Lecture 5
Radio Channel Modeling Using Stochastic Propagation Graphs

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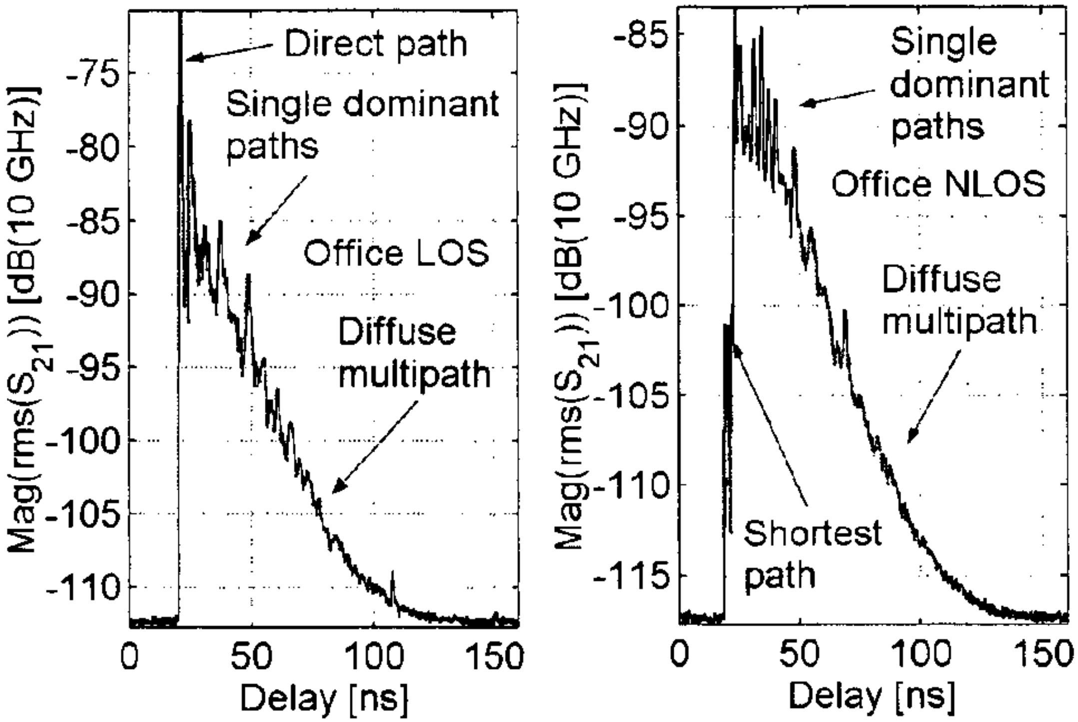
Motivation: Specular-to-Diffuse Transition

The specular-to-diffuse transition was noticed by Suzuki (1977) in an urban scenario and by Pamp&Kunisch (2002) in an indoor scenario.



Spatially averaged power delay profiles obtained from a line-of-sight scenario (left) and a non-line-of-sight scenario (right) [Pamp&Kunish2002].

- Not much attention has been paid to this transition effect.
- "Specular" and "diffuse" components are modeled as separate effects.
- The specular to diffuse transition appears to be "signature"-like pattern that is of importance for e.g. indoor positioning.

Philosophy, Goals, and Method

Philosophy:

- Model the *environment* and the *propagation mechanisms* instead of the *response* of the environment

Goals:

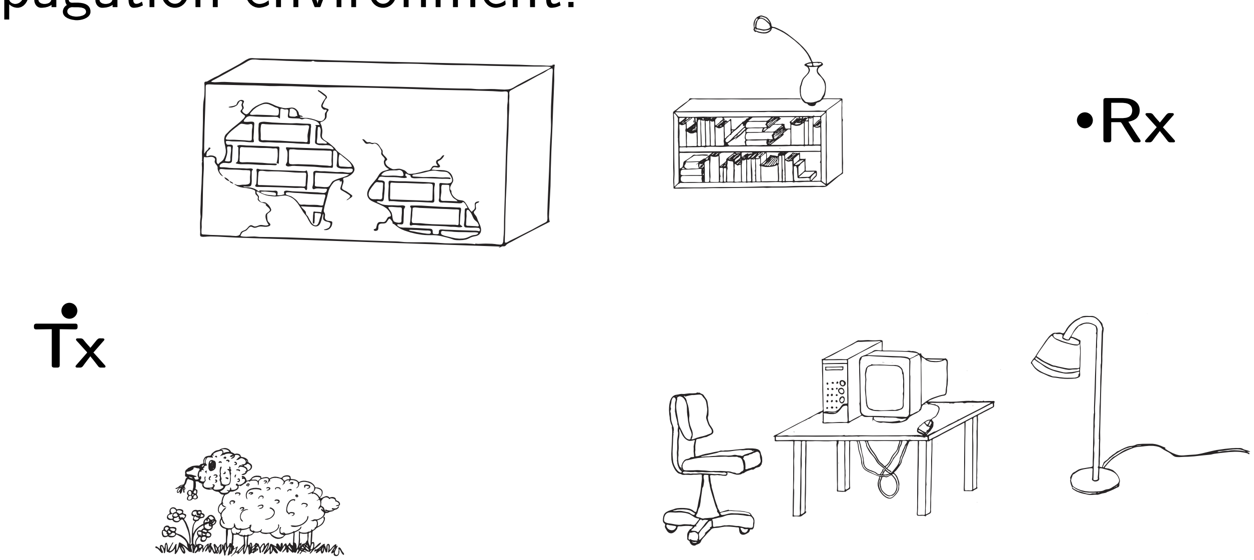
- The obtained response should exhibit an exponential power decay.
- A joint description of specular and diffuse signal components.
- Relation between the features of power delay profile and the propagation environment.

Method:

- Model a cluttered environment
- Model the propagation mechanisms in the environment
- Compute the response

Model of the Propagation Environment

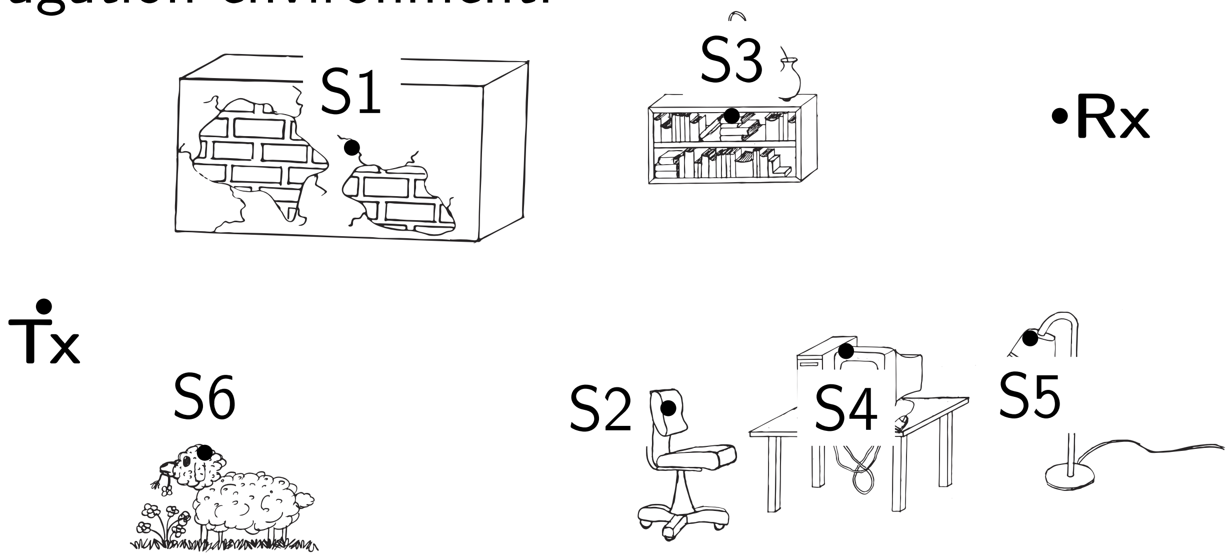
A “typical” propagation environment:



(The propagation environment is static.)

Model of the Propagation Environment

A "typical" propagation environment:

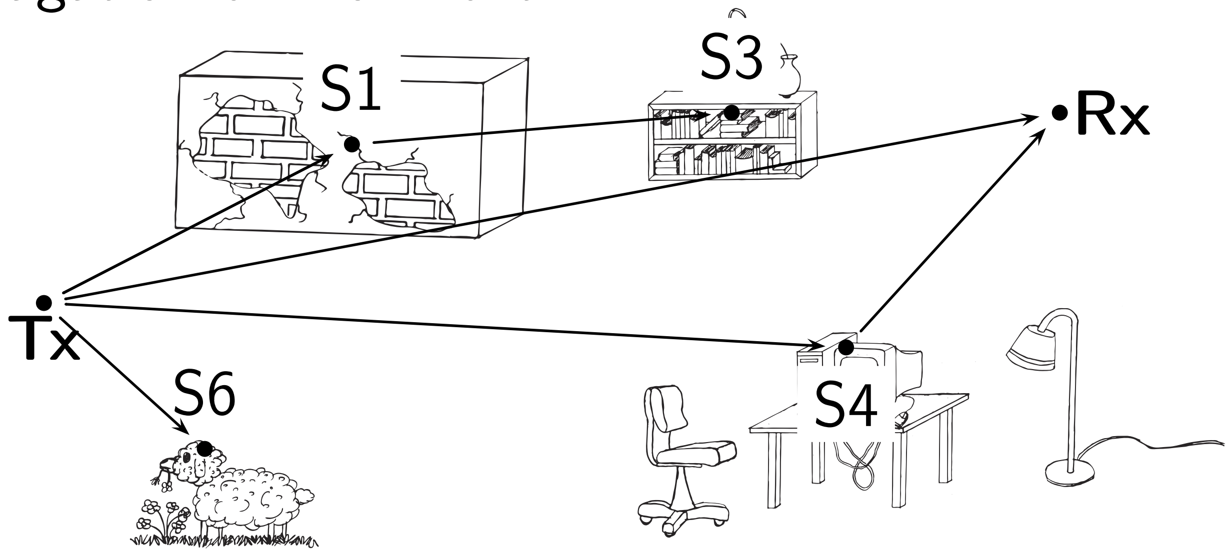


(The propagation environment is static.)

- We model scatterers as the vertices of a signal flow-graph.

Model of the Propagation Environment

A “typical” propagation environment:



(The propagation environment is static.)

- We model scatterers as the vertices of a signal flow-graph.
- The wave propagation between scatterers is modelled by the edges of the graph.

Modeling Propagation Using Graphs (1)

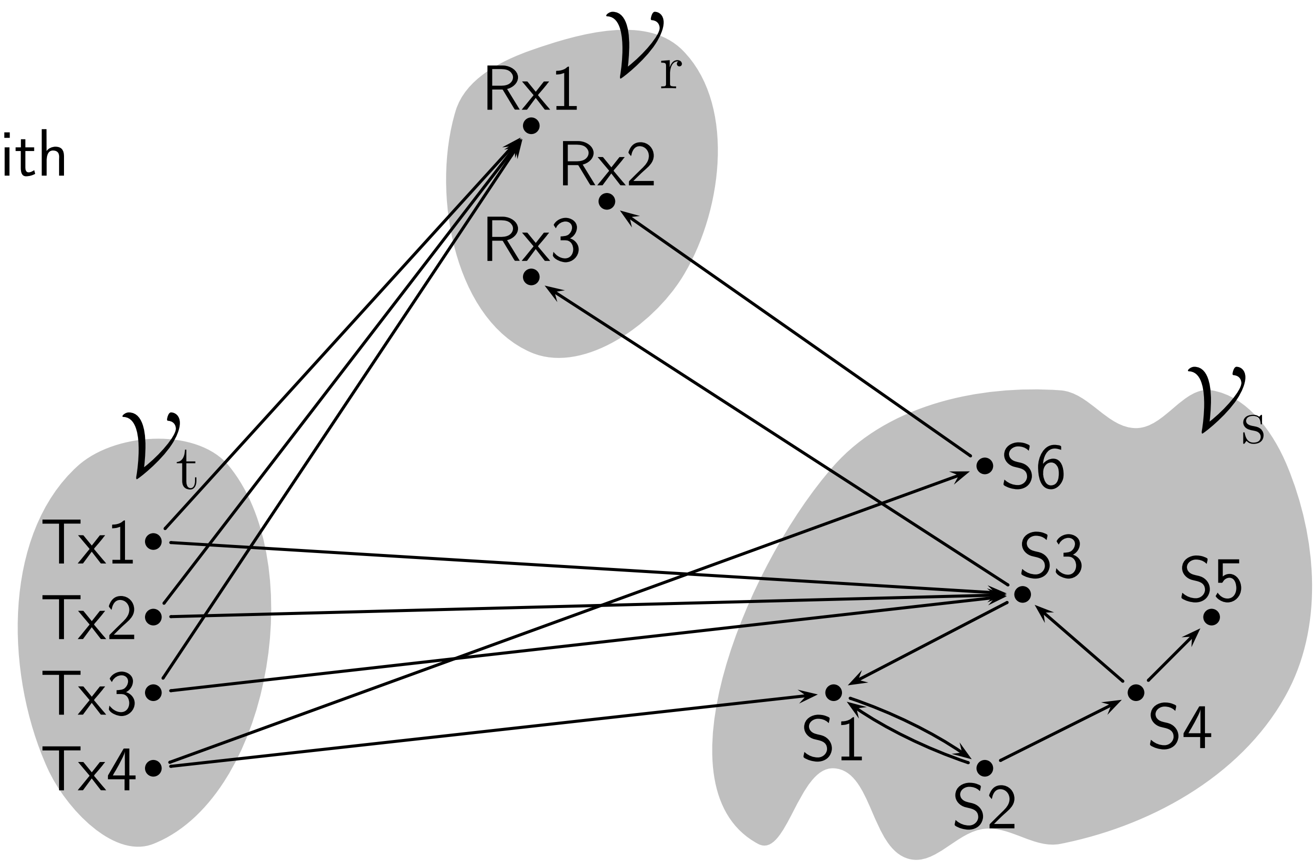
Notations:

We consider a simple directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$.

Vertex set $\mathcal{V}$: The transmitters, receivers, and scatterers are represented by vertices in the set: $\mathcal{V} = \mathcal{V}_{\mathrm{t}} \cup \mathcal{V}_{\mathrm{r}} \cup \mathcal{V}_{\mathrm{s}}$.

Edge set $\mathcal{E}$: Wave propagation between the vertices is modeled by edges in $\mathcal{E}$. Iff wave propagation from $v \in \mathcal{V}$ to $v' \in \mathcal{V}$ is possible, then $(v, v') \in \mathcal{E}$.

A propagation graph with four transmitters (Tx), three receivers (Rx), and six scatterers (S).



Modeling Propagation Using Graphs (2)

Signal propagation in the graph:

- The sum of signals impinging via the incoming edges of a scatterer are re-emitted via the outgoing edges.
- An edge $(v, v') \in \mathcal{E}$ transfers the signal from v to v' according to its transfer function

$$A_{(v,v')}(f) = \begin{cases} g_{(v,v')} \cdot \exp(-j2\pi\tau_{(v,v')}f), & (v,v') \in \mathcal{E} \\ 0 & \text{otherwise.} \end{cases}$$

$$\tau_{(v,v')} = \frac{|\mathbf{r}_v - \mathbf{r}_{v'}|}{c}, \qquad |g_{(v,v')}|^2 = \left(\frac{1}{1 + |\mathbf{r}_v - \mathbf{r}_{v'}|}\right)^2 \cdot \frac{|g|^2}{\mathrm{outdegree}(v)},$$

where

- we have assigned a position vector $\mathbf{r}_v \in \mathbb{R}^3$ to vertex v,
- $|g| < 1$ is a constant gain,
- $\mathrm{outdegree}(v)$ is the number of outgoing edges of vertex v, and
- c is the speed of light in vacuum.

Power Constraint

Check that the output power (at the "output" of the outgoing edges) is less than the power of the input signal $X_v(f)$:

$$\sum_{e \in \mathcal{E}_v} |g_e X_v(f)|^2 < |X_v(f)|^2 \Leftrightarrow \sum_{e \in \mathcal{E}_v} |g_e|^2 < 1$$

where $\mathcal{E}_v$ is the set of outgoing edges of vertex v.

It suffices to consider the case where $|\mathcal{E}_v| = \mathrm{outdegree}(v) \geq 1$. We upper bound $|g_e|^2$ as

$$|g_e|^2 = \left(\frac{1}{1 + |\mathbf{r}_v - \mathbf{r}_{v'}|}\right)^2 \cdot \frac{|g|^2}{\mathrm{outdegree}(v)} \leq \frac{|g|^2}{\mathrm{outdegree}(v)}$$

Since $|\mathcal{E}_v| = \mathrm{outdegree}(v)$ we obtain

$$\sum_{e \in \mathcal{E}_v} |g_e|^2 \leq \sum_{e \in \mathcal{E}_v} \frac{|g|^2}{\mathrm{outdegree}(v)} = |\mathcal{E}_v| \frac{|g|^2}{\mathrm{outdegree}(v)} = |g|^2 < 1.$$

Response of a Propagation Graph (1)

Relation between the input signal vector $\mathbf{X}(f)$ and the output signal vector $\mathbf{Y}(f)$ in the Fourier domain:

$$\mathbf{Y}(f) = \mathbf{H}(f)\mathbf{X}(f)$$

In the following we derive an expression for the transfer matrix $\mathbf{H}(f)$

(Four slides of math will follow. Sorry!)

Response of a Propagation Graph (2)

Define the state vector:

$$\mathbf{C}(f) = \begin{bmatrix} \mathbf{X}(f) \\ \mathbf{Y}(f) \\ \mathbf{Z}(f) \end{bmatrix}$$

where $\mathbf{Z}(f)$ is the vector of signals observed at the scatterers.

Decompose $\mathbf{C}(f)$ according to the number of edges k the signals have traversed:

$$\mathbf{C}(f) = \sum_{k=0}^{\infty} \mathbf{C}_k(f) = \sum_{k=0}^{\infty} \begin{bmatrix} \mathbf{X}_k(f) \\ \mathbf{Y}_k(f) \\ \mathbf{Z}_k(f) \end{bmatrix}$$

Obviously $\mathbf{X}_k(f) = \begin{cases} \mathbf{X}(f), & k = 0 \\ \mathbf{0}, & \text{otherwise.} \end{cases}$

Response of a Propagation Graph (3)

We have the following recursive equation

$$\mathbf{C}_0(f) = [\mathbf{X}(f)^{\mathsf{t}}, \mathbf{0}^{\mathsf{t}}, \mathbf{0}^{\mathsf{t}}]^{\mathsf{t}}$$
$$\mathbf{C}_{k+1}(f) = \mathbf{A}(f)\mathbf{C}_k(f), \quad k \geq 0$$

where $\mathbf{A}(f)$ is the weighted adjacency matrix of the graph:

$$[\mathbf{A}(f)]_{nn'} = \begin{cases} A_{(v_n, v_{n'})}(f), & (v_n, v_{n'}) \in \mathcal{E} \\ 0, & \text{otherwise} \end{cases}$$

By appropriate vertex indexing:

$$\mathbf{A}(f) = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{D}(f) & \mathbf{0} & \mathbf{R}(f) \\ \mathbf{T}(f) & \mathbf{0} & \mathbf{B}(f) \end{bmatrix}$$

$$\begin{aligned} \mathbf{D}(f) &: \quad \text{transmitters} \rightarrow \text{receivers} \\ \mathbf{R}(f) &: \quad \text{scatterers} \rightarrow \text{receivers} \\ \mathbf{T}(f) &: \quad \text{transmitters} \rightarrow \text{scatterers} \\ \mathbf{B}(f) &: \quad \text{scatterers} \rightarrow \text{scatterers.} \end{aligned}$$

Response of a Propagation Graph (4)

Obviously

$$\mathbf{Y}_1(f) = \mathbf{D}(f)\mathbf{X}(f).$$

By inspection of the series $\mathbf{A}^2(f), \mathbf{A}^3(f), \ldots$ we see

$$\mathbf{A}^k(f) = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{R}(f)\mathbf{B}^{k-2}(f)\mathbf{T}(f) & \mathbf{0} & \mathbf{R}(f)\mathbf{B}^{k-1}(f) \\ \mathbf{B}^{k-1}(f)\mathbf{T}(f) & \mathbf{0} & \mathbf{B}^k(f) \end{bmatrix}, \ k \geq 2.$$

Thus

$$\mathbf{Y}_k(f) = \mathbf{R}(f)\mathbf{B}^{k-2}(f)\mathbf{T}(f)\mathbf{X}(f), k \geq 2$$

Response of a Propagation Graph (5)

Summing up signal contributions we obtain:

$$\begin{aligned}\mathbf{Y}(f) &= \mathbf{D}(f)\mathbf{X}(f) + \sum_{k=2}^{\infty} \mathbf{R}(f)\mathbf{B}^{k-2}(f)\mathbf{T}(f)\mathbf{X}(f) \\ &= \left[\mathbf{D}(f) + \sum_{k'=0}^{\infty} \mathbf{R}(f)\mathbf{B}^{k'}(f)\mathbf{T}(f)\right]\mathbf{X}(f) \\ &= \underbrace{\left[\mathbf{D}(f) + \mathbf{R}(f)(\mathbf{I} - \mathbf{B}(f))^{-1}\mathbf{T}(f)\right]}_{\mathbf{H}(f)}\mathbf{X}(f).\end{aligned}$$

The sum converges due to the power constraint (we omit the proof here).

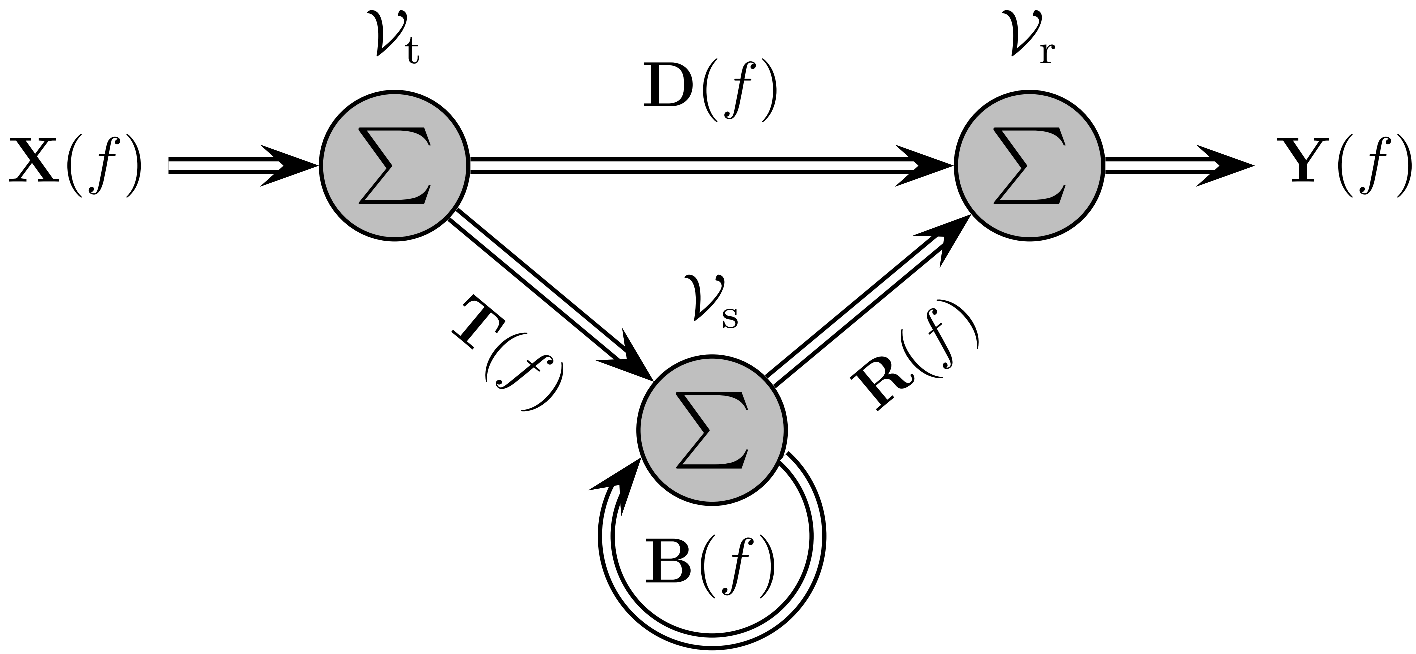
Transfer Matrix of a Propagation Graph

The relation between the input vector $\mathbf{X}(f)$ and the output vector $\mathbf{Y}(f)$ in the Fourier domain reads

$$\mathbf{Y}(f) = \mathbf{H}(f)\mathbf{X}(f),$$

where the transfer matrix $\mathbf{H}(f)$ is of the form [Pedersen&Fleury 2007]

$$\begin{aligned}\mathbf{H}(f) &= \mathbf{D}(f) + \mathbf{R}(f)(\mathbf{I} + \mathbf{B}(f) + \mathbf{B}(f)^2 + \mathbf{B}(f)^3 + \dots)\mathbf{T}(f) \\ &= \mathbf{D}(f) + \mathbf{R}(f)(\mathbf{I} - \mathbf{B}(f))^{-1}\mathbf{T}(f).\end{aligned}$$



How to Generate a Propagation Graph

A propagation graph can be obtained in different ways:

- From a deterministic environment (e.g. by ray-tracing).
- Generate a random environment (scatter locations and weights) and calculate visibilities.
- By randomly generating the vertices and the edges of the graph.

We focus on the third option.

Example:

1. Assume fixed $\mathbf{r}_{\mathsf{Tx}}$ and $\mathbf{r}_{\mathsf{Rx}}$.
2. Generate the scatterer positions according to a point process in a region $\mathcal{R} \subset \mathbb{R}^3$.
3. Generate the edges $(v, v') \in \mathcal{V}^2$ from a Bernoulli experiment with edge probability $P_{(v,v')}$

Stochastic Propagation Graphs

1. Fix the coordinates of the transmitters and receivers.
2. Draw the positions of N of scatterers according to a uniform distribution defined a solid volume $\mathcal{R}$.
3. Generate edges according to the edge occurrence probability:

$$\Pr((v,v') \in \mathcal{E}) = \begin{cases} P_{\mathrm{dir}} & \text{if } (v,v') = (\mathsf{Tx}, \mathsf{Rx}) \\ 0 & \text{if } v = v' \\ 0 & \text{if } v = \mathsf{Rx} \\ 0 & \text{if } v' = \mathsf{Tx} \\ P_{\mathsf{vis}} & \text{otherwise} \end{cases}$$

4. Compute $\mathbf{H}(f_{\min})$, $\mathbf{H}(f_{\min} + \Delta f), \ldots, \mathbf{H}(f_{\max})$.
5. Compute the channel impulse responses using the inverse discrete Fourier transform applying a Hanning window.

Simulation Scenario

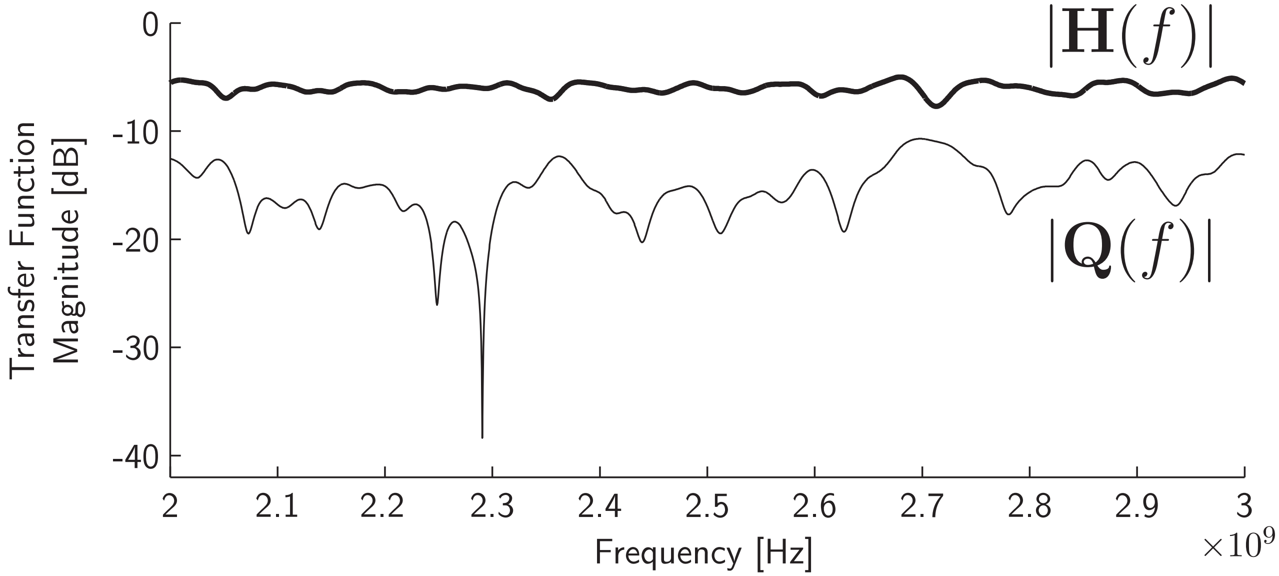
We consider a single-input single-output system and simulate:

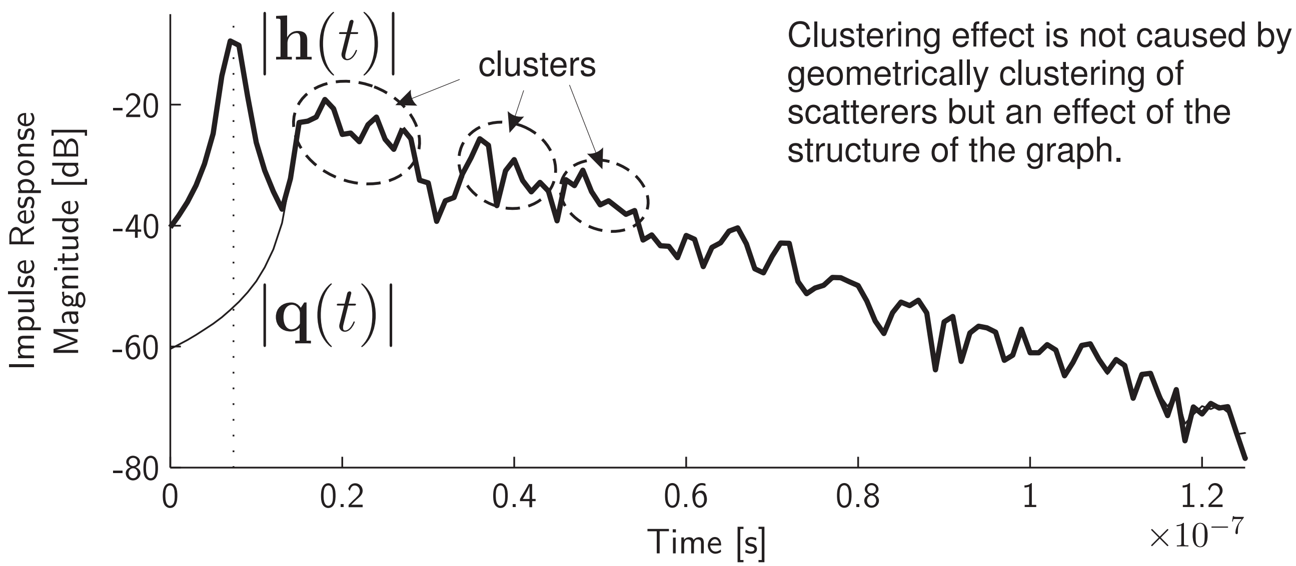
- $H(f)$ and its inverse Fourier transform $h(t)$ and
- the indirect term $Q(f) \triangleq \mathbf{R}(f)(\mathbf{I} - \mathbf{B}(f))^{-1}\mathbf{T}(f)$ and its inverse Fourier transform $q(t)$.

Parameters	Values
$\mathcal{R}$	$[0, 5] \times [0, 10] \times [0, 3.5]\,\mathrm{m}^3$
$\mathbf{r}_{\mathrm{Tx}}$	$[1.8,\ 2.0,\ 0.5]^{\mathrm{T}}\,\mathrm{m}$
$\mathbf{r}_{\mathrm{Rx}}$	$[1.0,\ 4.0,\ 1.0]^{\mathrm{T}}\,\mathrm{m}$
Number of scatterers	20
g	0.8
P_{vis}	0.8
P_{dir}	1
Δf	$0.5\,\mathrm{GHz}$

An Example Transfer Function

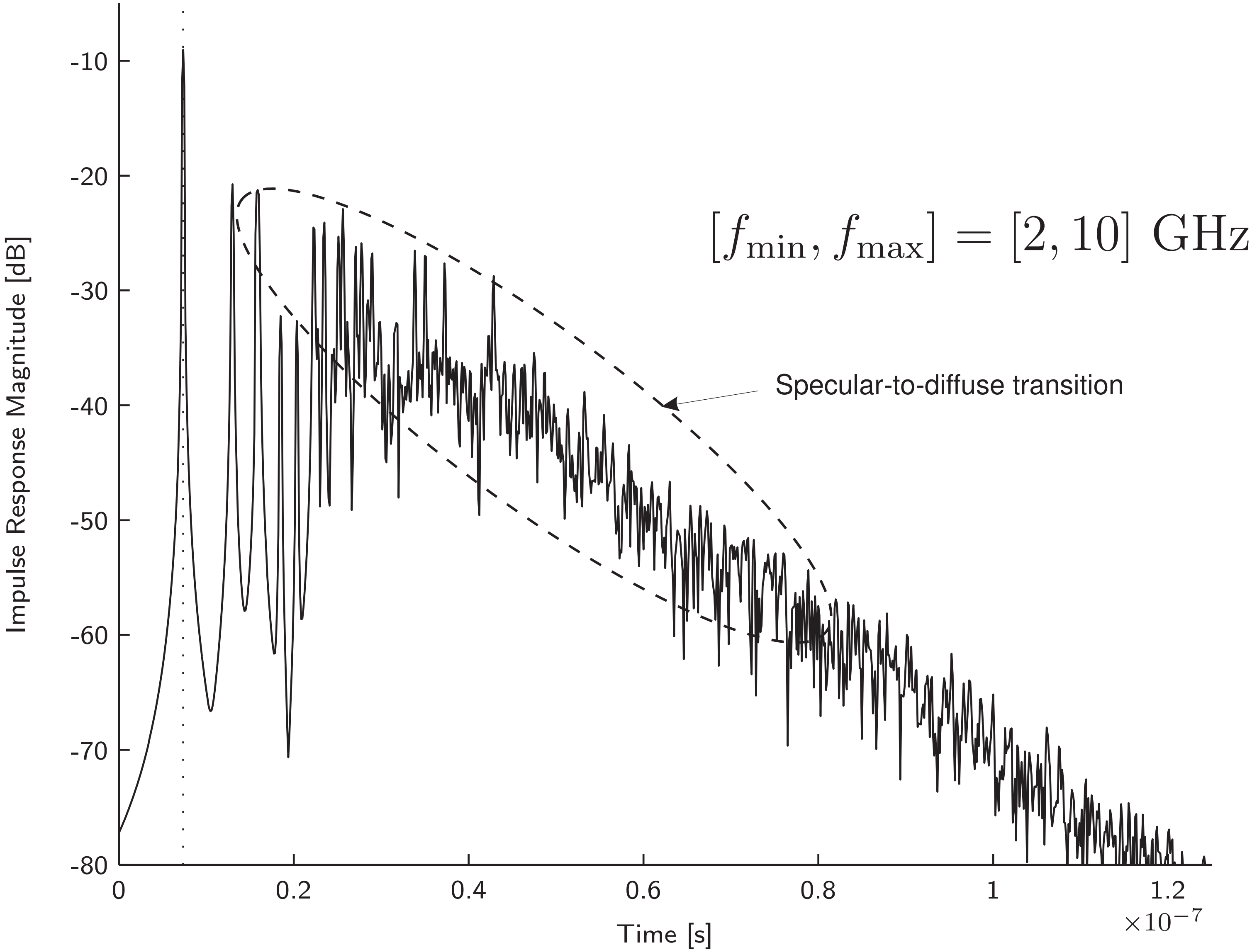
Transfer function (thick line) and indirect term (thin line).





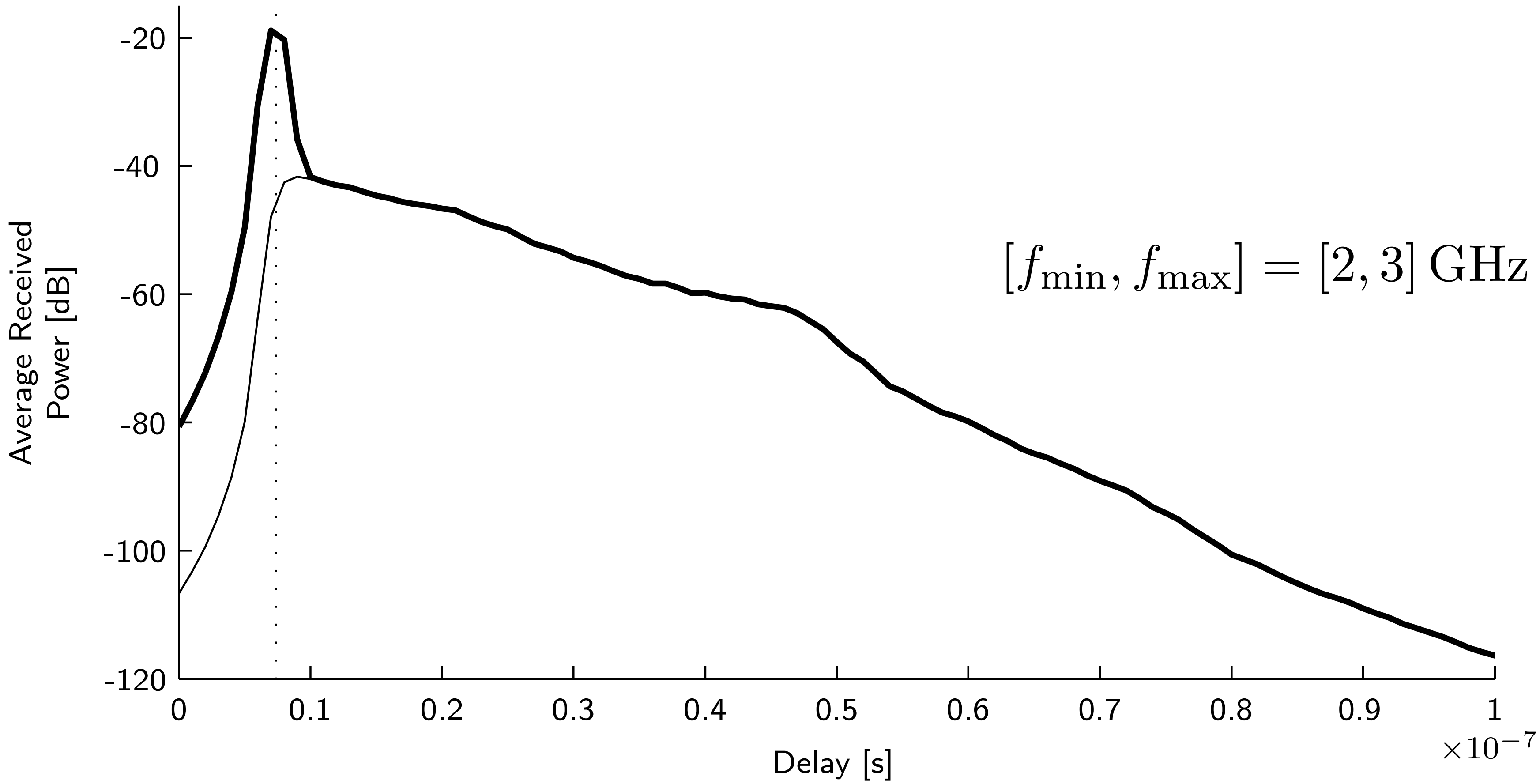
Specular-to-Diffuse Transition

An example of $|h(t)|$.



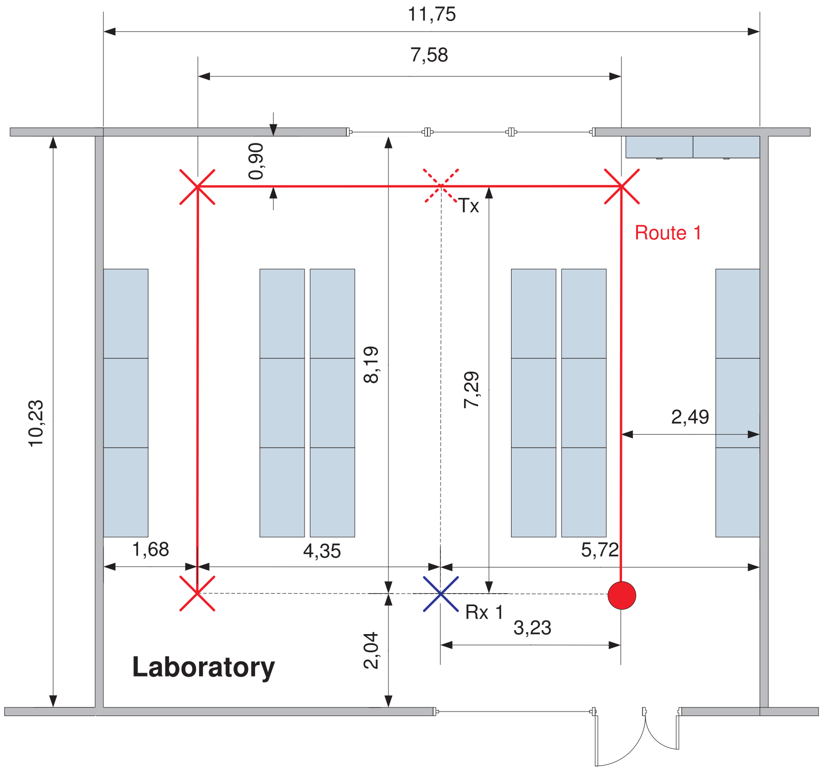
Estimated Delay-Power Spectrum

Delay-power spectrum estimate computed from 1000 realisations of $|h(t)|$ (thick line) and $|q(t)|$ (thin line).



Experimental Investigation (1)

- Carrier frequency: 5.25 GHz
- Code length : 255 chips
- Chip rate: 100 Mchip/s
- Sampling frequency: 200 MHz
- Laboratory environment

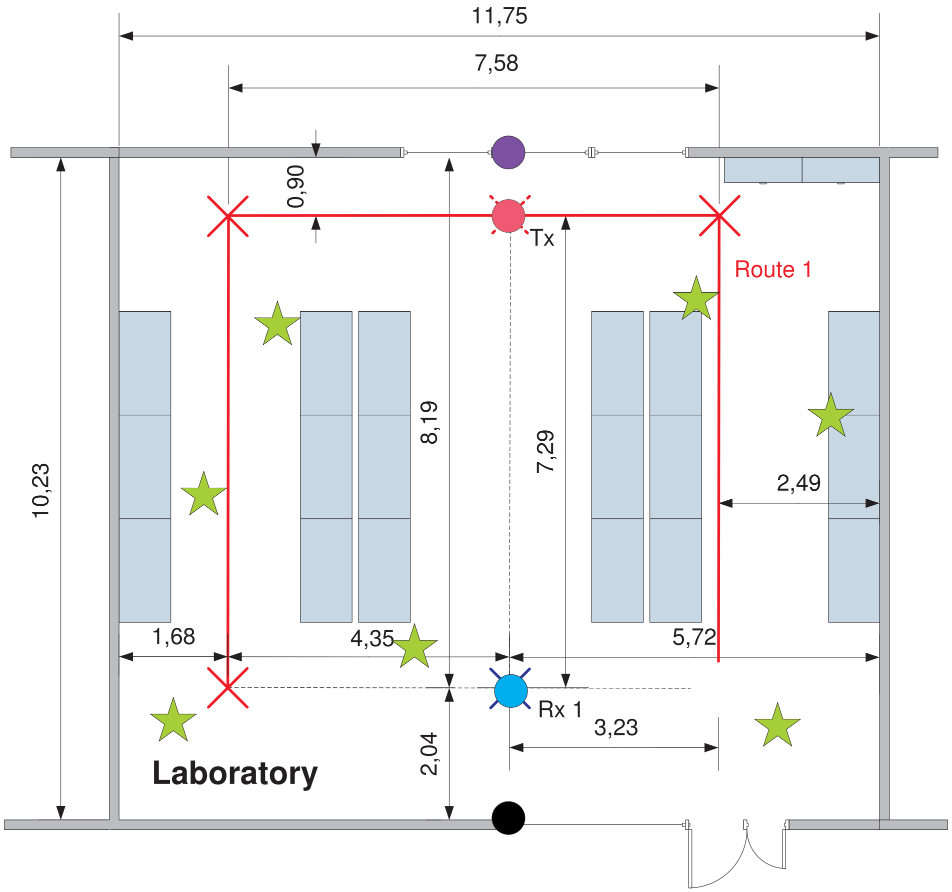


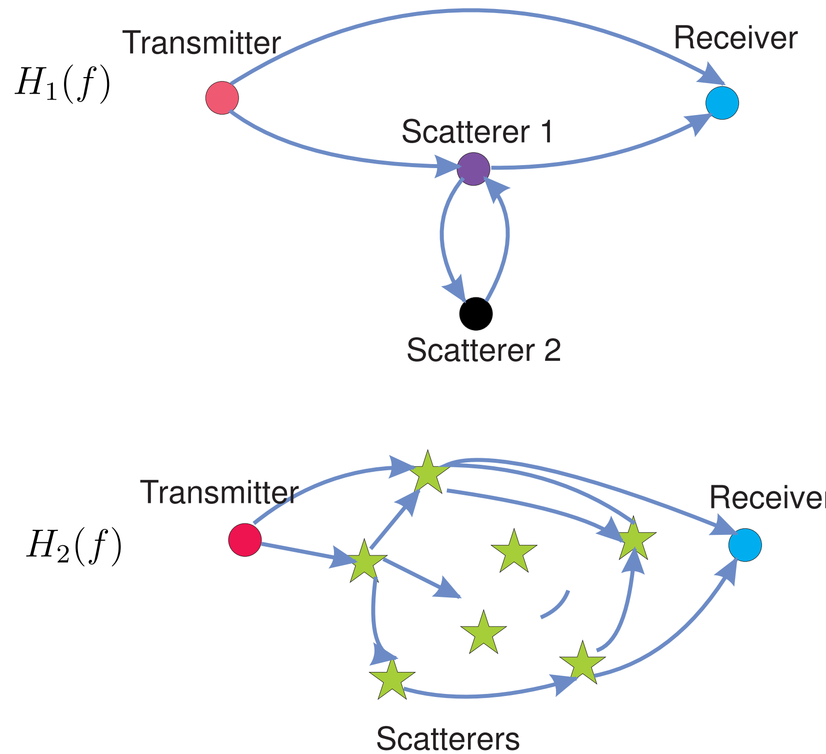
Tx Environment

Rx Environment

Experimental Investigation (2)

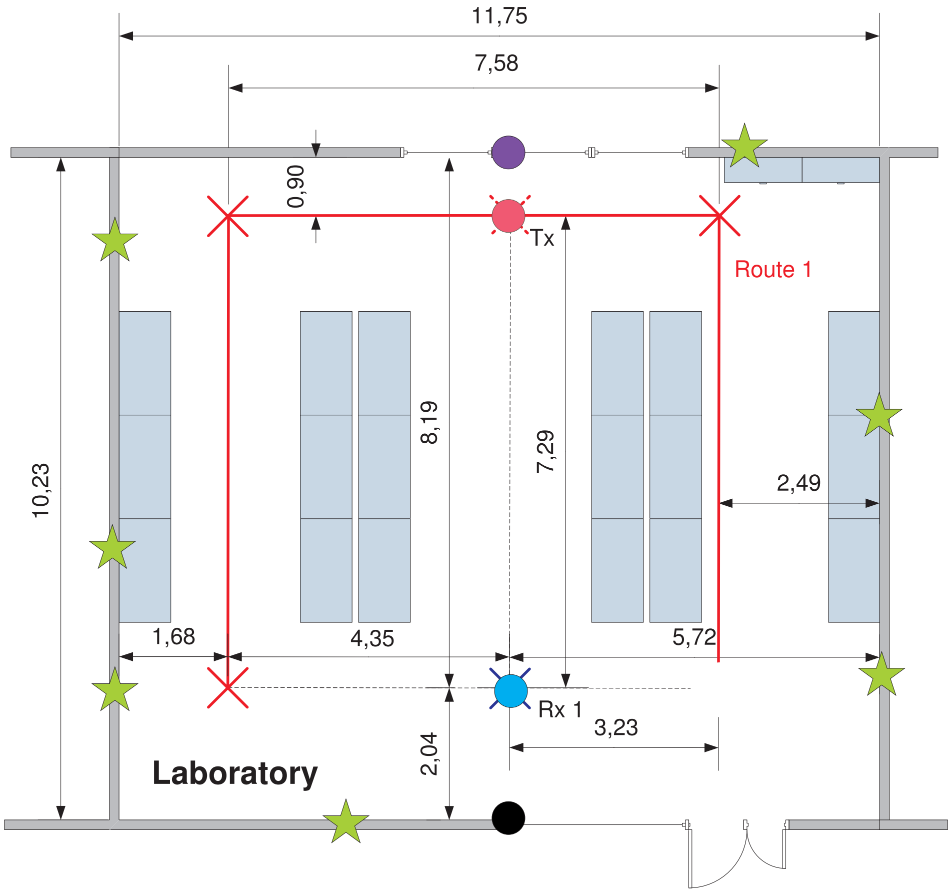
Construct propagation graphs based on the environment:
Transfer function: $H(f) = H_1(f) + H_2(f)$
Channel response: $h(t) = \mathcal{F}^{-1} H(f)$
Graph 1:

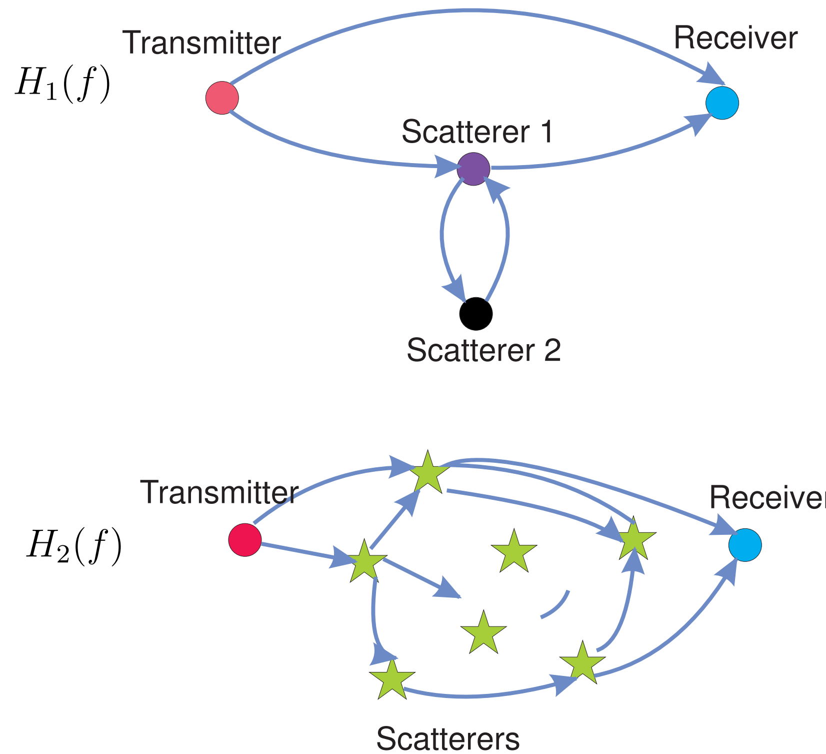




Experimental Investigation (3)

Construct propagation graphs based on the environment:
Transfer function: $H(f) = H_1(f) + H_2(f)$
Channel response: $h(t) = \mathcal{F}^{-1} H(f)$
Graph 2:



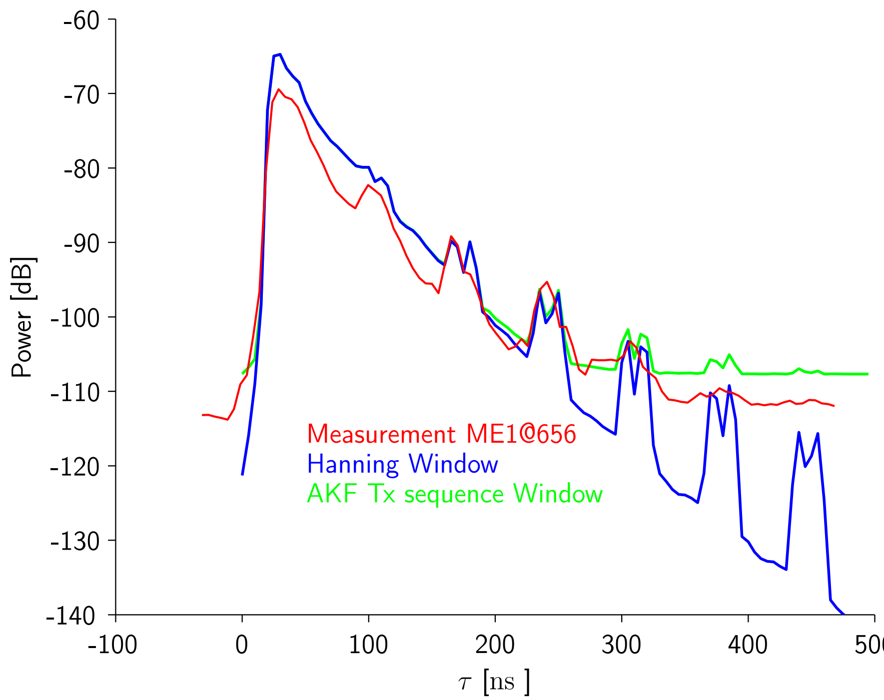


Experimental Investigation (4)

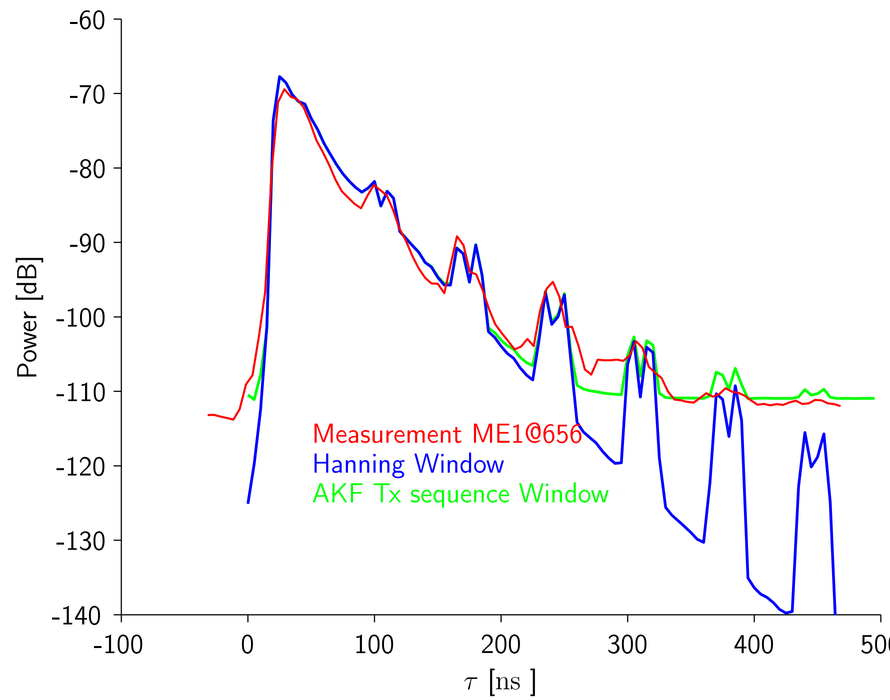
Comparison of the power delay profiles obtained with graphs:

Power spectral height is calculated by averaging 500 Monte-Carlo runs.

Graph 1



Graph 2

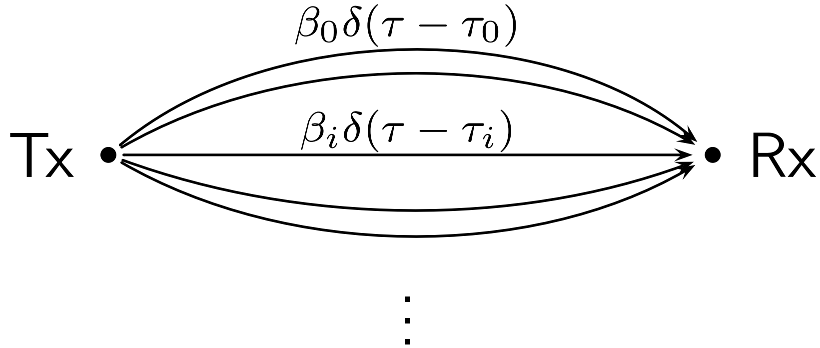


Discussions

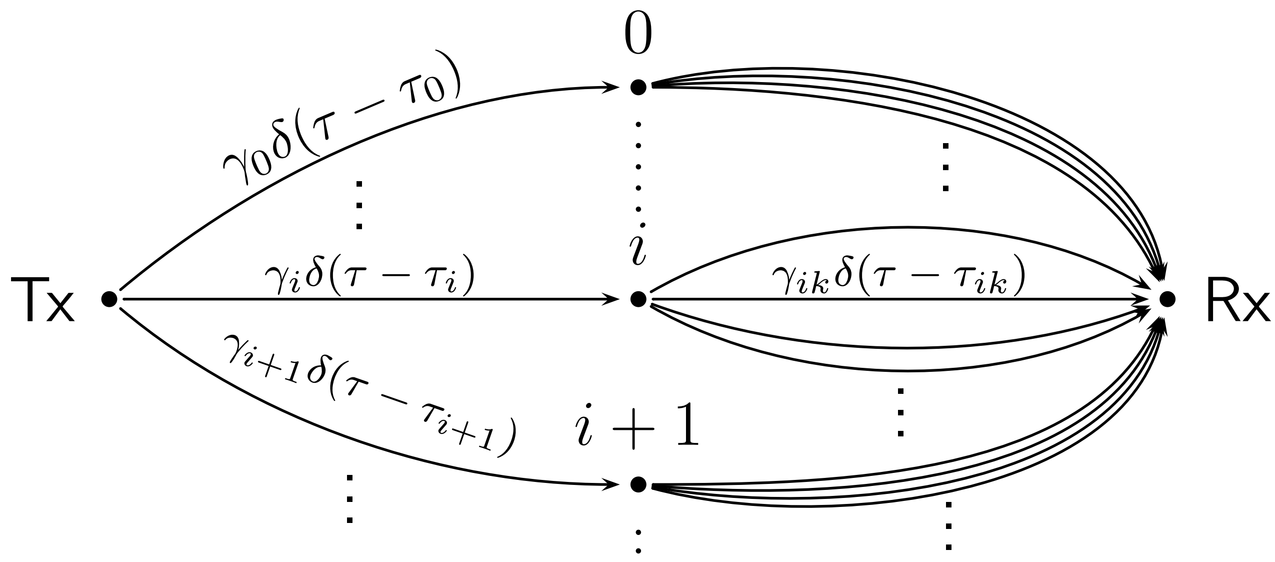
- Propagation graphs can be used to generate propagation path parameters for geometric based generic models.
- The gain, delay, direction of departure and arrival of a path can be generated.
- Non-stationary channels can be modelled by including movement of vertices and changing visibilities.
- Multiple and transmitter and receiver vertices can be included to accommodate multiuser and/or MIMO systems.
- Frequency-dependent scatterers can be included (UWB models).

Graph Representation of Existing Models

- The Turin model: $h(\tau) = \sum_{i=0}^{\infty} \beta_i \delta(\tau - \tau_i)$



- The Saleh-Valenzuela model: $h(t) = \sum_{i=0}^{\infty} \gamma_i \sum_{k=0}^{\infty} \gamma_{ik} \delta(\tau - (\tau_i + \tau_{ik}))$



Concluding Remarks

- A **graph-based radio channel model** was proposed.
- The model is described in [Pedersen&Fleury 2006] and a **closed form expression for the transfer matrix** is derived in [Pedersen&Fleury 2007].
- As an effect of the **recursive structure** of the model, the obtained impulse responses exhibit
  - a **transition** from **early specular** to **later diffuse** contributions, and
  - an **exponential** power decay.
- The propagation graph model can be easily extended to include dispersion in directions of departure and arrival.

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