preamble

Exercises for Chapter 1

Descriptive questions

- What is ALOHANET? Please list three properties of this network?
- What are the typical characteristics of the projects supported by DARPA?
- What are the standards for 3G wireless communications?
- What are the candidate standards for 4G wireless communications?
- How to categorize the satellite communication systems by the heights of the satellite?
- What are the advantages of GEO satellite communication networks?

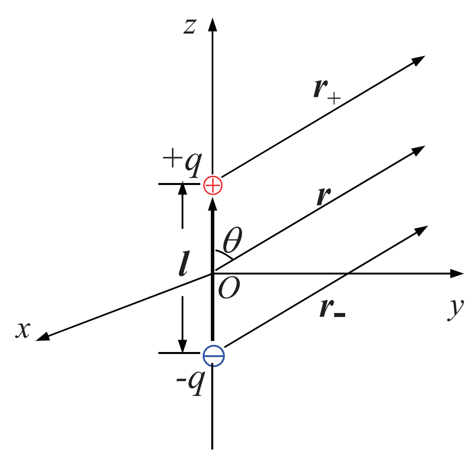
Exercises for Chapter 2 Preface

Descriptive questions:

- Please list at least four phenomena leading to the variation of propagation channels.
- List the dimensions the propagation channel can be characterized.
- What is the difference between the path loss and shadowing?
- What can lead to the variation of the path loss?
- What are the large-scale and small-scale propagation effects referred to?

Computational questions:

- Assume that there is a dipole antenna as illustrated in the following figure. Please calculate the constant η for the path loss in free space.



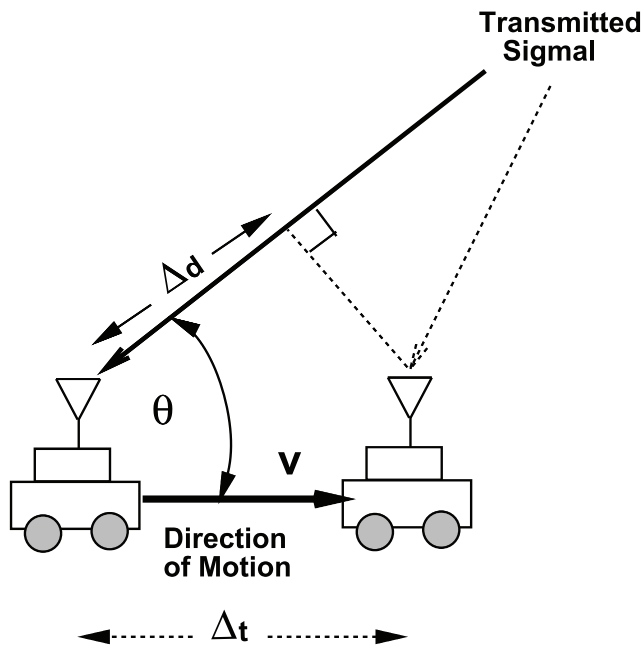
Chapter 2.1

Descriptive questions:

- What are the differences between deterministic modeling and stochastic modeling of propagation channel?
- What is a ray-tracing model? Please describe the advantages of disadvantages of the ray-tracing model?

Chapter 2.2

Computative questions:
Assuming that the transmitter is fixed, and the receiver is moving towards the transmitter at a speed of 5 m/s, as shown in the figure below. Please calculate the Doppler frequency experienced by the receiver when the carrier frequency equals 2.4 GHz.



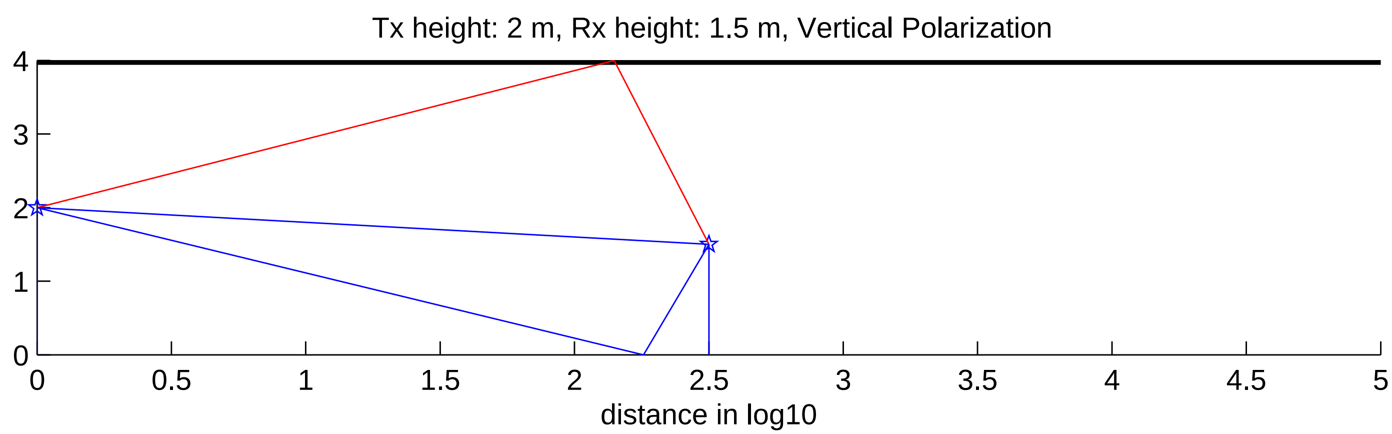
Chapter 2.3

Computational questions:
Considering a WiFi network working at carrier frequency $f_c = 2.4$ GHz. It is expected to cover the range of 50 meters, with a wall between the transmitter and the receiver, which gives the obstruction loss of 20 dB. Assuming that the noise level is 1 μW. If the minimum SNR for demodulating the signals is 10 dB, please calculate the required transmission power for the transmitter.

Exercise for Chapter 2.4

Descriptive question: In which cases the error of the ray-tracing approximation is smallest when $f_c = 30$ GHz: (a) Rural areas with trees and bushes; (b) Classroom with tables; (c) Corridor with plain walls, and (d) Hall with people standing.

Simulation questions (please solve it based on the program "two_ray_model.m"): Assuming that both Tx and Rx are under the same roof in an open indoor environment. A reflection from the ceiling is considered so as to form a three-ray model, as shown below:



Room height is 4 m, Tx and Rx 2 m, and 1.5 m respectively. Plot power falloff versus distance between the Tx and the Rx, and compare with the case where no reflection from the ceiling is considered.

Chapter 2.4.2

Descriptive questions:

- What average power falloff with distance do you expect for the ten-ray model? Why?

Computative questions:

- For the ten-ray model, assume that the transmitter and receiver are at the same height in the middle of a street of width 20 m. The transmitter-receiver separation is 500 m. Find the delay spread for this model.

Chapter 2.4.3

Descriptive questions:

- What are the possible approaches to model the diffraction? Which one is mostly used in practical ray-tracing? Why?

Computative questions:

- Consider a system with a transmitter, receiver, and scatterers, which are located at $(0,0)$, $(0,d)$ and $(0.5d, 0.5d)$ respectively. The transmitter and receiver are both at heights $h_t = h_r = 4$ m. Assume a radar cross-section of 20 dBm^2, $G_s = 1$, and $f_c = 900$ MHz. Find the path loss of the scattered signal for $d = 1, 10, 100$, and 1000 meters. Compare with the path loss at these distances if the signal is only reflected, with reflection coefficient $R = -1$.

Chapter 2.5

Descriptive questions:

- What is Hata model? When the height of the transmitter antenna increases, will the path loss decrease faster or slower when the mobile user moves away from the transmitter?

Chapter 2.6

Consider the set of empirical measurements of P_r/P_t given in the table below for an indoor system at 900MHz. Find the path loss exponent γ that minimizes the MSE between the simplified model and the empirical dB power measurements, assuming that $d_0 = 1$ m and K is determined from the free space path gain formula at this d_0. Find the received power at 150 m for the simplified path loss model with this path loss exponent and at transmit power of 1 mW (0dBm).

Distance from transmitter	P_r/P_t
10 m	-50 dB
15 m	- 65 dB
20 m	-75 dB
50 m	-90 dB
70 m	-100 dB
100 m	-110 dB
300 m	-143 dB

Chapter 2.7

Descriptive questions:
Please explain why the shadow fading follows normal distribution? Is there any relationship between the variance of the shadow fading and the delay spread? If yes, please explain the relationship.

Chapter 3 Preface

Descriptive questions:

- What are the narrowband channel and wideband channel? If the delay spread of the channel is 100 ns, how do you call this channel when the bandwidth of the transmitted signal equals 1 MHz and 100 MHz?

Chapter 3.2.1

Descriptive questions:

- Please comment on the statistical behavior of the three fading characteristics of the channel, i.e. the path loss, shadowing, and multipath fading (narrowband fading)

Computative questions:

- Please draw approximately the PSD $S_(r_I)(f)$ of $r_I(t)$ in the following cases:
  - where the scatterers are uniformly distributed within $[0, \pi]$
  - where the scatterers are uniformly distributed within $[\pi/2, 3\pi/2]$
  - where the scatterers are uniformly distributed within $[-\pi/2, \pi/2]$

Chapter 3.4

The power delay profile is often modeled as having a one-sided exponential distribution:

$$A_c(\tau) = \frac{1}{\bar{T}_m} e^{-\tau/\bar{T}_m}, \quad \tau \geq 0.$$

Show that the average delay spread is $\mu_{T_m} = \bar{T}_m$ and find the rms delay spread.

Solution:

It is straightforward to show that $\int_0^{+\infty} A_c(\tau)\mathrm{d}\tau = 1$. The average delay spread μ_{T_m} can be calculated as

$$\mu_{T_m} = \frac{1}{\bar{T}_m} \int_0^{+\infty} \tau e^{-\tau/\bar{T}_m} \mathrm{d}\tau = \bar{T}_m$$

Chapter 3.4

The power delay profile is often modeled as having a one-sided exponential distribution:

$$A_c(\tau) = \frac{1}{\bar{T}_m} e^{-\tau/\bar{T}_m}, \quad \tau \geq 0.$$

Show that the average delay spread is $\mu_{T_m} = \bar{T}_m$ and find the rms delay spread.

Solution: The rms delay spread is

$$\begin{aligned}
\sigma_{T_m} &= \sqrt{\frac{1}{\bar{T}_m}\int_0^{+\infty} (\tau - \mu_{T_m})^2 e^{-\tau/\bar{T}_m} \mathrm{d}\tau} \\
&= \sqrt{\frac{1}{\bar{T}_m}\int_0^{+\infty} \tau^2 e^{-\tau/\bar{T}_m} \mathrm{d}\tau - \mu_{T_m}^2} \\
&= \sqrt{2\bar{T}_m^2 - \bar{T}_m^2} \\
&= \bar{T}_m.
\end{aligned}$$

Chapter 3.5

Consider a wideband channel with multipath intensity profile

$$A_c(\tau) = \begin{cases} e^{-\tau/0.00001} & 0 \leq \tau \leq 20\mu s \\ 0 & \text{else} \end{cases}$$

Find the mean and rms delay spreads of the channel and find the maximum symbol rate such that a linearly modulated signal transmitted through this channel does not experience ISI.

Solution:

The average delay spread is

$$\mu_{T_m} = \frac{\int_0^{20\cdot10^{-6}} \tau e^{-\tau/0.00001} \mathrm{d}\tau}{\int_0^{20\cdot10^{-6}} e^{-\tau/0.00001} \mathrm{d}\tau} = 6.87 \cdot 10^{-6}$$

Chapter 3.5

The rms delay spread is

$$\sigma_{T_m} = \sqrt{\frac{\int_0^{20\cdot 10^{-6}} (\tau - \mu_{T_m})^2 e^{-\tau/0.00001} \mathrm{d}\tau}{\int_0^{20\cdot 10^{-6}} e^{-\tau/0.00001} \mathrm{d}\tau}} = 5.25 \cdot 10^{-6}$$

To avoid ISI it is required that the linear modulation to have a symbol period T_s that is large relative to σ_{T_m}, i.e. $T_s \gg 10 \cdot \sigma_{T_m}$, Thus, $T_s = 52.5 \cdot 10^{-6}$ seconds, i.e. the symbol rate R_s is

$$R_s = T_s^{-1} = 19.04 \text{ kilosymbols per second}$$

Example

In indoor environments $\sigma_{T_m} \approx 50$ ns whereas in outdoor microcells $\sigma_{T_m} \approx 30$ μs. Find the maximum symbol rate R_s for these environments such that a linearly modulated signal transmitted through them experiences negligible ISI.

Solution:

$R_s = 1/T_s$. To avoid ISI, $T_s \geq 10\sigma_{T_m}$, which leads to $R_s < 1/(10\sigma_{T_m})$. Therefore $R_s < 2$ Msymbols per second for the indoor, and < 3.33 Ksymbols per second for the microcell.

Comments:

- It is evident that these rates are less than the required: 50 Mbps for indoor, and 2.4 Mbps for outdoor
- Some form of ISI mitigation should be adopted
- ISI is less severe in the environments with smaller rms delay spread, e.g. in indoor

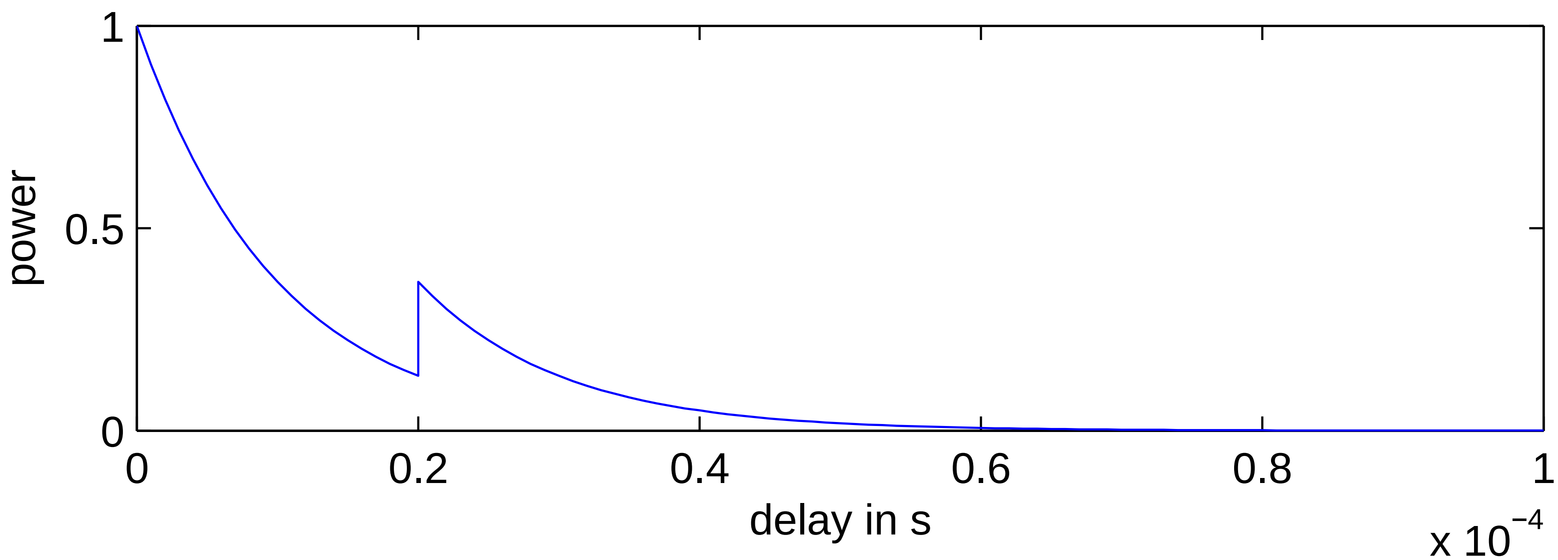
Chapter 3.5

Consider a wideband channel with power delay profile in the Saleh-Valenzeula model as

$$A_c(\tau) = \begin{cases} e^{-\tau/\gamma} & 0 \leq \tau \leq T_1 \\ e^{-T_1/\Gamma} e^{-(\tau - T_1)/\gamma} & \tau > T_1 \end{cases}$$

with model parameters:

- the intra-cluster exponent power decay constant $\gamma = 10\ \mu$s,
- the arrival time of the second cluster $T_1 = 20\ \mu$s, and
- the inter-cluster exponent power decay constant $\Gamma = 20\ \mu$s.



Exercise (Conti.)

Find by simulation (based on the program “plot_sv_model.m”):

- the mean delay spread of the channel
- the rms delay spread of the channel
- the approximate value for coherence bandwidth with correlation exceeding 0.2

and calculate

- the maximum symbol rate such that a linearly modulated signal transmitted through this channel does not experience ISI.

Chapter 3.5

For a channel with Doppler spread $B_D = 80$ Hz, find the time separation required in samples of the received signal in order for the samples to be approximately independent.

Solution:
It can be shown that the coherence time $T_c \approx 1/B_D = 0.0125$ s, i.e. the samples spaced by 12.5 ms apart are approximately uncorrelated. Thus, given the Gaussian properties of the underlying random process, these samples are approximately independent.

Chapter 3.3.4

Let a scattering function $S_c(\tau,\rho)$ be nonzero over $0.1 \le \tau \le 0.2$ ms and $-1 \le \rho \le 1$Hz. Assume that the power of the scattering function is approximately uniform over the range where it is nonzero.

- What are the multipath delay spread and the Doppler spread of the channel?
- Suppose you input to this channel two sinusoids separated in frequency by Δf. What is the minimum value of Δf for which the channel response to the first sinusoid is approximately independent of the channel response to the second sinusoid?
- For two sinusoidal inputs to the channel, i.e. $u_1(t) = \sin(2\pi f t), u_2(t) = \sin(2\pi f t + \Delta t)$, find the minimum value of Δt for which the channel response to $u_1(t)$ is approximately independent of the channel response to $u_2(t)$.
- Will this channel exhibit flat fading or frequency-selective fading for a typical voice channel with a 3-kHz bandwidth? For a cellular channel with a 30-kHz bandwidth?

Chapter 3.4

Descriptive questions:

- What is Turin's discrete-time model? what would be the parameters of a Turin's discrete-time model?

Chapter 3.5

Computative questions:

- Assuming that there are three paths with the same attenuation α, coming to the receiver, with AoA (azimuth of arrival) $\theta = -5°$, $0°$, and $5°$. Please calculate the mean AoA and AoA spread.

Chapter 5 Preface

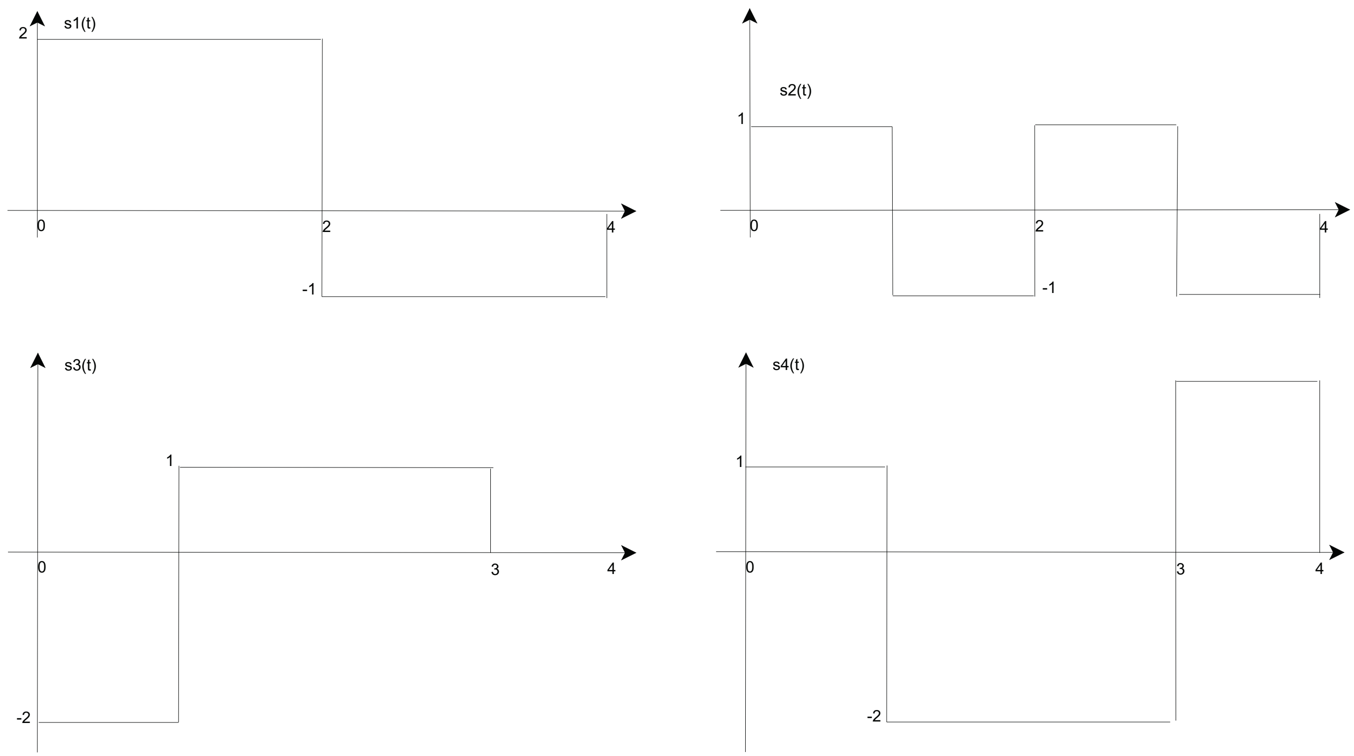
Descriptive questions:

- What is the difference between linear modulation and nonlinear modulation?
- What is spectral efficiency? How can we increase spectral efficiency?
- What are the advantages and disadvantages of choosing large signal constellation?

Chapter 5.1.3

Consider the four signal waveforms as shown in the figure below

- Determine the dimensionality of the waveforms and a set of basis functions.
- Use the basis functions to represent the four waveforms by vectors.
- Determine the minimum distance between all the vector pairs.



Chapter 5.1.4

Computative/analytical problems:

- For BPSK modulation, find decision regions Z_1 and Z_2 corresponding to constellation $s_1 = A/2$ and $s_2 = -A$ for $A > 0$
- For BPSK modulation, find decision regions Z_1 and Z_2 corresponding to constellation $s_1 = A$ and $s_2 = -A$ for $A > 0$, with $N_0 = A/10$, $p(\boldsymbol{s}_1) = 1/4$, and $p(\boldsymbol{s}_2) = 3/4$

Simulation problem:

You receive a sequence, which is the Morse code distorted by white Gaussian noise. We use -1 for spot, and 1 for bar, and 0 for the spacing between two symbols. Please detect the message in the sequence.

摩尔斯电码表

字符	电码符号	字符	电码符号	字符	电码符号
A	• —	N	— •	1	• — — — —
B	— • • •	O	— — —	2	• • — — —
C	— • — •	P	• — — •	3	• • • — —
D	— • •	Q	— — • —	4	• • • • —
E	•	R	• — •	5	• • • • •
F	• • — •	S	• • •	6	— • • • •
G	— — •	T	—	7	— — • • •
H	• • • •	U	• • —	8	— — — • •
I	• •	V	• • • —	9	— — — — •
J	• — — —	W	• — —	0	— — — — —
K	— • —	X	— • • —	?	• • — — • •
L	• — • •	Y	— • — —	/	— • • — •
M	— —	Z	— — • •	( )	— • — — • —
				—	— • • • • —
				•	• — • — • —

Chapter 5.1.5

Consider a signal constellation in $\mathcal{R}^2$ defined by $s_1 = (A, 0)$, $s_2 = (0, A)$, $s_3 = (-A, 0)$ and $s_4 = (0, -A)$. Assume $A/\sqrt{N_0} = 4$. Find the minimum distance and the union bound, looser bound, closed form bound and nearest neighbor approximation on P_e for this constellation set.

Solution:

- $d_{\min} = d_{12} = d_{23} = d_{34} = d_{41} = \sqrt{2A^2}$, $d_{13} = d_{24} = 2A$
- By symmetry, $P_e(m_i \text{ sent}) = P_e(m_j \text{ sent}), j \neq i$
- Union bound:

$$P_e^u \leq \sum_{i=1}^{M} p(m_i) P_e(m_i \text{ sent}) = \frac{1}{M} \sum_{i=1}^{M} \sum_{k=1, k\neq i}^{M} Q(\frac{d_{ik}}{\sqrt{2N_0}})$$

$$= \sum_{j=2}^{4} Q(\frac{d_{1j}}{\sqrt{2N_0}}) = 2Q(A/\sqrt{N_0}) + Q(\sqrt{2}A/\sqrt{N_0})$$

$$= 2Q(4) + Q(\sqrt{32})$$

$$= 3.1679 \times 10^{-5}$$

Chapter 5.1.5 (Conti.)

Consider a signal constellation in $\mathcal{R}^2$ defined by $s_1 = (A, 0)$, $s_2 = (0, A)$, $s_3 = (-A, 0)$ and $s_4 = (0, -A)$. Assume $A/\sqrt{N_0} = 4$. Find the minimum distance and the union bound, looser bound, closed form bound and nearest neighbor approximation on P_e for this constellation set.

Solution:

- Looser bound:

$$
\begin{aligned}
P_e^l &\le (M-1)Q(\frac{d_{\min}}{\sqrt{2N_0}}), \text{with } d_{\min} = \min_{i,k} d_{i,k} \\
&= (4-1)Q(\frac{\sqrt{2}A}{\sqrt{2N_0}}) \\
&= 3Q(4) = 9.5 \times 10^{-5}
\end{aligned}
$$

Chapter 5.1.5 (Conti.)

Consider a signal constellation in $\mathcal{R}^2$ defined by $s_1 = (A, 0)$, $s_2 = (0, A)$, $s_3 = (-A, 0)$ and $s_4 = (0, -A)$. Assume $A/\sqrt{N_0} = 4$. Find the minimum distance and the union bound, looser bound, closed form bound and nearest neighbor approximation on P_e for this constellation set.

Solution:

- Closed-form bound:

$$
\begin{aligned}
P_e^c &\leq \frac{M-1}{d_{\min}\sqrt{\pi/N_0}} \exp\left[\frac{-d_{\min}^2}{4N_0}\right] \\
&= \frac{M-1}{\sqrt{2}A\sqrt{\pi/N_0}} \exp\left[\frac{-2A^2}{4N_0}\right] \\
&= \frac{3}{4\sqrt{2\pi}} \exp\{-8\} \\
&= 1 \times 10^{-4}
\end{aligned}
$$

Chapter 5.1.5 (Conti.)

Consider a signal constellation in $\mathcal{R}^2$ defined by $s_1 = (A, 0)$, $s_2 = (0, A)$, $s_3 = (-A, 0)$ and $s_4 = (0, -A)$. Assume $A/\sqrt{N_0} = 4$. Find the minimum distance and the union bound, looser bound, closed form bound and nearest neighbor approximation on P_e for this constellation set.

Solution:

- Nearest neighbor approximation

$$
\begin{aligned}
P_e^n &\approx M_{d_{\min}} Q(\frac{d_{\min}}{\sqrt{2N_0}}) \\
&= 2Q(4) \\
&= 3.1671 \times 10^{-5}
\end{aligned}
$$

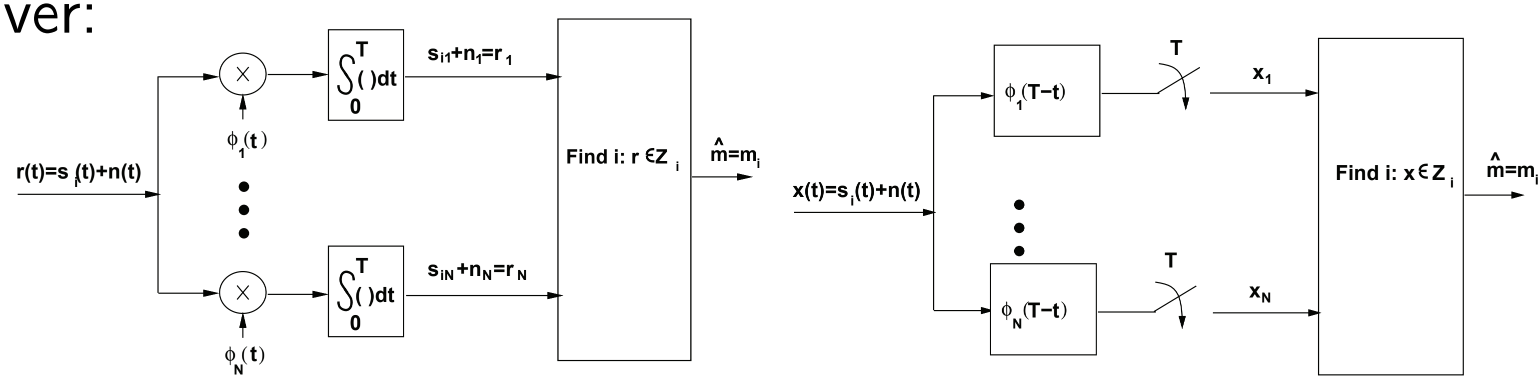
Compare:

$$
\begin{aligned}
P_e^u &= 3.1679 \times 10^{-5} \\
P_e^l &= 9.5 \times 10^{-5} \\
P_e^c &= 1 \times 10^{-4} \\
P_e^n &= 3.1671 \times 10^{-5}
\end{aligned}
$$

Chapter 5.1.5 (Homework)

Descriptive questions (5-10):

- Show that the ML receiver is equivalent to the matched filter receiver:



Computative questions:

- (5-9) Find the matched filters $g(T-t)$, $0 \leq t \leq T$, and find $\int_0^T g(t)g(T-t)dt$ for the following waveforms:
  - Rectangular pulse: $g(t) = \sqrt{2/T}$;
  - Sinc pulse: $g(t) = \text{sinc}(t)$;
  - Gaussian pulse: $g(t) = (\sqrt{\pi}/\alpha)e^{-\pi^2 t^2/\alpha^2}$
- (5-11) Compute the union-bound, looser-bound, close-form bound, and nearest approximation bounds for an asymmetric signal constellation $s_1 = (A_c, 0)$, $s_2 = (0, 2A_c)$, $s_3 = (-2A_c, 0)$, and $s_4 = (0, -A_c)$, assuming that $A_c/\sqrt{N_0} = 4$.

Example 5.4

For $g(t) = \sqrt{2/T_s}(0 \leq t < T_s)$ a rectangular pulse shape, find the average energy of 4-PAM modulation.

Solution:

For 4-PAM the A_i values are $A - i = \{-3d, -d, d, 3d\}$, so the average energy is

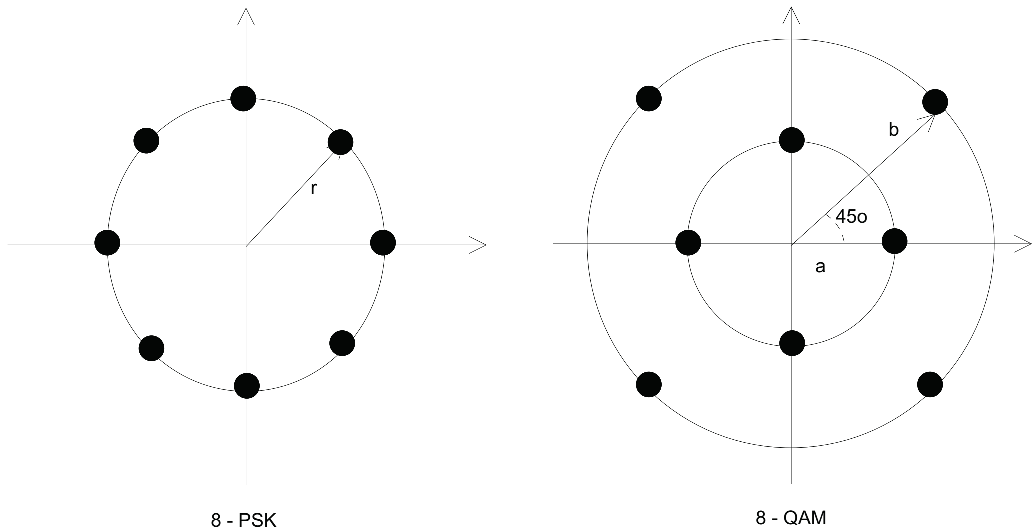
$$\bar{E}_s = \frac{d^2}{4}(9 + 1 + 1 + 9) = 5d^2.$$

Chapter 5.3

Computational problems:

- Exercise 17

Consider the octal signal point constellation in the figure shown below



(a) The nearest neighbor signal points in the 8-QAM signal constellation are separated in distance by A. Determine the radii a and b of the inner and outer circles.

(b) The adjacent signal points in the 8-PSK are separated by a distance of A. Determine the radius r of the circle.

(c) Determine the average transmitter powers for the two signal constellations and compare the two powers. What is the relative power advantage of one constellation over the other? (Assume that all signal points are equally probable.)

(d) Is it possible to assign three data bits to each point of the signal constellation such that nearest (adjacent) points differ in only one bit position?

(e) Determine the symbol rate if the desired bit rate is 90 Mbps.

Exercise

Problem: Find the sequence of symbols transmitted using DPSK for the bit sequence 101110 starting at the kth symbol time assuming the transmitted symbol at the $k-1$th symbol time was $s(k-1) = Ae^{j\pi}$

Solution:

- The first bit, a 1, results in a phase transition of π, so $s(k) = Ae^{j(\pi+\pi)} = A$.
- The next bit, a 0, results in no transition, so $s(k) = A$.
- The next bit, a 1, results in another transition of π, so $s(k+1) = Ae^{j\pi}$.
- $\ldots$
- The full symbol sequence corresponding to 101110 is $A, A, Ae^{j\pi}, A, Ae^{j\pi}, Ae^{j\pi}$.

Chapter 5.3.4

Computational problems:

- For MPSK with differential modulation, let $\Delta\phi$ denote the phase drift of the channel over a symbol time T_s. In the absence of noise, how large must $\Delta\phi$ be to make a detection error? What would be the formula for computing the maximum speed of moving when the base station is fixed, and a mobile phone is moving?
- Find the sequence of symbols transmitted using DQPSK for the bit sequence 1011100100 starting at the kth symbol time assuming the transmitted symbol at the $k-1$th symbol time was $s(k-1) = Ae^{j\pi}$

Chapter 5.3.4 (Conti.)

Simulation problems:

- Please use Matlab to accomplish the following tasks:
  - The BER of QPSK modulation with AWGN channel at SNR from 0 dB to 30 dB
  - The BER of QQPSK modulation with AWGN channel at SNR from 0 dB to 30 dB

Hints:

- AWGN generation: “randn”
- Noise with N_0 as the variance: $n = sqrt(N_0/2) * randn(1, M)$, with M being the symbol length.
- SNR: $\gamma = E_s/N_0$, which is A^2/N_0 in the QPSK scenario.
- It would be nice to put these two curves in one figure

Exercises for Chapter 6

Descriptive questions:

- How to calculate the SNR;
- What is the relationship between the γ_b and γ_s

Computative questions:

- Compare the exact P_s and the approximation of it by using difference bounds, for MPSK, MQAM, etc.