Lecture 1
Radio Channel Characterization for MIMO System Applications

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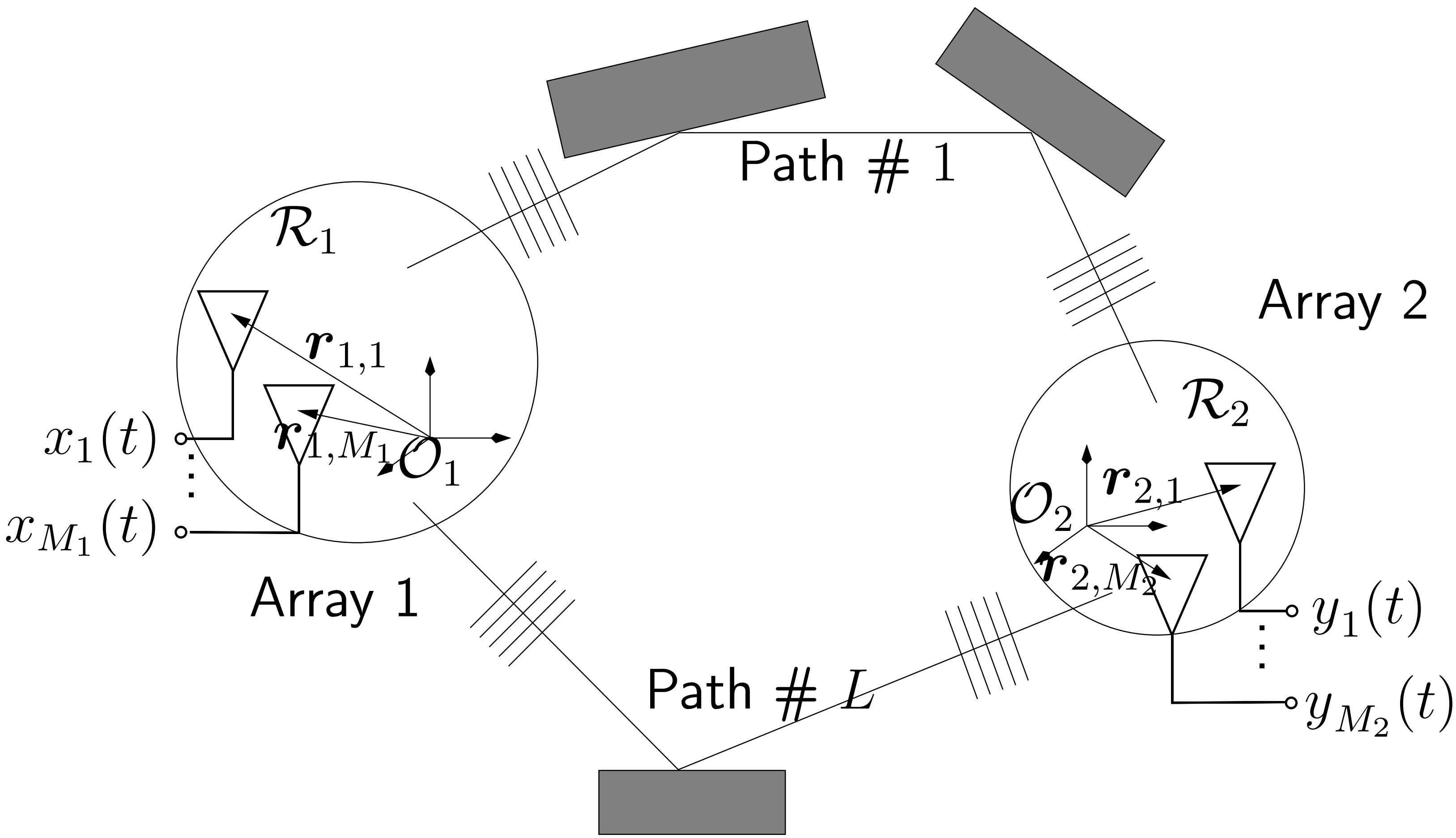
Content

MIMO systems



Transfer matrix of MIMO systems

Signal at the output of the nth Rx antenna:

$$y_n = \sum_{m=1}^{M_1} H_{nm} x_m + w_n \qquad n = 1, \ldots, M_2.$$

In matrix form:

$$\underbrace{\begin{bmatrix} y_1 \\ \vdots \\ y_{M_2} \end{bmatrix}}_{\boldsymbol{y} \doteq} = \underbrace{\begin{bmatrix} H_{11} & \ldots & H_{1M_1} \\ \vdots & & \vdots \\ H_{M_2 1} & \ldots & H_{M_2 M_1} \end{bmatrix}}_{\boldsymbol{H} \doteq} \underbrace{\begin{bmatrix} x_1 \\ \vdots \\ x_{M_1} \end{bmatrix}}_{\boldsymbol{x} \doteq} + \underbrace{\begin{bmatrix} w_1 \\ \vdots \\ w_{M_2} \end{bmatrix}}_{\boldsymbol{w} \doteq}$$

$\boldsymbol{y} = \boldsymbol{H}\boldsymbol{x} \ + \ \boldsymbol{w}$, $\quad \boldsymbol{w}$: spatially white complex noise

Singular value decomposition

The transfer matrix $\boldsymbol{H}$ can be decomposed according to

$$\boldsymbol{H} = \boldsymbol{U}\boldsymbol{F}\boldsymbol{V}^{\mathrm{H}}$$

- $(\cdot)^{\mathrm{H}}$ is the Hermitian operator
- $\boldsymbol{U}$ is a $M_2 \times M_2$ complex unitary matrix,
- $\boldsymbol{V}$ is a $M_1 \times M_1$ complex unitary matrix,
- and

$$\boldsymbol{F} = \begin{bmatrix} \sqrt{\gamma_1} & & & & & \\ & \ddots & & & & \\ & & \sqrt{\gamma_K} & & & \\ & & & 0 & & \\ & & & & \ddots & \\ & & & & & 0 \end{bmatrix}$$

K is the rank of $\boldsymbol{H}$ and $\gamma_1, \cdots, \gamma_K$ are the eigenvalues of $\boldsymbol{H}\boldsymbol{H}^{\mathrm{H}}$.

Eigenchannels of a MIMO system

Multiplying $\boldsymbol{U}^{\mathrm{H}}$ on both sides of $\boldsymbol{y} = \boldsymbol{H}\boldsymbol{x} + \boldsymbol{w}$

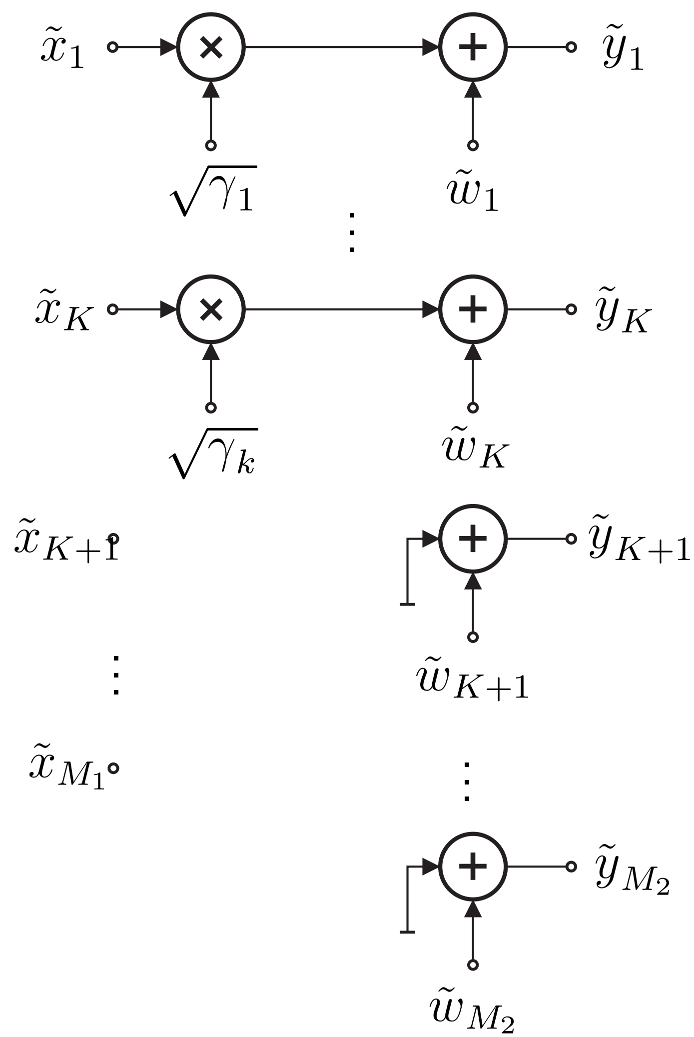
$$
\begin{aligned}
\boldsymbol{U}^{\mathrm{H}}\boldsymbol{y} &= \boldsymbol{U}^{\mathrm{H}}\boldsymbol{H}\boldsymbol{x} + \boldsymbol{U}^{\mathrm{H}}\boldsymbol{w} \\
\boldsymbol{U}^{\mathrm{H}}\boldsymbol{y} &= \boldsymbol{U}^{\mathrm{H}}\boldsymbol{U}\boldsymbol{F}\boldsymbol{V}^{\mathrm{H}}\boldsymbol{x} + \boldsymbol{U}^{\mathrm{H}}\boldsymbol{w} \\
\underbrace{\boldsymbol{U}^{\mathrm{H}}\boldsymbol{y}}_{\tilde{\boldsymbol{y}}\doteq} &= \boldsymbol{F}\underbrace{\boldsymbol{V}^{\mathrm{H}}\boldsymbol{x}}_{\tilde{\boldsymbol{x}}\doteq} + \underbrace{\boldsymbol{U}^{\mathrm{H}}\boldsymbol{w}}_{\tilde{\boldsymbol{w}}\doteq}
\end{aligned}
$$

yields

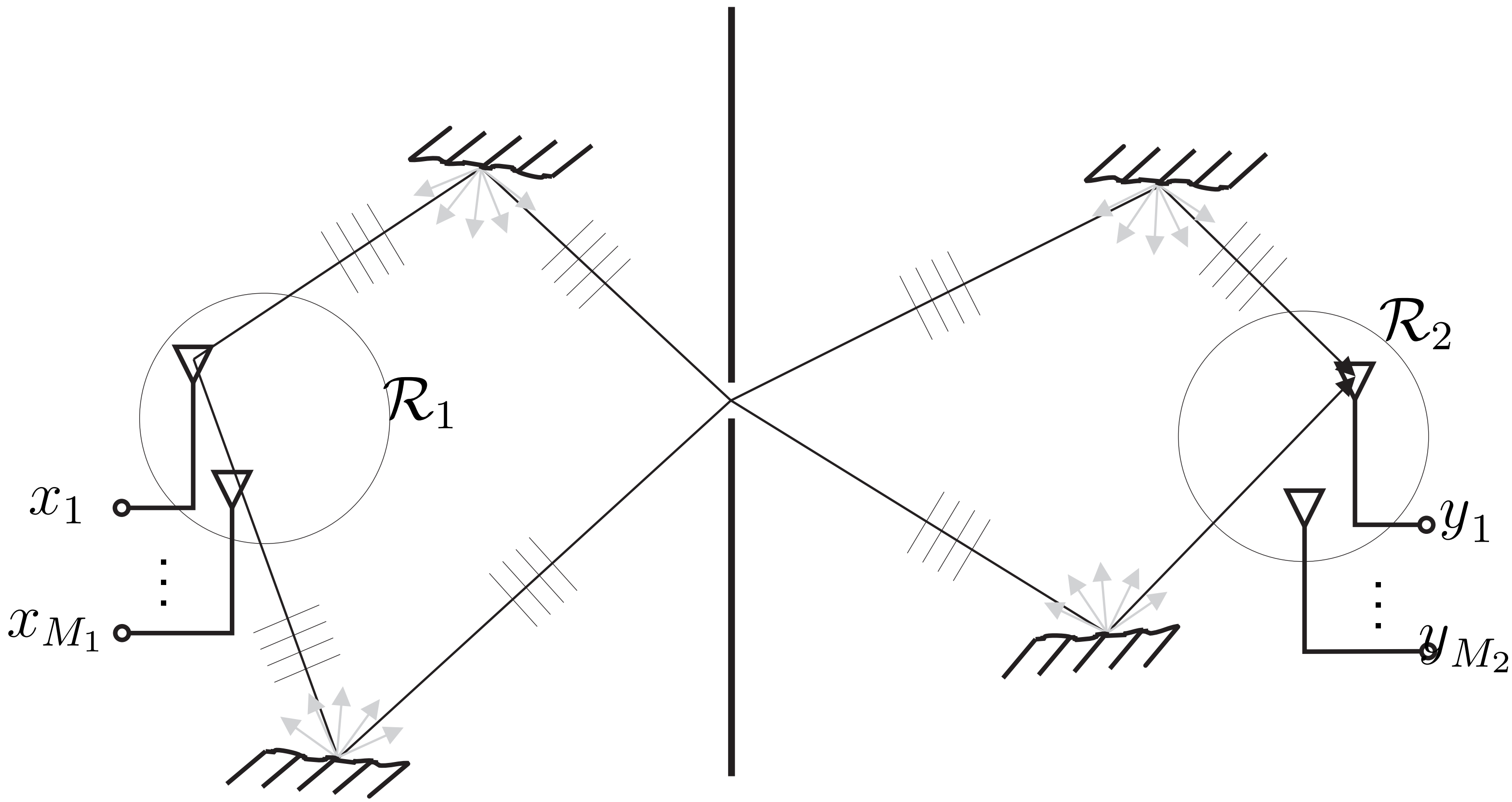
$$
\tilde{\boldsymbol{y}} = \boldsymbol{F}\tilde{\boldsymbol{x}} + \tilde{\boldsymbol{w}}, \quad \text{i.e.}
$$

$$
\begin{bmatrix} \tilde{y}_1 \\ \vdots \\ \tilde{y}_K \\ \vdots \\ \tilde{y}_{M_2} \end{bmatrix} = \begin{bmatrix} \sqrt{\gamma_1} & & & \\ & \ddots & & \\ & & \sqrt{\gamma_K} & \\ & & & 0 \\ & & & & \ddots \end{bmatrix} \begin{bmatrix} \tilde{x}_1 \\ \vdots \\ \tilde{x}_K \\ \vdots \\ \tilde{x}_{M_1} \end{bmatrix} + \begin{bmatrix} \tilde{w}_1 \\ \vdots \\ \tilde{w}_K \\ \vdots \\ \tilde{w}_{M_2} \end{bmatrix}
$$

Eigenchannels of a MIMO system



Key-hole effect



Key-hole MIMO system

Facts:

- If the antenna elements are sufficiently spaced, the correlation between any two entries of $\boldsymbol{H}$ nearly vanishes.
- The singular value decomposition of transfer matrix $\boldsymbol{H}$ of a key-hole MIMO system reads

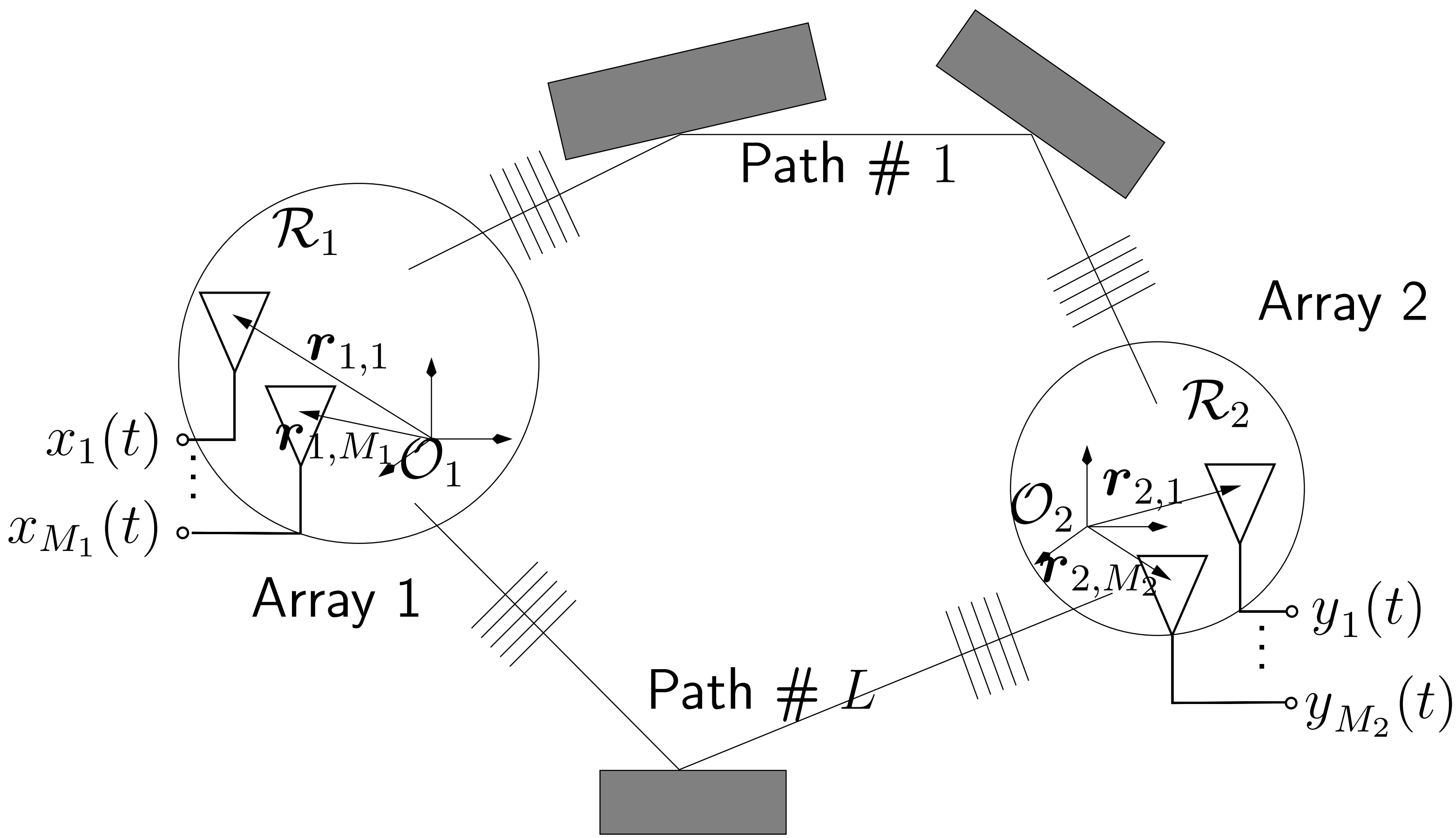
$$\boldsymbol{H} = \boldsymbol{u}_1 \sqrt{\gamma_1} \boldsymbol{v}_1^{\mathrm{H}}.$$

The rank of $\boldsymbol{H}$ is *ONE*.

Hence, the transfer matrix $\boldsymbol{H}$ of a key-hole MIMO system is a matrix with nearly uncorrelated entries and with rank one.

Propagation Constellation

- Relationship between the entries of the transfer matrix $\boldsymbol{H}$ of a MIMO system and the underlying propagation constellation.
- Relationship between the correlation properties of the entries of $\boldsymbol{H}$ and the underlying propagation constellation.



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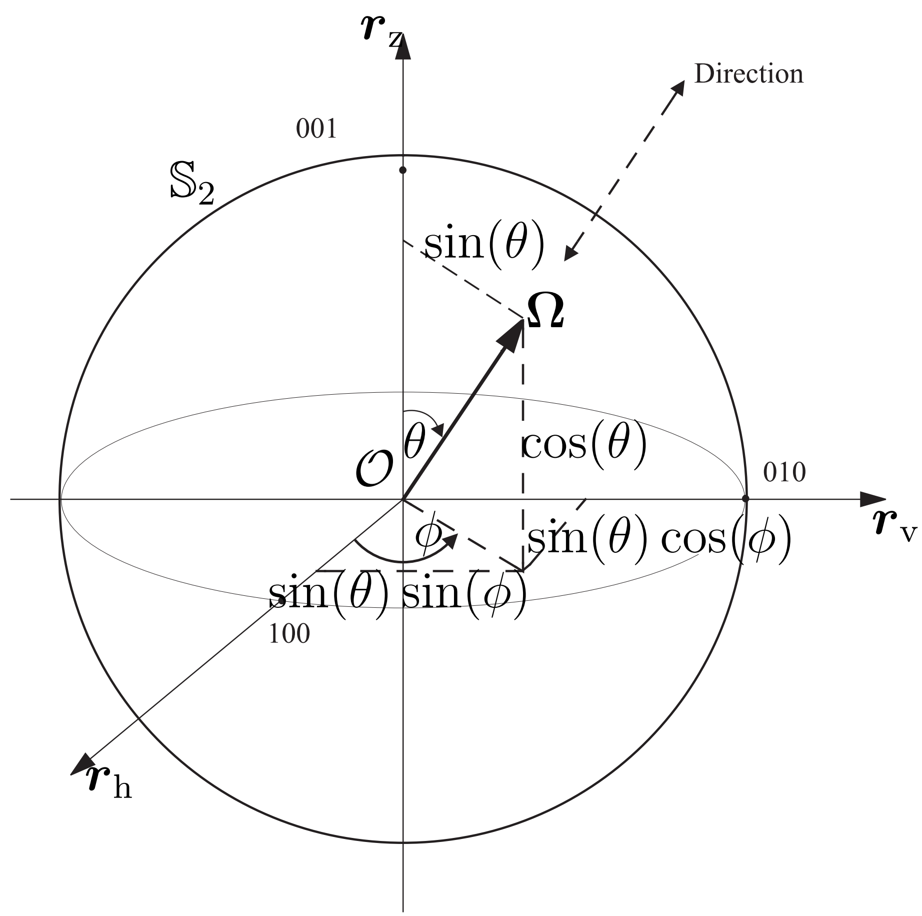
Transfer matrix of a MIMO system

The entry $\boldsymbol{H}_{n,m}$ can be expressed as

$$\begin{aligned}\boldsymbol{H}_{n,m} &= \int_{\mathbb{S}_2}\int_{\mathbb{S}_2} f_{1,m}(\boldsymbol{\Omega}_1)\exp\{\jmath 2\pi\lambda_0^{-1}(\boldsymbol{\Omega}_1\cdot\boldsymbol{r}_{1,m})\}\\ &\quad\cdot f_{2,n}(\boldsymbol{\Omega}_2)\exp\{\jmath 2\pi\lambda_0^{-1}(\boldsymbol{\Omega}_2\cdot\boldsymbol{r}_{2,n})\}\\ &\quad\cdot h(\boldsymbol{\Omega}_1,\boldsymbol{\Omega}_2)\mathrm{d}\boldsymbol{\Omega}_1\mathrm{d}\boldsymbol{\Omega}_2.\end{aligned}$$

- λ_0 is the wavelength.
- $f_{i,m}(\boldsymbol{\Omega})$ is the field pattern of the mth element of the ith array, $m = 1,\ldots,M_i$, $i = 1,2$.
- The complex function $h(\boldsymbol{\Omega}_1,\boldsymbol{\Omega}_2)$ with definition domain $\mathbb{S}_2\times\mathbb{S}_2$ is referred to as the *bidirection spread function* in $\mathcal{R}_1$ and $\mathcal{R}_2$ of the propagation channel.

Characterization of a direction



$$\boldsymbol{\Omega} = \boldsymbol{e}(\phi, \theta) \doteq [\cos(\phi)\sin(\theta), \sin(\phi)\sin(\theta), \cos(\theta)]^\mathrm{T} \in \mathbb{S}_2$$

Direction spread functions

Bidirection spread function in $\mathcal{R}_1$ and $\mathcal{R}_2$:

$$h(\mathbf{\Omega}_1, \mathbf{\Omega}_2)$$

$h(\mathbf{\Omega}_1, \mathbf{\Omega}_2)$ describes direction dispersion jointly in $\mathcal{R}_1$ and $\mathcal{R}_2$.

Direction spread function in $\mathcal{R}_1$:

$$h_1(\mathbf{\Omega}_1) \doteq \int_{\mathbb{S}_2} h(\mathbf{\Omega}_1, \mathbf{\Omega}_2) \mathrm{d}\mathbf{\Omega}_2$$

$h_1(\mathbf{\Omega}_1)$ describes direction dispersion in $\mathcal{R}_1$ only.

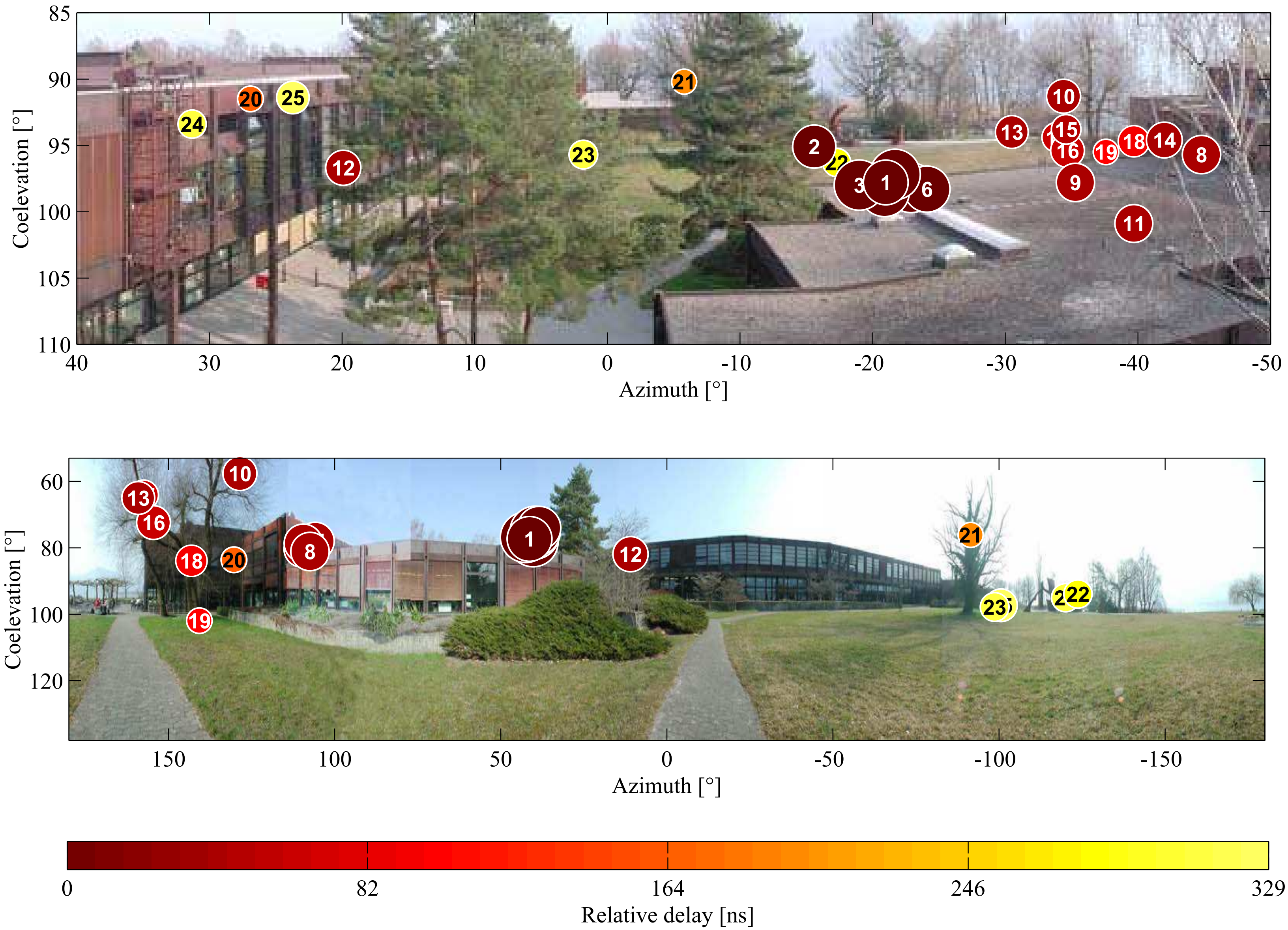
Direction spread function in $\mathcal{R}_2$:

$$h_2(\mathbf{\Omega}_2) \doteq \int_{\mathbb{S}_2} h(\mathbf{\Omega}_1, \mathbf{\Omega}_2) \mathrm{d}\mathbf{\Omega}_1$$

$h_2(\mathbf{\Omega}_2)$ describes direction dispersion in $\mathcal{R}_2$ only.

Direction spread function

Example of estimated direction spread functions:



Transfer matrix of a MIMO system

Responses of the antenna arrays:

$$\boldsymbol{c}_i(\boldsymbol{\Omega}_i) \doteq \begin{bmatrix} f_{i,1}(\boldsymbol{\Omega}_i)\exp\{\jmath 2\pi\lambda_0^{-1}(\boldsymbol{\Omega}_i\cdot\boldsymbol{r}_{i,1})\}, \\ \vdots \\ f_{i,M_i}(\boldsymbol{\Omega}_i)\exp\{\jmath 2\pi\lambda_0^{-1}(\boldsymbol{\Omega}_i\cdot\boldsymbol{r}_{i,M_i})\} \end{bmatrix}, \quad i=1,2$$

With the above definition, the entry $\boldsymbol{H}_{n,m}$ can be recast as

$$\boldsymbol{H}_{n,m} = \int_{\mathbb{S}_2}\int_{\mathbb{S}_2}[\boldsymbol{c}_1(\boldsymbol{\Omega}_1)]_m[\boldsymbol{c}_2(\boldsymbol{\Omega}_2)]_n h(\boldsymbol{\Omega}_1,\boldsymbol{\Omega}_2)\mathrm{d}\boldsymbol{\Omega}_1\mathrm{d}\boldsymbol{\Omega}_2.$$

Transfer matrix of a MIMO system

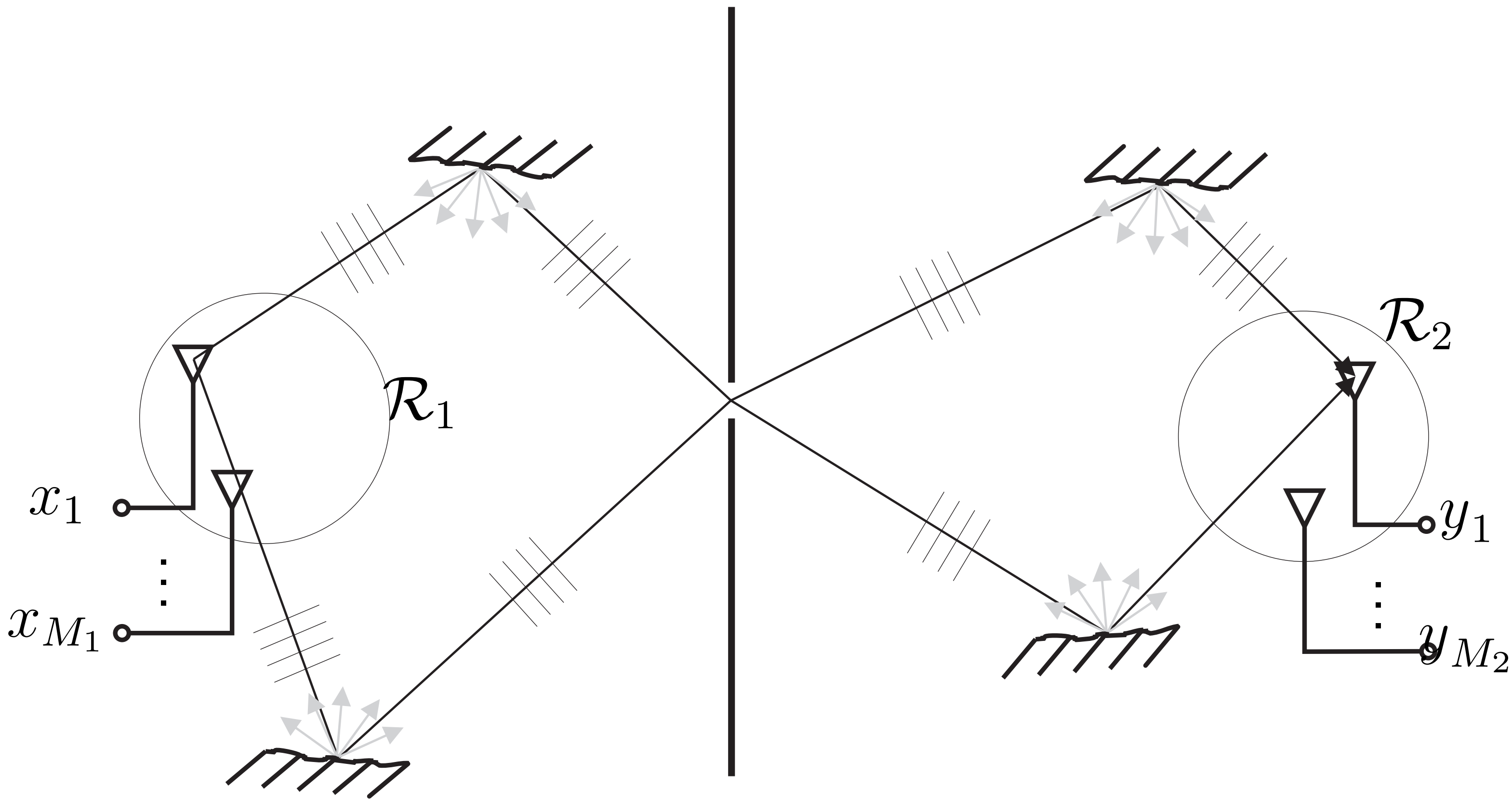
The transfer matrix $\boldsymbol{H}$ can be expressed as

$$\boldsymbol{H} = \int_{\mathbb{S}_2}\int_{\mathbb{S}_2} \boldsymbol{c}_1(\boldsymbol{\Omega}_1)^{\mathrm{T}} \otimes \boldsymbol{c}_2(\boldsymbol{\Omega}_2) h(\boldsymbol{\Omega}_1, \boldsymbol{\Omega}_2) \mathrm{d}\boldsymbol{\Omega}_1 \mathrm{d}\boldsymbol{\Omega}_2$$

The above equation relates the transfer matrix of the MIMO system

- *to the propagation constellation via the bidirection spread function* $h(\boldsymbol{\Omega}_1, \boldsymbol{\Omega}_2)$,
- *to the array characteristics, i.e. layouts and field patterns, via the array responses* $\boldsymbol{c}_i(\boldsymbol{\Omega})$, $i = 1, 2$.

Key-hole effect



$$h(\mathbf{\Omega}_1, \mathbf{\Omega}_2) = h_1(\mathbf{\Omega}_1) h_2(\mathbf{\Omega}_2)$$

Correlation matrix of a MIMO system

Vectorize the transfer matrix $\boldsymbol{H}$:

$$\boldsymbol{H}^{\mathrm{s}} \doteq [H_{1,1}, \ldots, H_{M_2,1}, \ldots, H_{1,M_1}, \ldots, H_{M_2,M_1}]^{\mathrm{T}}$$

Covariance matrix of a MIMO system:

$$\boldsymbol{R}_{\boldsymbol{H}} \doteq \mathsf{E}[(\boldsymbol{H}^{\mathrm{s}} - \mathsf{E}[\boldsymbol{H}^{\mathrm{s}}])((\boldsymbol{H}^{\mathrm{s}} - \mathsf{E}[\boldsymbol{H}^{\mathrm{s}}]))^{\mathrm{H}}]$$

where

- $\mathsf{E}[\cdot]$ is the expectation operator.

Correlation matrix of a MIMO system

WSS/US (Wide-Sense-Stationary/Uncorrelated Scattering) assumption: We assume that the bidirection spread function is a zero-mean uncorrelated process, i.e.

- $\mathsf{E}\Big[h(\mathbf{\Omega}_1, \mathbf{\Omega}_2)\Big] = 0$
- $\mathsf{E}\Big[h^*(\mathbf{\Omega}_1, \mathbf{\Omega}_2)h(\mathbf{\Omega}'_1, \mathbf{\Omega}'_2)\Big] = P(\mathbf{\Omega}_1, \mathbf{\Omega}_2)\delta(\mathbf{\Omega}'_1 - \mathbf{\Omega}_1)\delta(\mathbf{\Omega}'_2 - \mathbf{\Omega}_2)$

where

$$P(\mathbf{\Omega}_1, \mathbf{\Omega}_2) \doteq \mathsf{E}\Big[|h(\mathbf{\Omega}_1, \mathbf{\Omega}_2)|^2\Big]$$

is called the *bidirection power spectrum* in $\mathcal{R}_1$ and $\mathcal{R}_2$.

$P(\mathbf{\Omega}_1, \mathbf{\Omega}_2)$ characterizes the way the average power propagating from $\mathcal{R}_1$ to $\mathcal{R}_2$ is distributed with respect to the launching direction $\mathbf{\Omega}_1$ and the incident direction $\mathbf{\Omega}_2$.

Correlation matrix of a MIMO system

If the WSS/US assumption holds, the correlation matrix of the MIMO system reads

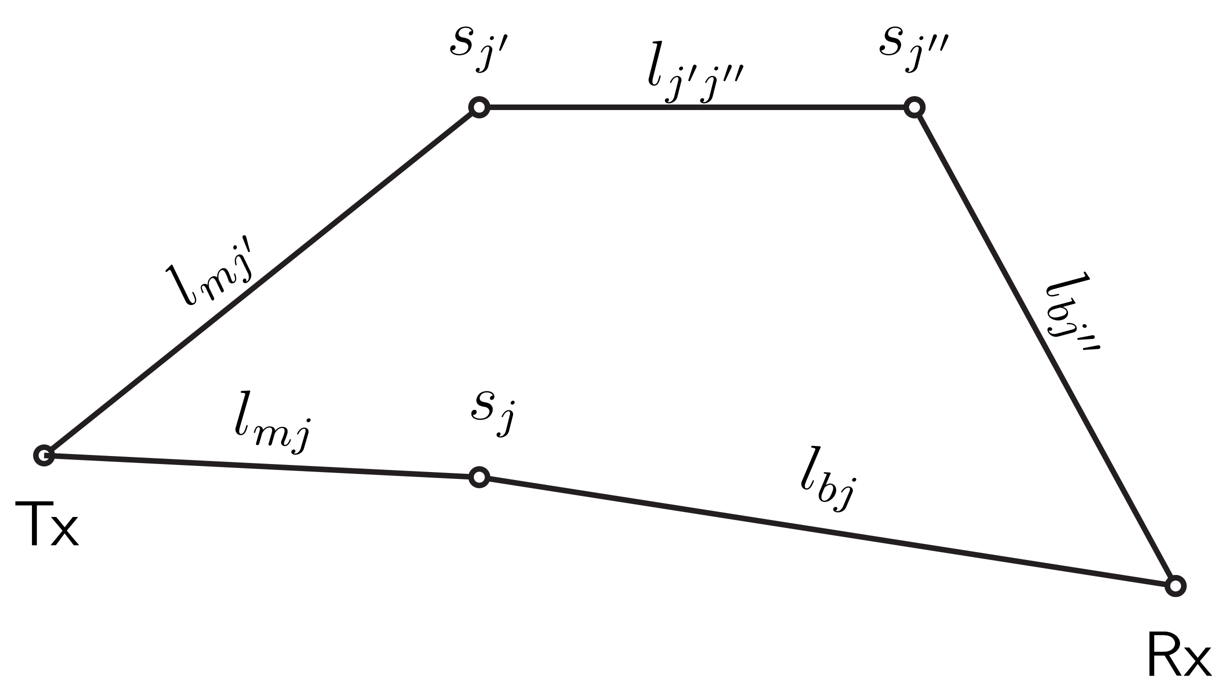
$$\boldsymbol{R}_{\boldsymbol{H}} = \int_{\mathbb{S}_2}\int_{\mathbb{S}_2} \left[\boldsymbol{c}_1(\boldsymbol{\Omega}_1)\boldsymbol{c}_1(\boldsymbol{\Omega}_1)^{\mathrm{H}}\right] \otimes \left[\boldsymbol{c}_2(\boldsymbol{\Omega}_2)\boldsymbol{c}_2(\boldsymbol{\Omega}_2)^{\mathrm{H}}\right] P(\boldsymbol{\Omega}_1, \boldsymbol{\Omega}_2)\mathrm{d}\boldsymbol{\Omega}_1\mathrm{d}\boldsymbol{\Omega}_2.$$

The above equation relates the correlation matrix of the MIMO system

- *to the propagation constellation via the bidirection power spectrum* $P(\boldsymbol{\Omega}_1, \boldsymbol{\Omega}_2)$*,*
- *to the array characteristics, i.e. layouts and field patterns, via the array responses* $\boldsymbol{c}_i(\boldsymbol{\Omega})$*,* $i = 1, 2$*.*

Simulation of propagation scenarios

One- and two-bounce scattering:

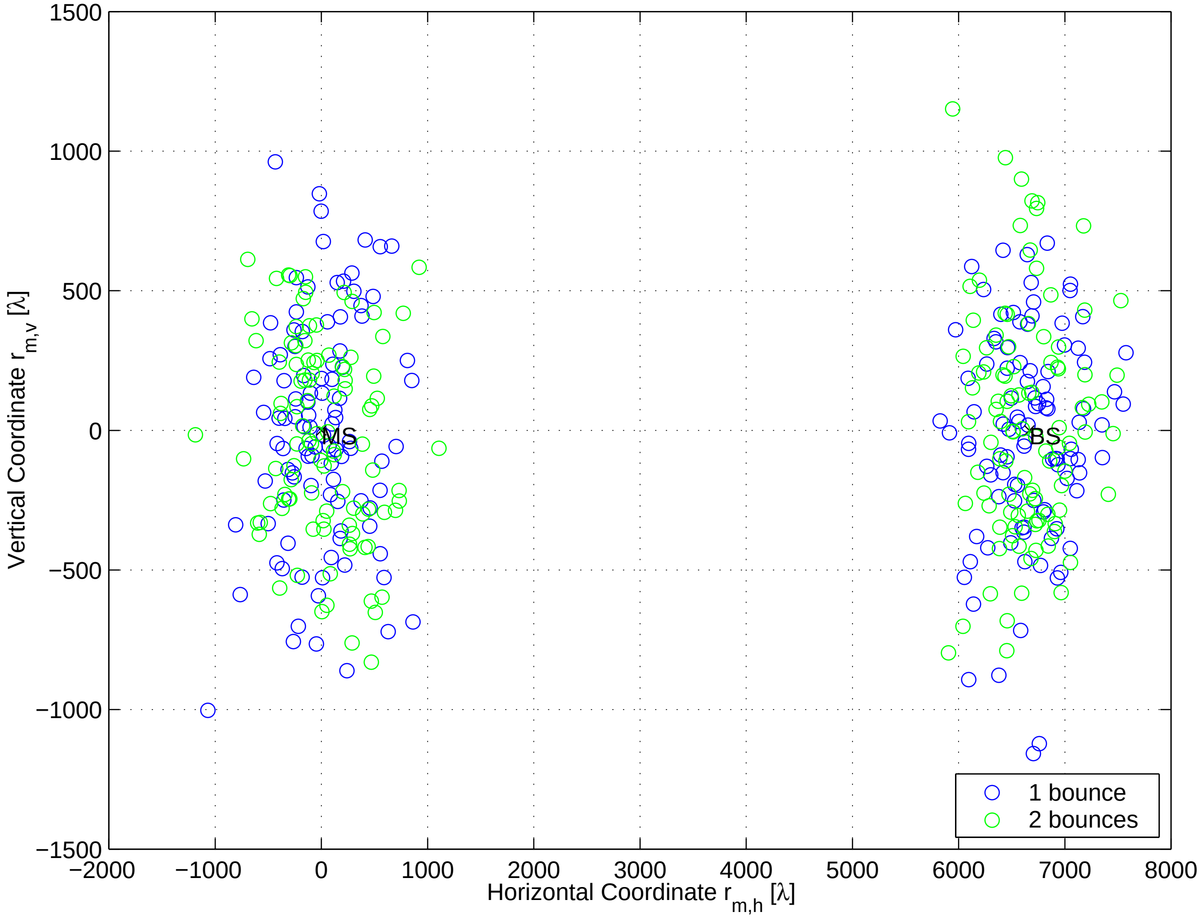


Path weight:

- One-bounce scattering: $\alpha_\ell = s_j(l_{\mathrm{m}j} l_{\mathrm{b}j})^{-1}$
- Two-bounce scattering: $\alpha_{\ell'} = s_{j'} s_{j''}(l_{\mathrm{m}j} l_{jj'} l_{\mathrm{b}j'})^{-1}$

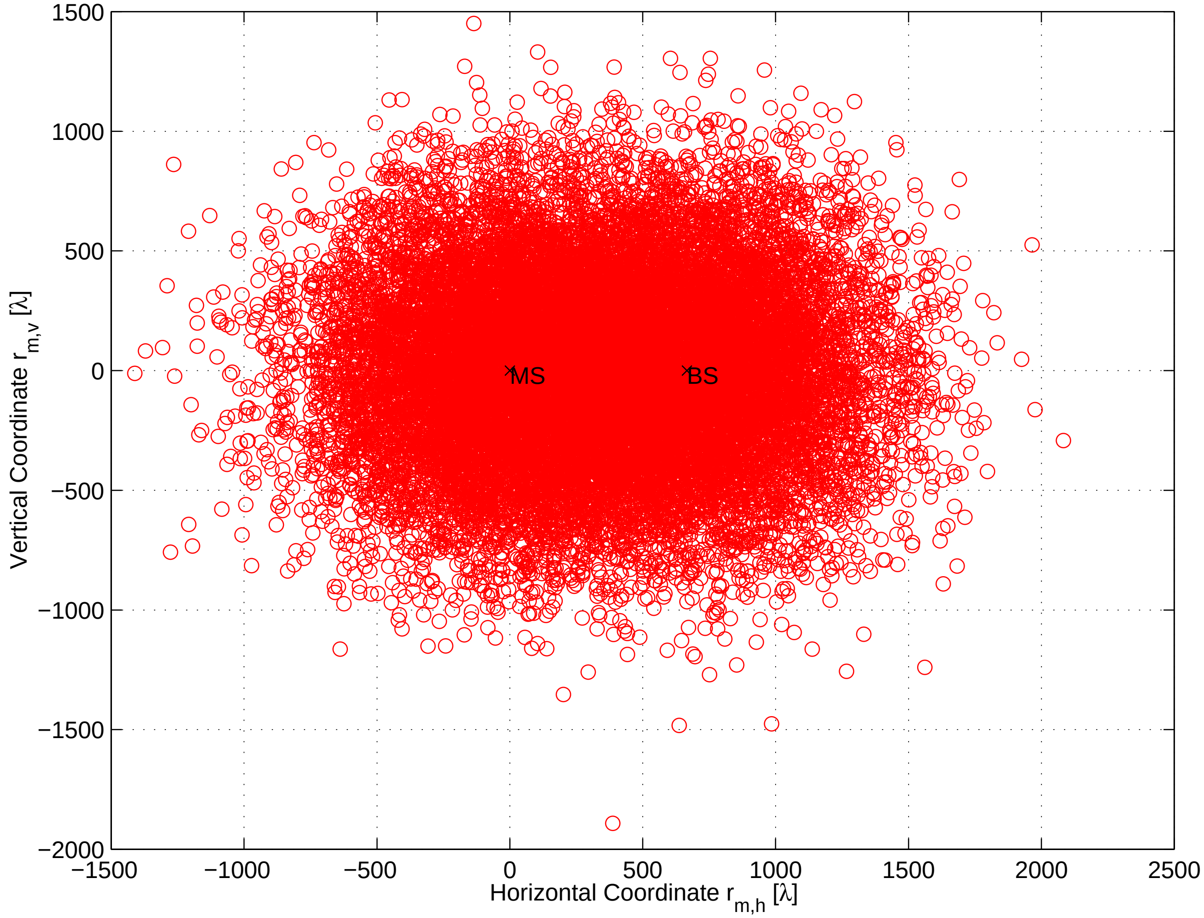
Microcellular propagation scenario

One- and two-bounce scatterers generated in one simulation run:



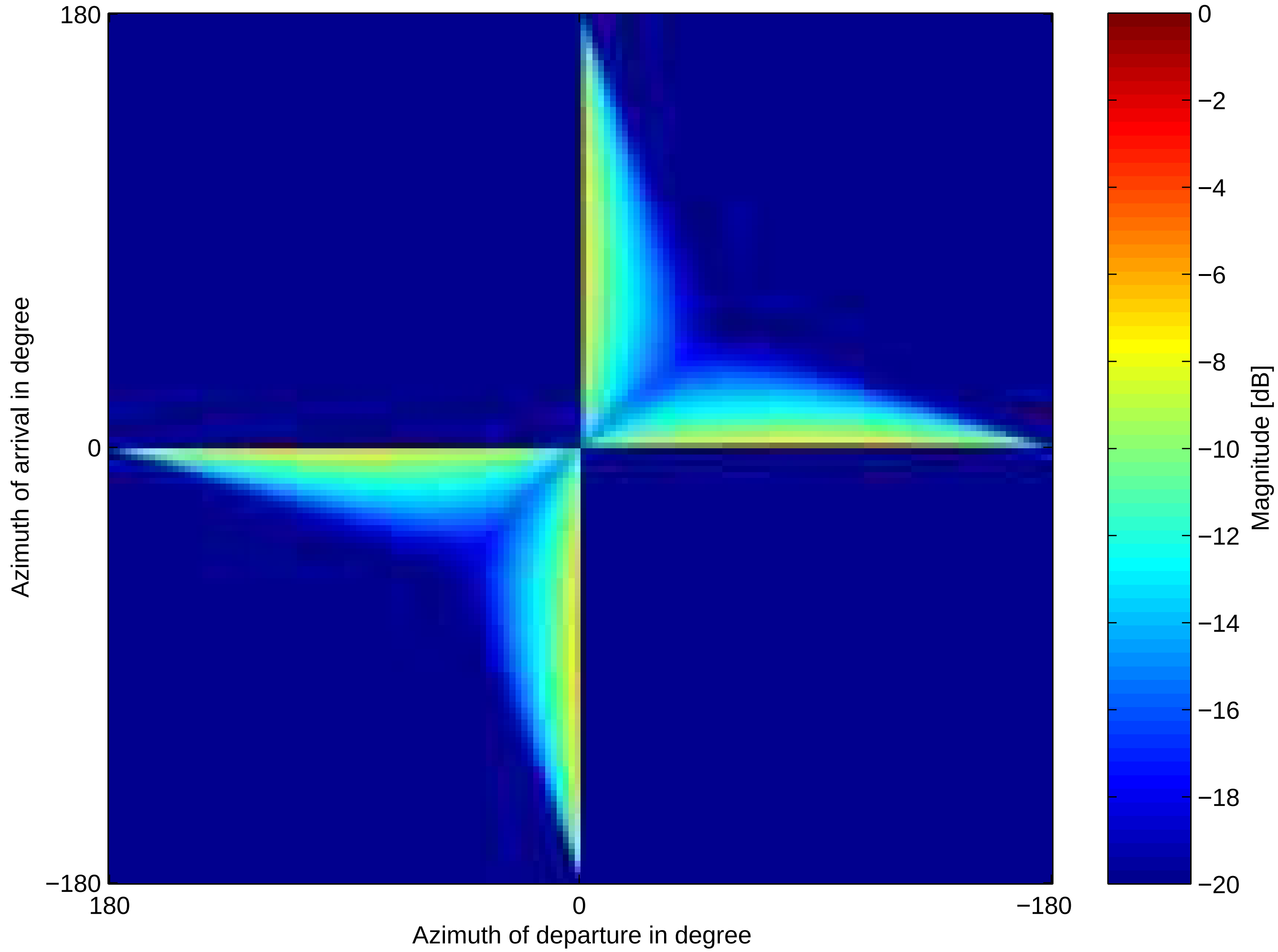
Microcellular propagation scenario

One- and two-bounce scatterers generated in all simulation runs:



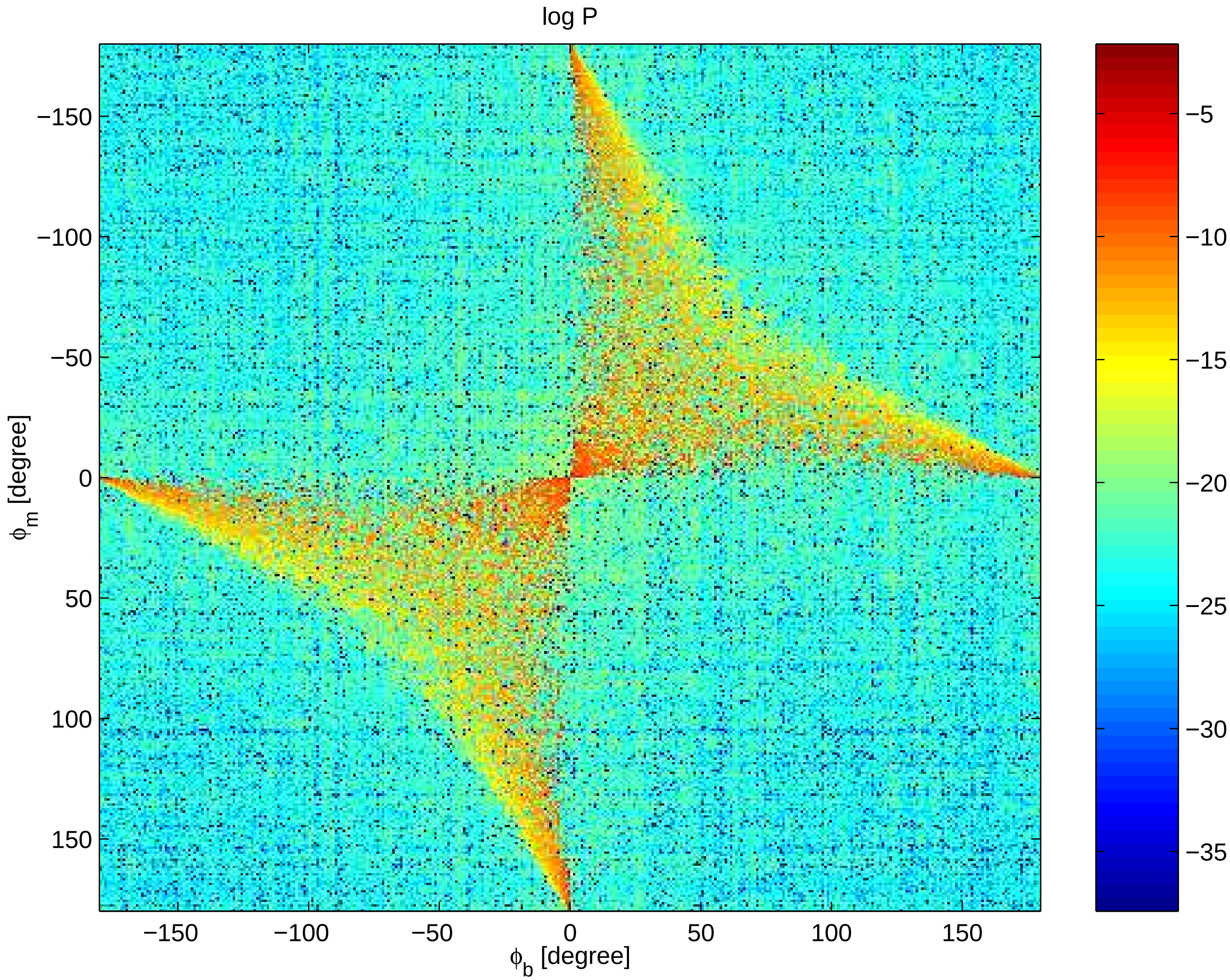
Simulated Biazimuth power spectrum $<|h(\phi_1, \phi_2)|^2>$

Only one-bounce scattering is considered.



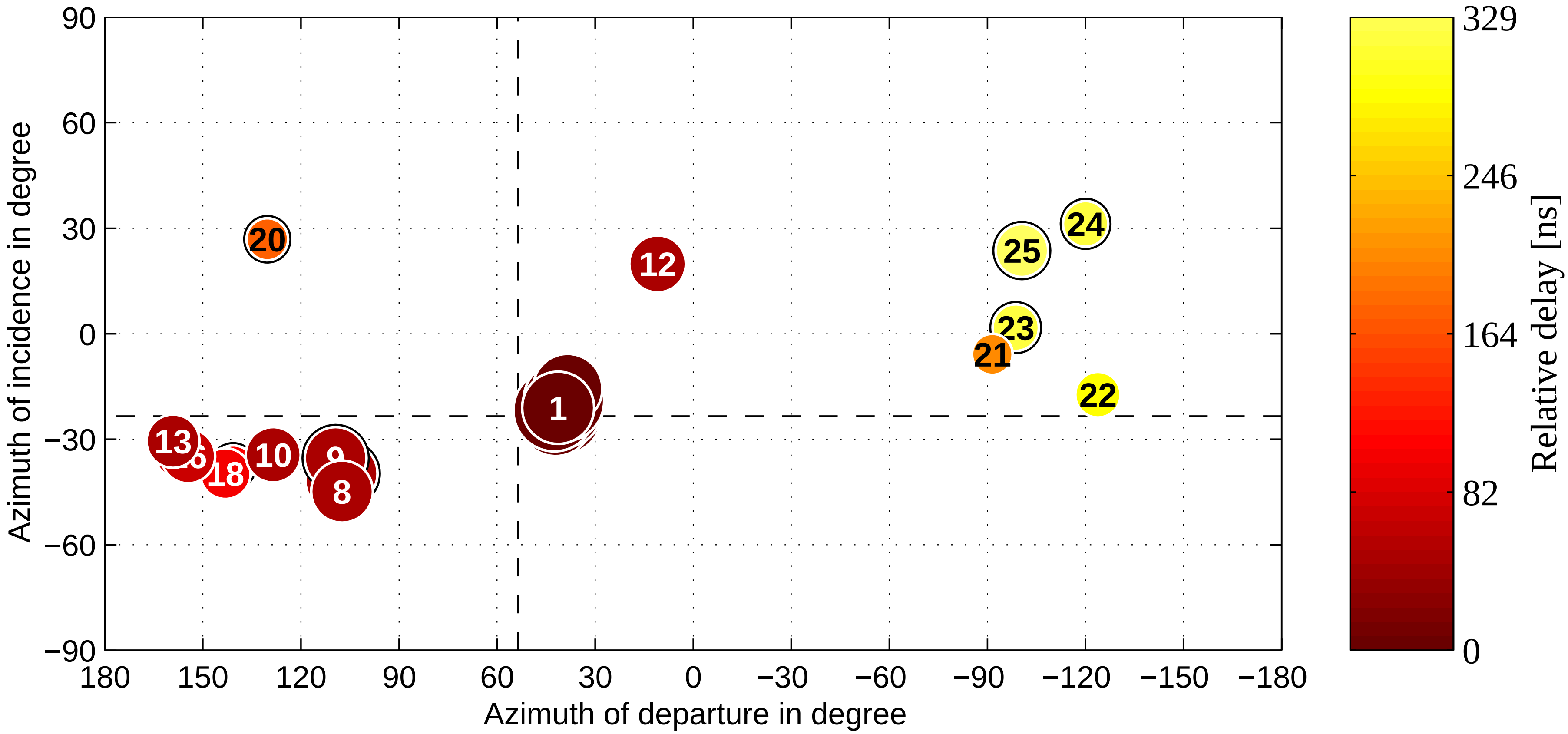
Simulated Biazimuth power spectrum $<|h(\phi_1, \phi_2)|^2>$

Both one- and two-bounce scattering are considered.



Measured Biazimuth Power Spectrum $<|h(\phi_1, \phi_2)|^2>$

In a none-line-of-sight (NLOS) scenario in an outdoor environment



Conclusions

- Relationship between the entries of the transfer matrix $\boldsymbol{H}$ of a MIMO system and the underlying propagation constellation. $\surd$

$$\boldsymbol{H} = \iint \boldsymbol{c}_1(\boldsymbol{\Omega}_1)^{\mathrm{T}} \otimes \boldsymbol{c}_2(\boldsymbol{\Omega}_2) h(\boldsymbol{\Omega}_1, \boldsymbol{\Omega}_2) \mathrm{d}\boldsymbol{\Omega}_1 \mathrm{d}\boldsymbol{\Omega}_2$$

- Relationship between the correlation properties of the entries of $\boldsymbol{H}$ and the underlying propagation constellation. $\surd$

$$\boldsymbol{R}_{\boldsymbol{H}} = \iint \left[\boldsymbol{c}_1(\boldsymbol{\Omega}_1)\boldsymbol{c}_1(\boldsymbol{\Omega}_1)^{\mathrm{H}}\right] \otimes \left[\boldsymbol{c}_2(\boldsymbol{\Omega}_2)\boldsymbol{c}_2(\boldsymbol{\Omega}_2)^{\mathrm{H}}\right] P(\boldsymbol{\Omega}_1, \boldsymbol{\Omega}_2) \mathrm{d}\boldsymbol{\Omega}_1 \mathrm{d}\boldsymbol{\Omega}_2.$$

- Explanation of the key-hole effect within this theory. $\surd$

$$h(\boldsymbol{\Omega}_1, \boldsymbol{\Omega}_2) = h_1(\boldsymbol{\Omega}_1) h_2(\boldsymbol{\Omega}_2)$$

- General *unifying* theory for characterizing MIMO systems. $\surd$