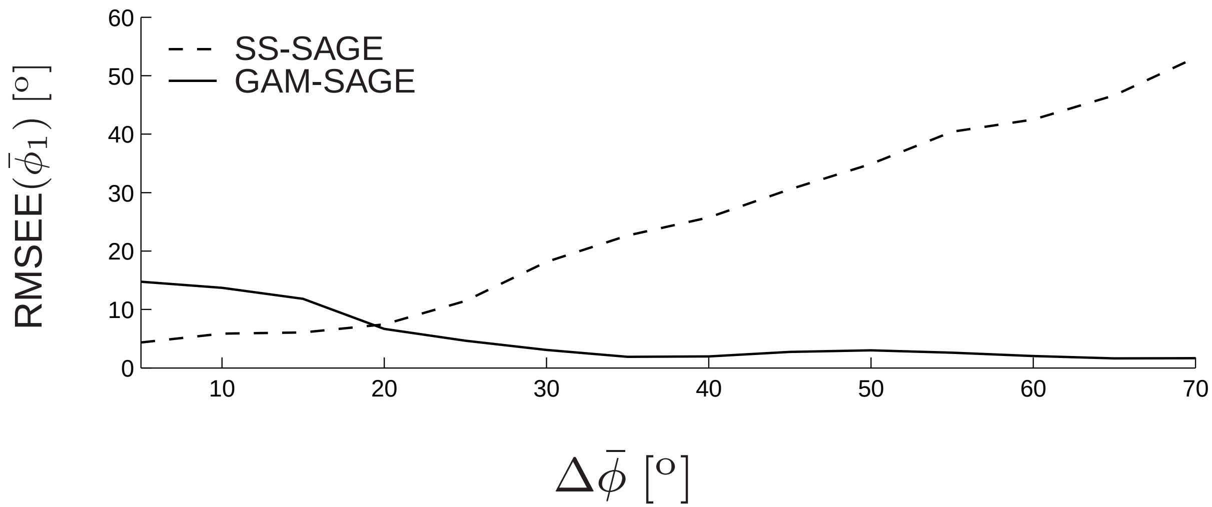


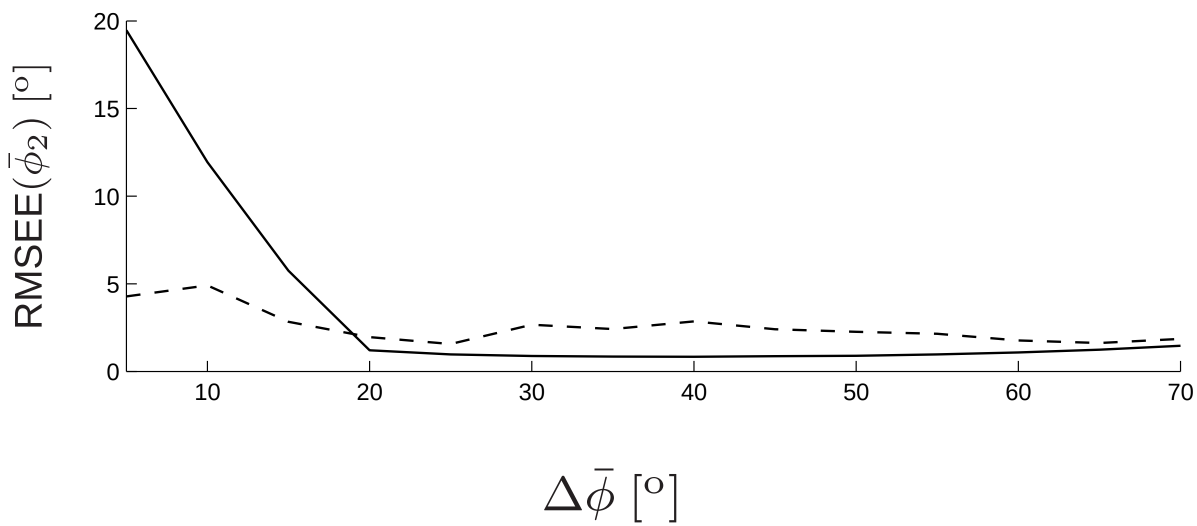
Simulation Studies

RMSEE for the nominal azimuth:

Simulation settings:

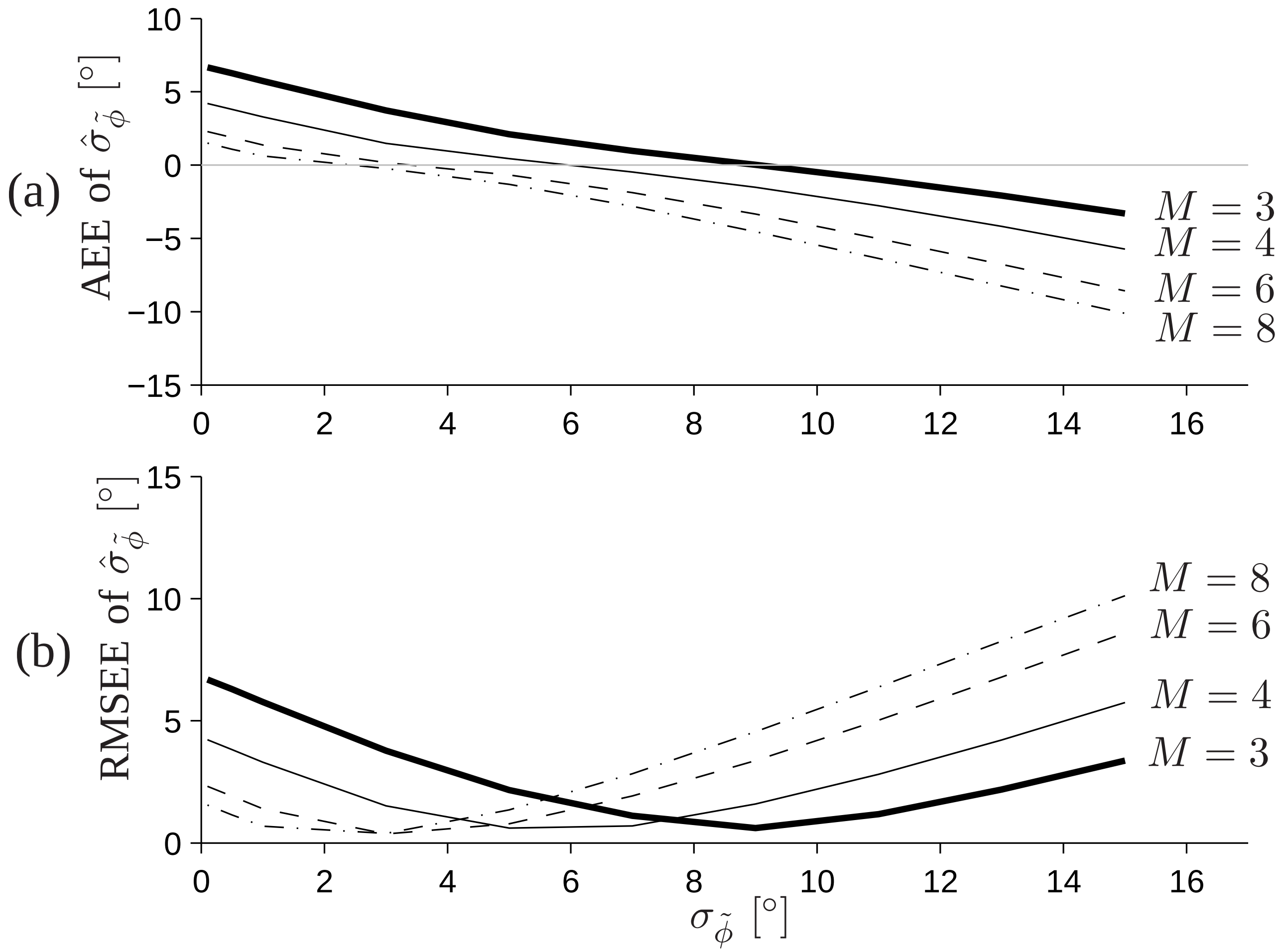
- two SDSs:
  $L = 50,$
  $\bar{\phi}_1 = 110^o,$
  $\bar{\phi}_2 = 110^o - \Delta\bar{\phi},$
  $\bar{\theta}_1 = \bar{\theta}_2 = 110^o,$
  $\Delta\bar{\phi} = 5^\circ, \ldots, 70^\circ,$
  $\sigma_{\tilde{\phi}_1} = \sigma_{\tilde{\phi}_2} = 5^\circ.$
- Output SNR:
  20 dB for SDS_1,
  29 dB for SDS_2.
- $N = 20$
- 6×6 uniform planar array
- 200 sim. runs





Azimuth Spread Estimation

Azimuth spread estimator is found to be biased, due to the mismatch between the effective model and the GAM model.



Azimuth Spread Estimation

We proposed an Array Size Adaptation (ASA) technique:

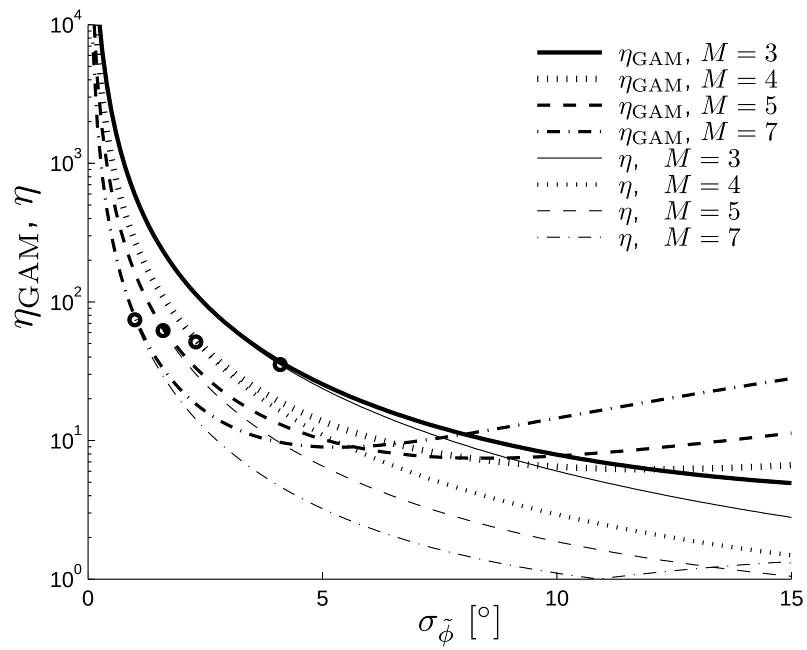
- It adjusts the array size, e.g. the number of antennas in uniform linear arrays, in such a way that the model mismatch is reduced.
- It can be combined with the SAGE algorithm for estimation of the parameters of individual path components.

Gerschgorin Radii

- Calculation of the Gerschgorin Radii [Wu, Yan & Chen 1995]:
  - Partition the covariance matrix: $\boldsymbol{\Sigma}_{\boldsymbol{yy}} = \begin{bmatrix} \boldsymbol{\Sigma}_1 & \boldsymbol{\varphi} \\ \boldsymbol{\varphi}^{\mathrm{H}} & \varepsilon \end{bmatrix}$,
    where $\boldsymbol{\Sigma}_1 \in \mathbb{C}^{(M-1)\times(M-1)}, \boldsymbol{\varphi} \in \mathbb{C}^{(M-1)\times 1}$ and $\varepsilon \in \mathbb{C}$.
  - Perform eigenvalue decomposition of $\boldsymbol{\Sigma}_1$:
    $\boldsymbol{\Sigma}_1 = \boldsymbol{U}\boldsymbol{\Lambda}\boldsymbol{U}^{\mathrm{H}}$ with $\boldsymbol{U} = [\boldsymbol{e}_1, \ldots, \boldsymbol{e}_{M-1}]$.
  - Calculate the Gerschgorin Radii (GR):
    $r_m = |\boldsymbol{e}_m^H \boldsymbol{\varphi}|,\ m = 1, \ldots, M-1.$
- Properties of the GR:
  Asymptotically,
  - GR associated with noise eigenvectors equal zero.
  - GR associated with signal eigenvectors are nonzero and independent of noise.

Array Size Adaptation Technique

Compute the ratio η and η_{GAM} of the largest to the 2nd largest GR obtained using covariance matrix $\boldsymbol{\Sigma}_{\boldsymbol{yy}}$ and $\boldsymbol{\Sigma}_{\boldsymbol{y}_{\mathrm{GAM}}\boldsymbol{y}_{\mathrm{GAM}}}$:



- η and η_{GAM} depend on array size M.
- Above a certain threshold value η_{th}, η and η_{GAM} are close.

Close agreement of η and η_{GAM} is an indicator that $\boldsymbol{y}_{\mathrm{GAM}}(t)$ provides a close approximation to $\boldsymbol{y}(t)$.

Array Size Adaptation Technique (Cont.)

The ASA technique for reducing the model mismatch:

> Adjust M such that the ratio between the largest GR and the next largest GR calculated using $\boldsymbol{\Sigma}_{\boldsymbol{yy}}$ is greater than or equal to η_{th}.

Remarks:

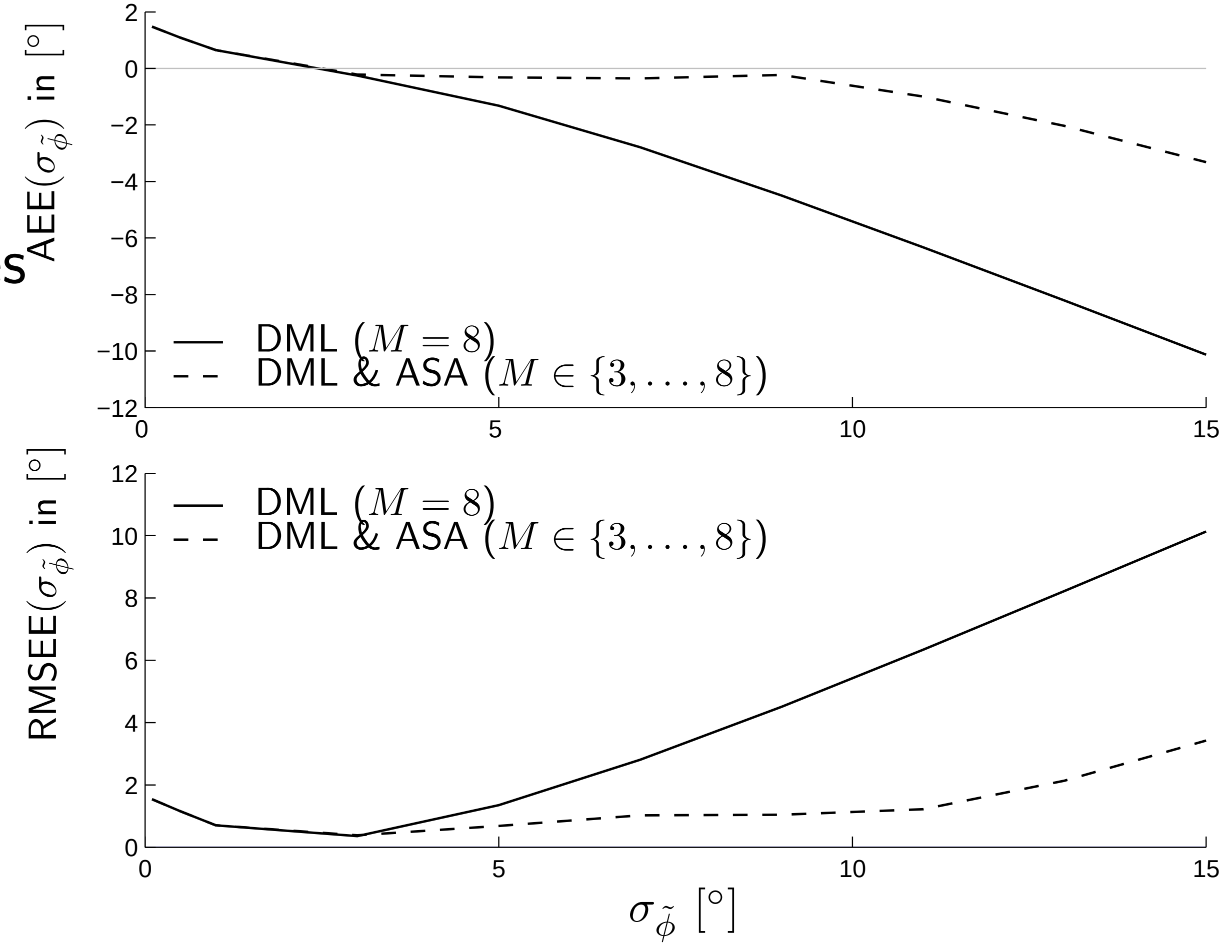
- This technique can selectively adjust the array size for individual SDSs.
- It can be extended to arrays with arbitrary layout.

Simulation Study

Average Estimation Error (AEE) and RMSEE of the azimuth spread:

Simulation settings

- $\bar{\phi} = 20^\circ$
- Input SNR 10 dB
- M-elements ULA
- 50 observation samples
- 200 simulation runs

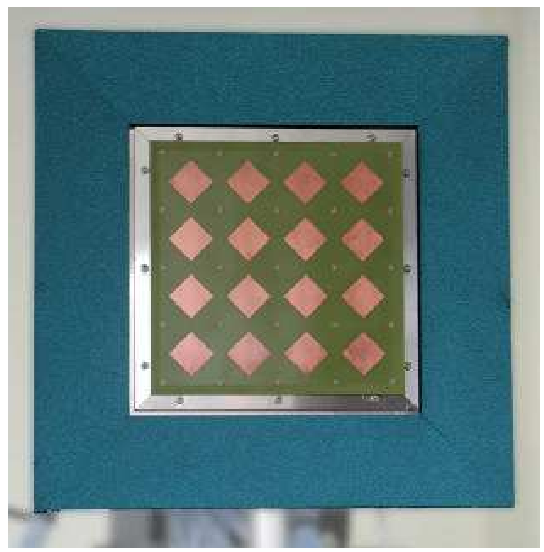


Experimental Investigation

Characteristics of the measurement set-up:

- TDM-MIMO channel sounder, PROPSound
- 3×8 dual-polarized ODA Tx array (M_1=54), (a)
- 4×4 dual-polarized planar Rx array (M_2=32), (b)
- 100MHz Signal bandwidth

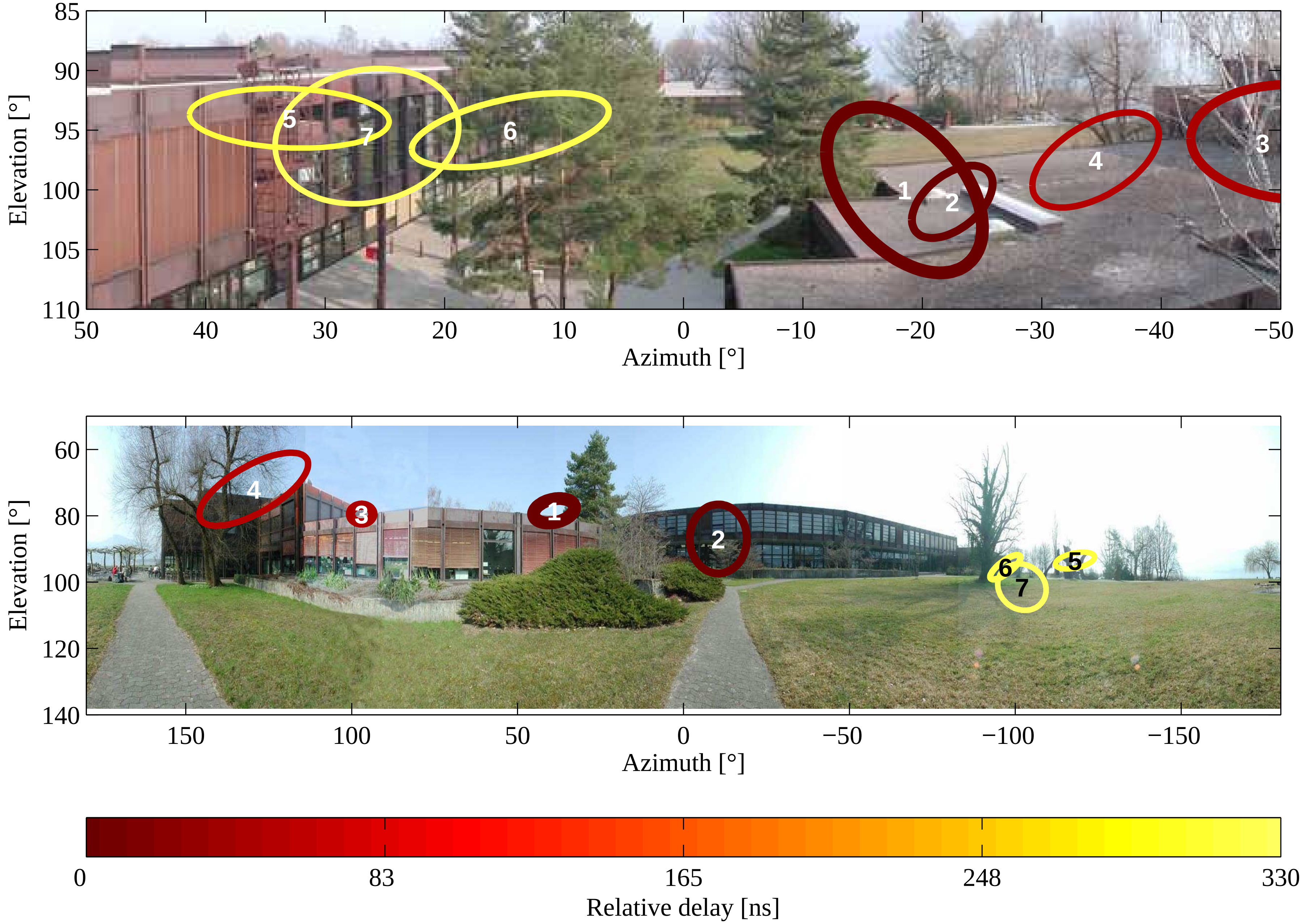
(a)



(b)

Experimental Investigation

Estimated slightly distributed scatterers:



Experimental Investigation

Estimated slightly distributed scatterers versus the estimated specular paths:

Estimated Dispersive path	Estimated Specular paths
1	1 to 7
3	8, 9, 11, 14
4	10, 13, 15, 16, 18
5	22, 24
6	25

Summary and Conclusions

- The GAM model-based estimators outperform the estimators derived using the specular-scatterer model in term of lower mean square estimator error.
- The array size adaptation (ASA) technique can be used to improve the performance of azimuth spread estimation.
- Experimental investigations showed that the SAGE algorithm derived using the generalized array manifold model can be used for estimation of parameter of slightly dispersed path components.