Lecture 4
Characterization and estimation of dispersed components in the channel response

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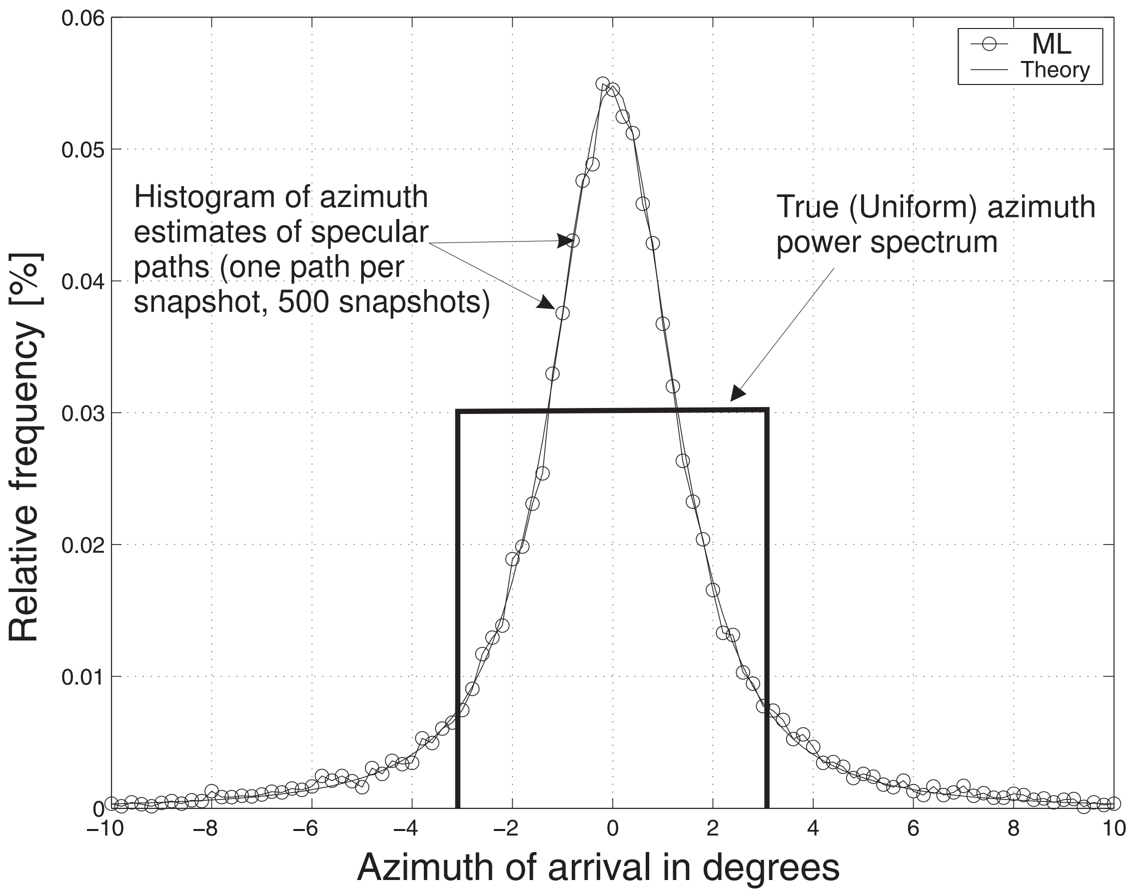
Estimation of Path Dispersion

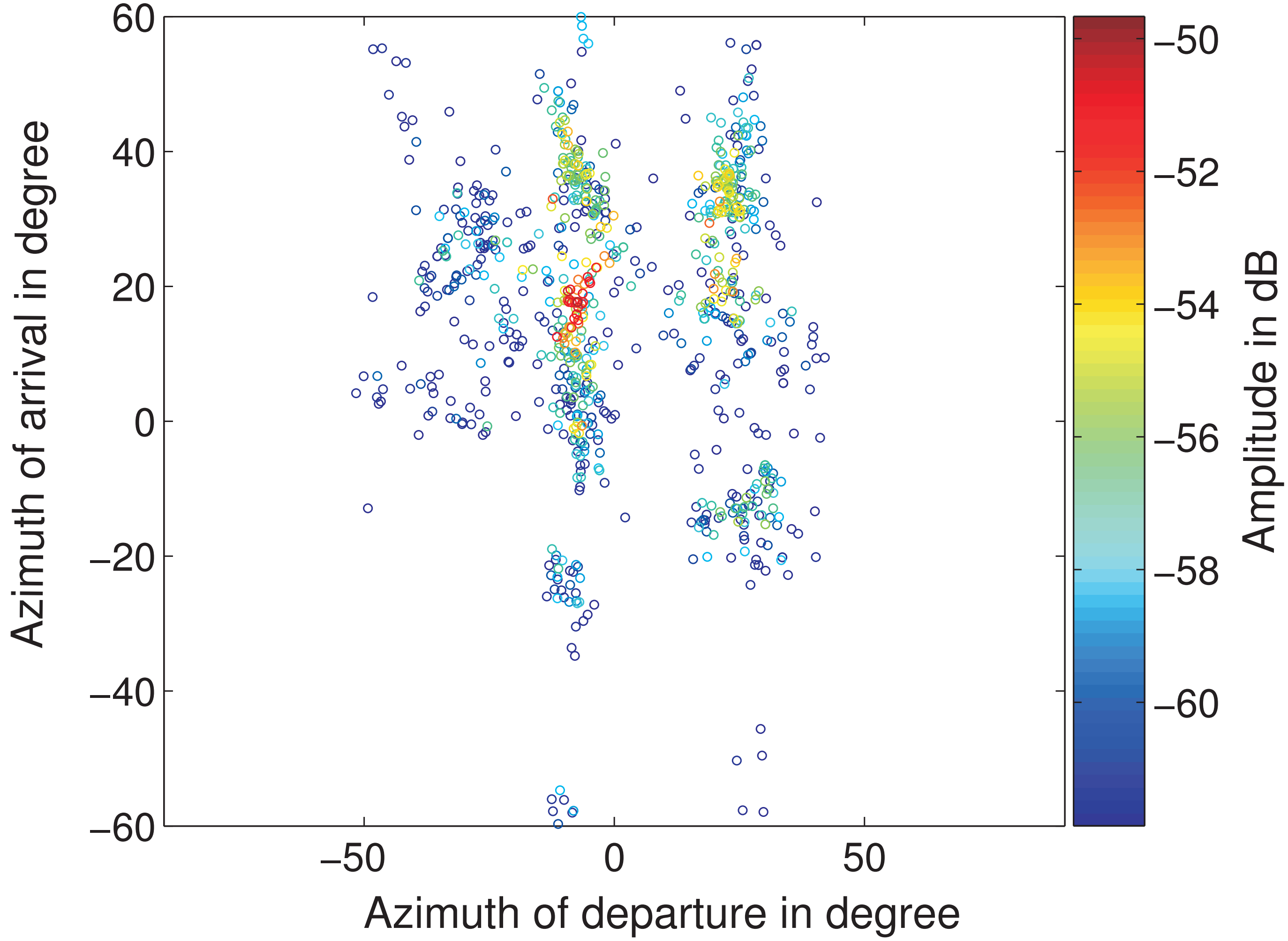
- Beamformers, such as Bartlett and Capon beamformers
- Clustering of the estimated dispersion parameters of specular components
- Estimate the parameters of (simple) parametric models characterizing path dispersion

Performance of specular-based estimators in dispersive scenarios



An Experimental Example of Specular Path Estimates in an Indoor Environment

- Wideband channel sounder RUSK ATM
- A SAGE algorithm is used to estimate multiple specular paths.



Dispersion of Individual Components

- Probability density functions (pdfs) are used to characterize the power spectral density function (psdf) of individual components:
  - Azimuth: Von-Mises pdf
  - Azimuth-elevation: Fisher-Bingham-5 pdf
  - Azimuth-elevation-delay: Von-Mises-Fisher pdf
  - Azimuth of arrival - azimuth of departure - delay: Von-Mises-Fisher pdf
- These pdfs maximize the entropy under the constraint that
  - their first moments (center of gravity) are fixed
  - their second moments characterizing dispersion are fixed.

Dispersion in Azimuth

We seek for a density function with

- specified first moment $\boldsymbol{\mu}_{\boldsymbol{\Omega}} \doteq \int \boldsymbol{\Omega} f(\boldsymbol{\Omega}) \mathrm{d}\boldsymbol{\Omega}$
- the entropy maximized.

This function is the density function of the von-Mises distribution [Mardia, 1975]:

$$f(\boldsymbol{\Omega}) = c(\kappa) \exp\{\kappa \bar{\boldsymbol{\Omega}}^{\mathrm{T}} \boldsymbol{\Omega}\}$$

with

- $\bar{\boldsymbol{\Omega}} = \|\boldsymbol{\mu}_{\boldsymbol{\Omega}}\|^{-1} \boldsymbol{\mu}_{\boldsymbol{\Omega}}$: the mode of the distribution
- κ: concentration parameter
- $c(\kappa)$: normalization factor.

Horizontal-only propagation: $\boldsymbol{\Omega} = \boldsymbol{e}(\phi) \doteq [\cos(\phi) \sin(\phi)]^{\mathrm{T}}$.
Via the mapping $\phi \mapsto \boldsymbol{e}(\phi)$, we obtain the azimuth density function:

$$f(\phi) = \frac{1}{2\pi I_0(\kappa)} \exp\{\kappa \cos(\phi - \bar{\phi})\}.$$

Dispersion in Biazimuth (AoA-AoD)

The entropy-maximizing density function $f(\boldsymbol{\Omega}_1, \boldsymbol{\Omega}_2)$ with specified

- first moments $\boldsymbol{\mu}_{\boldsymbol{\Omega}_1}$, $\boldsymbol{\mu}_{\boldsymbol{\Omega}_2}$
- second moments in $\int \boldsymbol{\Omega}_1 \boldsymbol{\Omega}_2^{\mathrm{T}} f(\boldsymbol{\Omega}_1, \boldsymbol{\Omega}_2) \mathrm{d}\boldsymbol{\Omega}_1 \mathrm{d}\boldsymbol{\Omega}_2$

is the von-Mises-Fisher density function [Mardia, 1975]

$$f(\boldsymbol{\Omega}_1, \boldsymbol{\Omega}_2) \propto \exp\{\boldsymbol{a}_1^{\mathrm{T}} \boldsymbol{\Omega}_1 + \boldsymbol{a}_2^{\mathrm{T}} \boldsymbol{\Omega}_2 + \boldsymbol{\Omega}_1^{\mathrm{T}} \boldsymbol{A} \boldsymbol{\Omega}_2\},$$

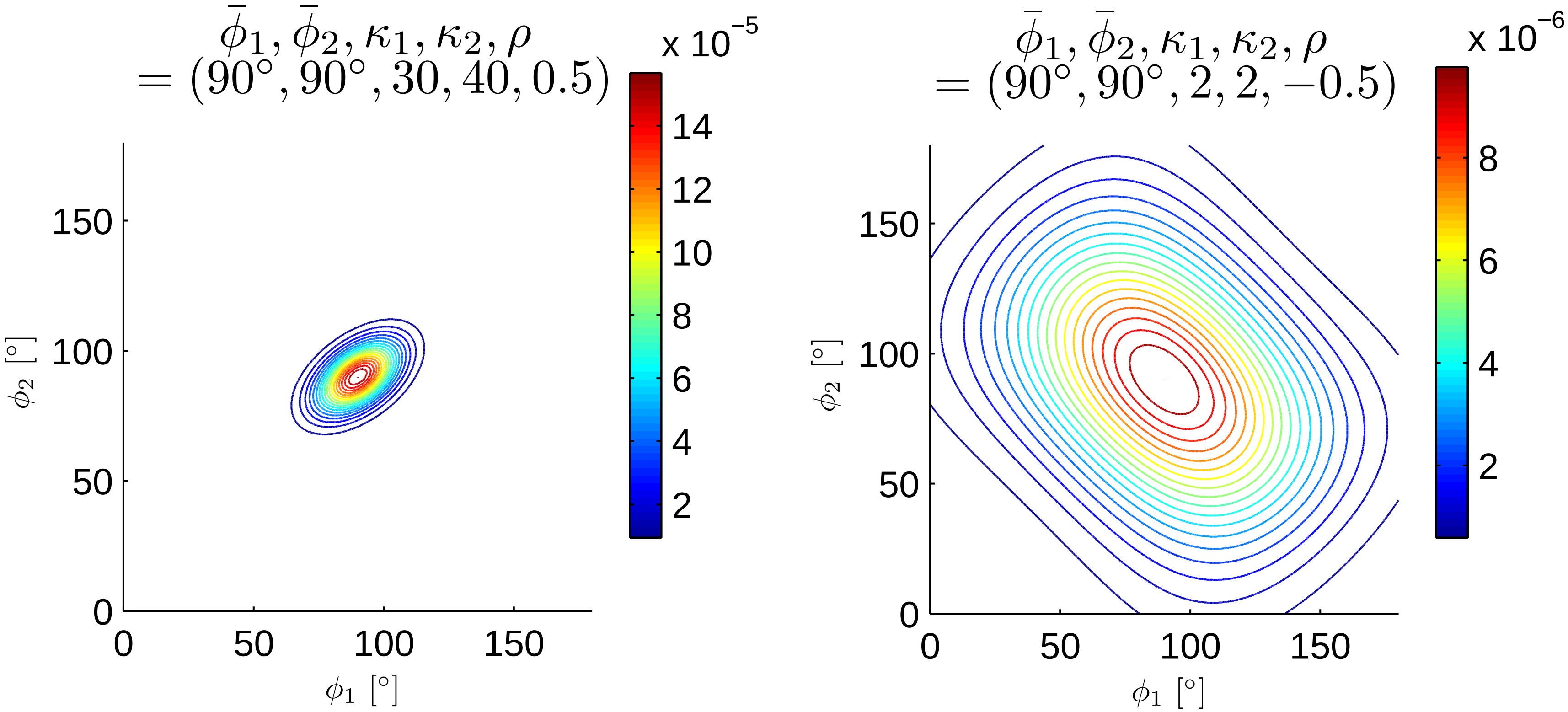
with $\boldsymbol{a}_1$, $\boldsymbol{a}_2$ and $\boldsymbol{A}$ being free parameters.

A biazimuth density function $f(\phi_1, \phi_2)$ derived via the mapping $[\phi_1, \phi_2] \mapsto [\boldsymbol{e}(\phi_1)^{\mathrm{T}}, \boldsymbol{e}(\phi_2)^{\mathrm{T}}]$ reads,

$$\begin{aligned} f(\phi_1, \phi_2) &= c(\kappa_1, \kappa_2, \rho) \cdot \exp\Bigg\{ \left(\frac{\kappa_1 - \rho\sqrt{\kappa_1 \kappa_2}}{1 - \rho^2} \right) \cos(\phi_1 - \bar{\phi}_1) \\ &\quad + \left(\frac{\kappa_2 - \rho\sqrt{\kappa_1 \kappa_2}}{1 - \rho^2} \right) \cos(\phi_2 - \bar{\phi}_2) + \frac{\rho\sqrt{\kappa_1 \kappa_2}}{1 - \rho^2} \cos[(\phi_1 - \bar{\phi}_1) - (\phi_2 - \bar{\phi}_2)] \Bigg\} \end{aligned}$$

Dispersion in Biazimuth (Cont.)

Contour plots of the biazimuth density function:



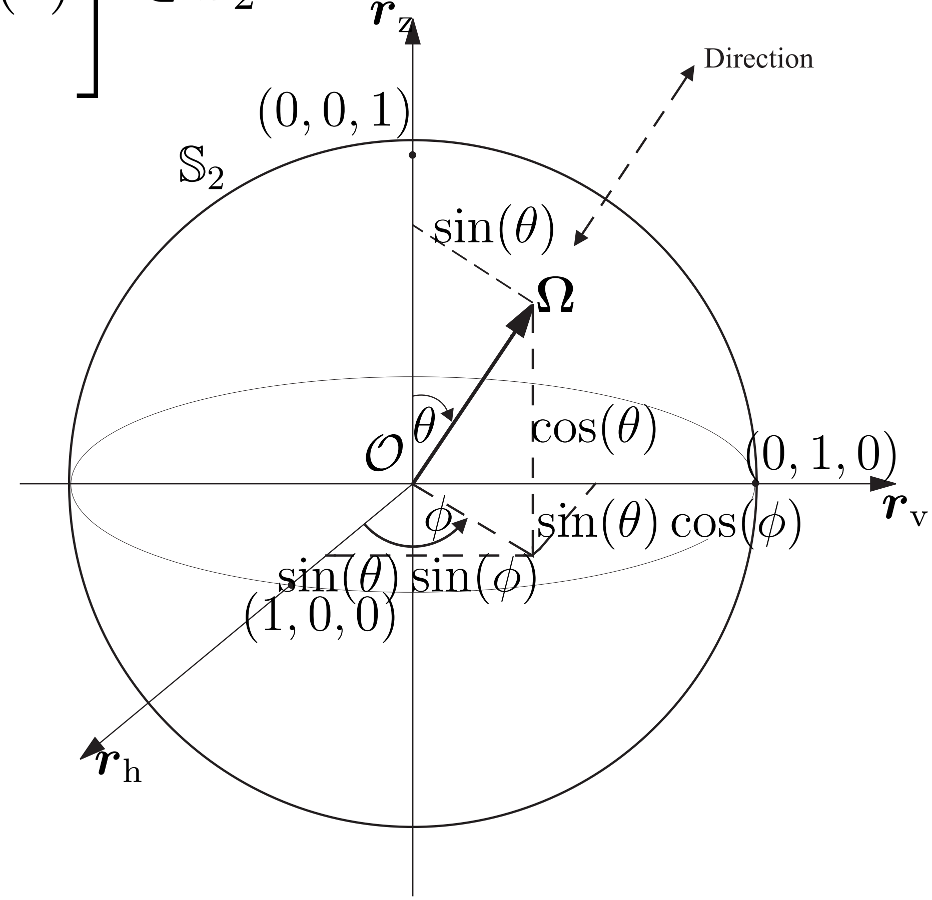
Dispersion in Direction (Azimuth-Elevation)

Definition of a direction:

$$\boldsymbol{\Omega} = \boldsymbol{e}(\phi, \theta) \doteq \begin{bmatrix} \cos(\phi)\sin(\theta) \\ \sin(\phi)\sin(\theta) \\ \cos(\theta) \end{bmatrix} \in \mathbb{S}_2$$

where

- $\mathbb{S}_2$: unit sphere
- ϕ: azimuth
- θ: elevation



Dispersion in Direction (Azimuth-Elevation)

The entropy-maximizing density function on $\mathbb{S}_2$ with specified

- first moment (mean direction) $\mu_{\boldsymbol{\Omega}} \doteq \int \boldsymbol{\Omega} f(\boldsymbol{\Omega})\mathrm{d}\boldsymbol{\Omega}$
- second moment matrix $\int \boldsymbol{\Omega}\boldsymbol{\Omega}^{\mathrm{T}} f(\boldsymbol{\Omega})\mathrm{d}\boldsymbol{\Omega}$

is the density function of the Fisher-Bingham 5 distribution [Kent 1982]:

$$f(\boldsymbol{\Omega}) = c(\kappa, \beta) \cdot \exp\left\{\kappa \boldsymbol{\gamma}_1^{\mathrm{T}}\boldsymbol{\Omega} + \beta[(\boldsymbol{\gamma}_2^{\mathrm{T}}\boldsymbol{\Omega})^2 - (\boldsymbol{\gamma}_3^{\mathrm{T}}\boldsymbol{\Omega})^2]\right\},$$

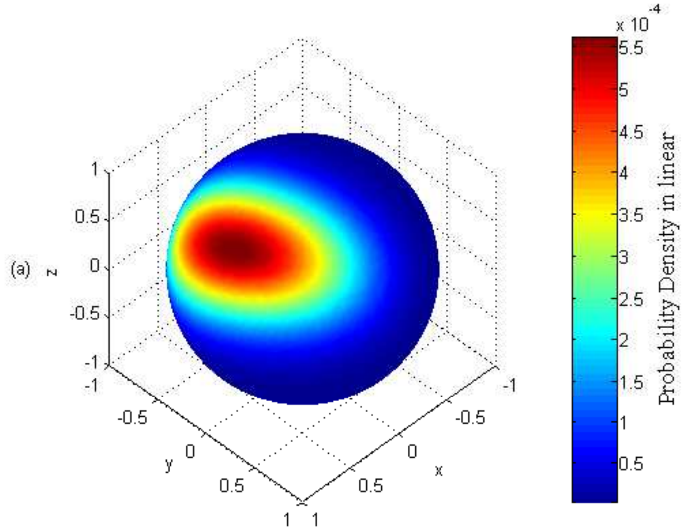
where

- κ: concentration parameter
- β: ovalness parameter
- $\boldsymbol{\gamma}_1, \boldsymbol{\gamma}_2, \boldsymbol{\gamma}_3$: orthonormal vectors determined by angles $\bar{\phi}, \bar{\theta}, \alpha$
- $\bar{\phi}$: azimuth of the mean direction $\mu_{\boldsymbol{\Omega}}$
- $\bar{\theta}$: elevation of $\mu_{\boldsymbol{\Omega}}$
- α : angle describing how the distribution is tilted on $\mathbb{S}_2$.

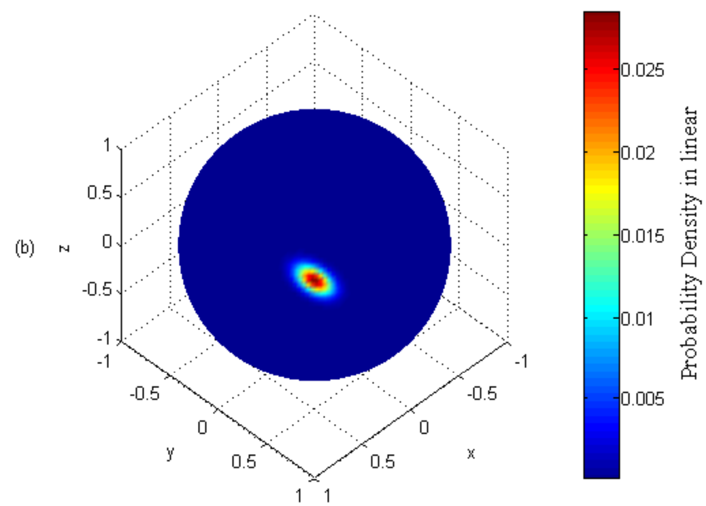
Dispersion in Direction (Azimuth-Elevation) (Cont.)

Two examples of the direction density function $f(\mathbf{\Omega})$:

$$(\bar{\phi}, \bar{\theta}, \alpha, \kappa, \beta) = (0^\circ, 45^\circ, 160^\circ, 5, 1.5)$$



$$(\bar{\phi}, \bar{\theta}, \alpha, \kappa, \beta) = (45^\circ, 70^\circ, 35^\circ, 200, 100)$$



The azimuth-elevation density function $f(\phi, \theta)$ induced by $f(\mathbf{\Omega})$ reads

$$f(\phi, \theta) = c(\kappa, \beta) \cdot \exp\left\{\kappa \boldsymbol{\gamma}_1^{\mathrm{T}} \mathbf{\Omega} + \beta[(\boldsymbol{\gamma}_2^{\mathrm{T}} \mathbf{\Omega})^2 - (\boldsymbol{\gamma}_3^{\mathrm{T}} \mathbf{\Omega})^2]\right\} \cdot \sin(\theta).$$

Dispersion in Biazimuth and Delay

Define $\boldsymbol{\psi} \doteq [\boldsymbol{\Omega}_1^{\mathrm{T}}, \boldsymbol{\Omega}_2^{\mathrm{T}}, \tau]^{\mathrm{T}}$.

The entropy-maximizing density function $f(\boldsymbol{\psi})$ with specified

- first moments $\boldsymbol{\mu}_{\psi}$
- second moments $\int \boldsymbol{\psi}\boldsymbol{\psi}^{\mathrm{T}} f(\boldsymbol{\psi})\mathrm{d}\boldsymbol{\psi}$

is [Mardia, 1975]

$$f(\boldsymbol{\psi}) \propto \exp\{\boldsymbol{b}^{\mathrm{T}}\boldsymbol{\psi} + \boldsymbol{\psi}^{\mathrm{T}}\boldsymbol{B}\boldsymbol{\psi}\},$$

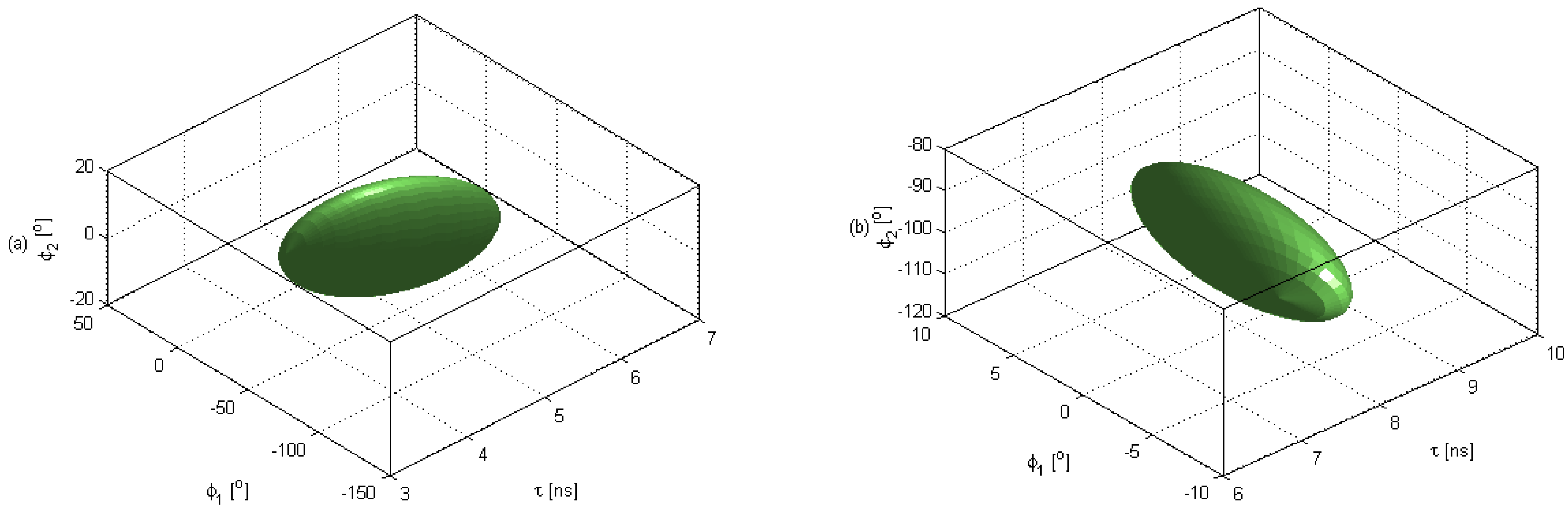
where $\boldsymbol{b} \in \mathbb{R}^{5\times 1}$ and $\boldsymbol{B} \in \mathbb{R}^{5\times 5}$ are free parameters.

Via the mapping $[\phi_1, \phi_2, \tau] \mapsto [\boldsymbol{e}(\phi_1)^{\mathrm{T}}, \boldsymbol{e}(\phi_2)^{\mathrm{T}}, \tau]$, the biazimuth-delay density function $f(\phi_1, \phi_2, \tau)$ can be induced by $f(\boldsymbol{\Omega}_1, \boldsymbol{\Omega}_2, \tau)$,

$$\begin{aligned} f(\phi_1, \phi_2, \tau) = C \exp\Big\{ & c_1 \cos(\phi_1 - \bar{\phi}_1) + c_2 \cos(\phi_2 - \bar{\phi}_2) \\ & + (\tau - \bar{\tau})[c_3 \sin(\phi_1 - \bar{\phi}_1) + c_4 \sin(\phi_2 - \bar{\phi}_2)] \\ & + c_5(\tau - \bar{\tau})^2 + c_6 \cos[(\phi_1 - \bar{\phi}_1) - (\phi_2 - \bar{\phi}_2)]\Big\}. \end{aligned}$$

Dispersion in Biazimuth and Delay (Cont.)

3dB-spread surfaces $\left\{(\phi_1, \phi_2, \tau) : f(\phi_1, \phi_2, \tau) = \frac{1}{2} f(\bar{\phi}_1, \bar{\phi}_2, \bar{\tau})\right\}$:



	$\bar{\tau}$ [ns]	$\bar{\phi}_1$ [°]	$\bar{\phi}_2$ [°]	σ_τ [ns]	κ_1	σ_{ϕ_1} [°]	κ_2	σ_{ϕ_2} [°]	ρ_{12}	ρ_1	ρ_2
(a)	5	-40	0	1	5	25.6	10	18.1	-0.4	-0.3	-0.3
(b)	8	0	-100	0.5	50	8.1	30	10.5	-0.5	0.6	-0.2

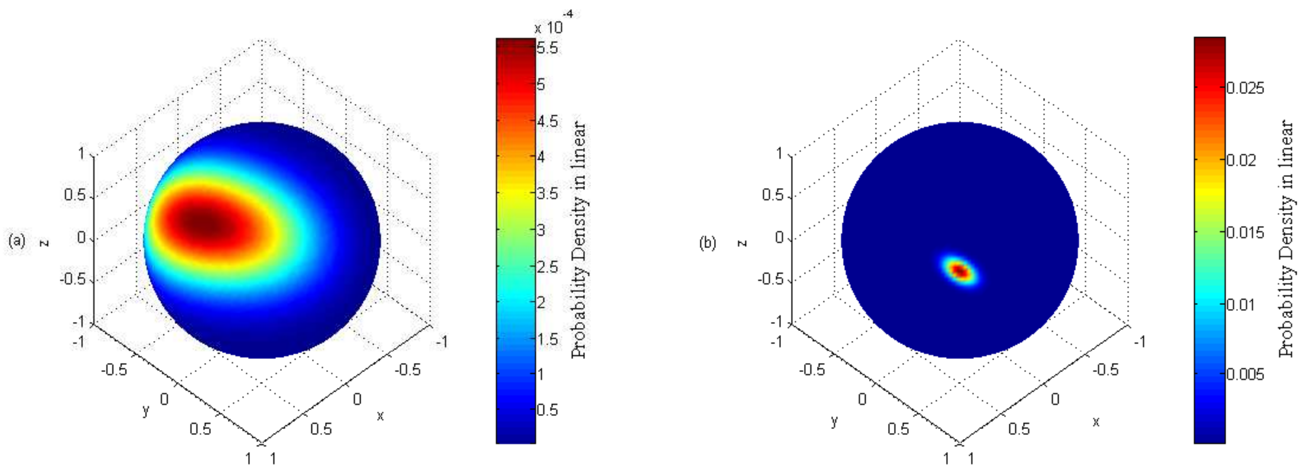
Direction-Delay Dispersion

The entropy-maximizing pdf of direction $\boldsymbol{\Omega}$ and delay τ with specified first moment and second moment is of the form

$$f_{\mathrm{ME}}(\boldsymbol{\Omega},\tau) \propto \exp\left\{ \begin{bmatrix} \boldsymbol{\Omega}-\bar{\boldsymbol{\Omega}} \\ \tau-\bar{\tau} \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} \boldsymbol{A} & \boldsymbol{c} \\ \boldsymbol{c}^{\mathrm{T}} & -b \end{bmatrix} \begin{bmatrix} \boldsymbol{\Omega}-\bar{\boldsymbol{\Omega}} \\ \tau-\bar{\tau} \end{bmatrix} \right\}.$$

Two conditions are used to derive the expressions of the parameters:

- The conditional pdf of direction is Fisher-Bingham-5 (FB5)
  Two examples of FB5 pdf:



- The conditional pdf of delay is Gaussian.

Direction-Delay Dispersion (Cont.)

The direction-delay pdf is calculated to be

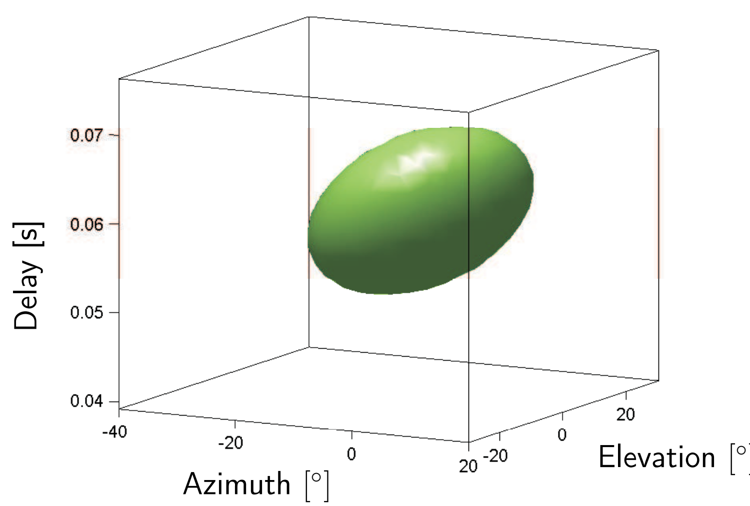
$$f_{\mathrm{ME}}(\mathbf{\Omega},\tau) = C\cdot\exp\Big(\kappa\bar{\mathbf{\Omega}}^{\mathrm{T}}\mathbf{\Omega}+\mathbf{\Omega}^{\mathrm{T}}\boldsymbol{A}(\tau,\zeta,\alpha,\beta)\mathbf{\Omega}-b(\tau-\bar{\tau})^2 - 2\eta\boldsymbol{g}^{\mathrm{T}}(\mathbf{\Omega}-\bar{\mathbf{\Omega}})(\tau-\bar{\tau})\Big),$$

where

- C: Constant for normalization
- $\bar{\Omega}$: Mean direction (equivalently $\bar{\phi},\bar{\theta}$)
- $\bar{\tau}$: Mean delay
- κ: Concentration parameter in direction
- b: Concentration parameter in delay
- ζ: Ovalness parameter in direction
- η: Dependence between the spread in direction and in delay
- α,β: Two angles describing how the pdf is tilted on $\mathbb{S}_2$
- $\boldsymbol{g}$: Vector jointly determined by $\bar{\Omega}$ and β

3-dB spread surface of $f_{\mathrm{ME}}(\mathbf{\Omega},\tau)$ with parameter setting

$\bar{\theta}$ [°]	$\bar{\phi}$ [°]	$\bar{\tau}$ [ms]	κ	ζ
0	0	58	100	0.01
α [°]	β [°]	η	b	
60	270	1×10^3	4×10^4	



Experimental Result I: Estimate of Direction (Azimuth-Elevation) Dispersion

Signal Model for SIMO Channel Sounding (Narrowband)

Received signal in a narrowband SIMO scenario

$$\boldsymbol{Y}(t) = \int_{-\pi}^{+\pi}\int_{-\pi/2}^{+\pi/2} \boldsymbol{c}(\phi,\theta)s(t)h(t;\phi,\theta)\mathrm{d}\phi\mathrm{d}\theta + \boldsymbol{W}(t),$$

where

- $\boldsymbol{Y}(t) \in \mathbb{C}^{M\times 1}$: output signals of the Rx array
- $\boldsymbol{c}(\phi,\theta) \in \mathbb{C}^{M\times 1}$: Rx antenna array response
- $s(t)$: complex envelope of the transmitted signal
- $h(t;\phi,\theta) \in \mathbb{C}^{1\times 1}$: (time-variant) AoA-EoA spread function of the propagation channel.
- $\boldsymbol{W}(t) \in \mathbb{C}^{M\times 1}$: complex white Gaussian noise.

Signal Model for SIMO Channel Sounding (Narrowband)

In a scenario with D path components:

$$h(t;\phi,\theta) = \sum_{d=1}^{D} h_d(t;\phi,\theta).$$

The azimuth-elevation power spectrum of the radio channel under the WSS uncorrelated scattering assumption reads

$$P(\phi,\theta) \doteq \mathrm{E}[|h(t;\phi,\theta)|^2] = \sum_{d=1}^{D} P_d(\phi,\theta),$$

where $P_d(\phi,\theta)$ is the power spectrum of the dth path component, i.e.

$$P_d(\phi,\theta) \doteq \mathrm{E}[|h_d(t;\phi,\theta)|^2] = P_d \cdot f_d(\phi,\theta)$$

with P_d being the average power, and $f_d(\phi,\theta)$ the spectral density of the path component.

Signal Model for SIMO Channel Sounding (Narrowband)

We assume

$$f_d(\phi, \theta) = f(\phi, \theta; \boldsymbol{\theta}_d)$$

with

- $f(\phi, \theta; \boldsymbol{\theta}_d)$: the Fisher-Bingham-5 density function
- $\boldsymbol{\theta}_d \doteq [\bar{\phi}_d, \bar{\theta}_d, \kappa_d, \beta_d, \alpha_d]$: parameter vector

of the dth path component.

Parameter vector for the D-path-component scenario:

$$\boldsymbol{\Theta} \doteq [P_d, \boldsymbol{\theta}_d; d = 1, \ldots, D].$$

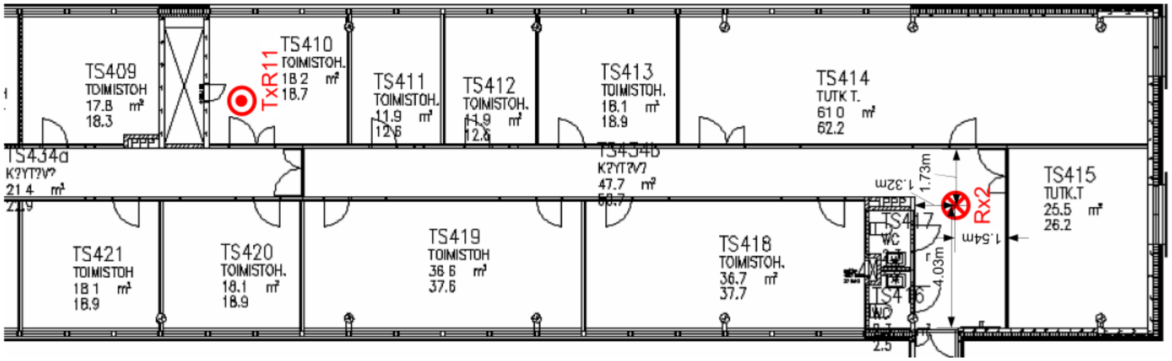
The SAGE algorithm can be used as an approximation of the maximum-likelihood estimate $\hat{\boldsymbol{\Theta}}$ (See the paper for the details of the algorithm.)

Experimental Investigations

(a) Surroundings of the Tx.

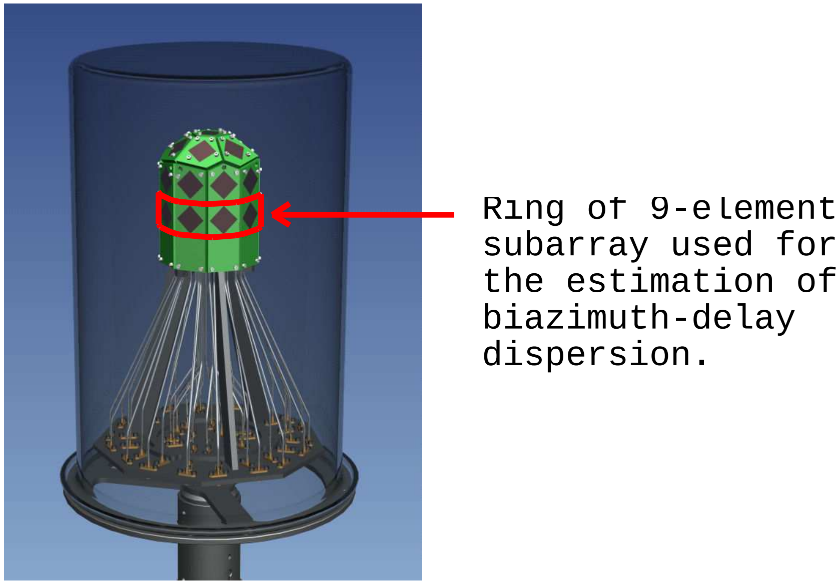
(b) Surroundings of the Rx.

(c) Map of the premises.

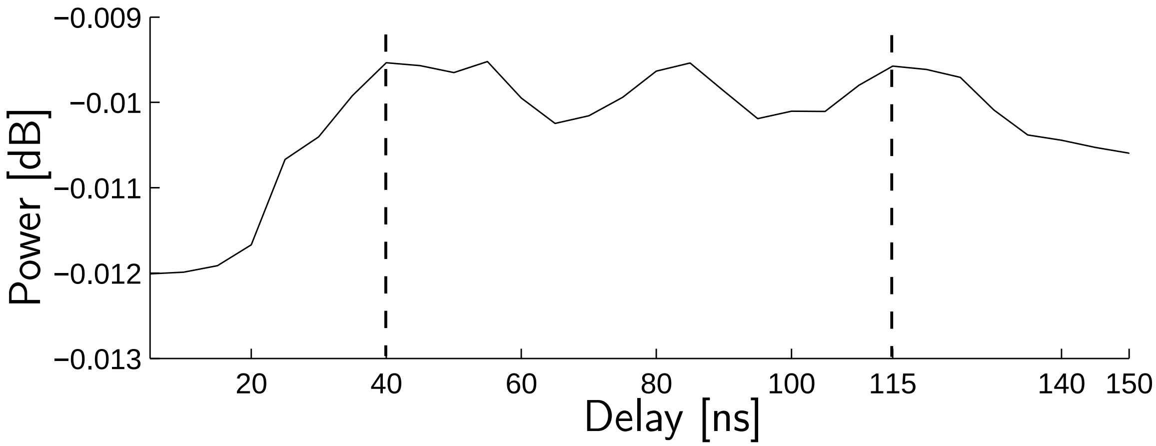


Experimental Investigations (Cont.)

- 1×9 SIMO system with Rx array consisting of dual-polarized antennas in the lower ring of the array



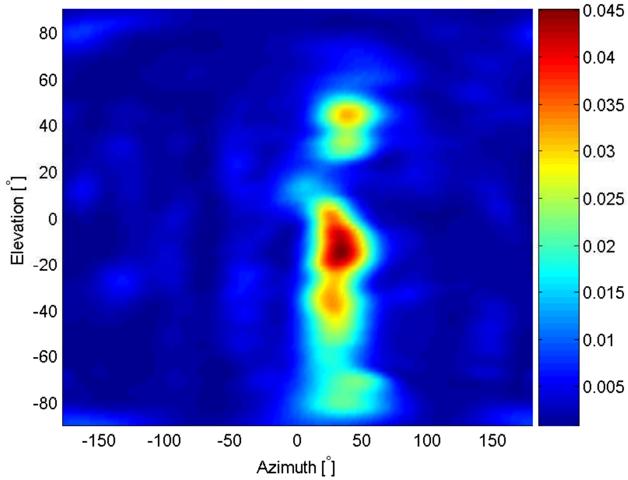
- Estimated delay power spectrum



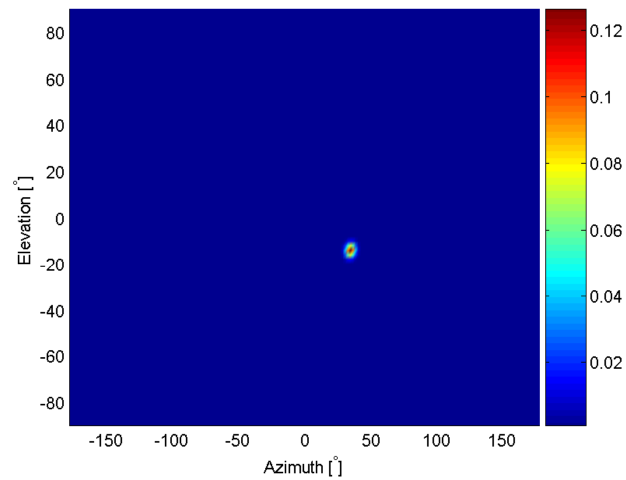
Estimate of Azimuth-Elevation Power Spectrum

$\tau = 40$ ns

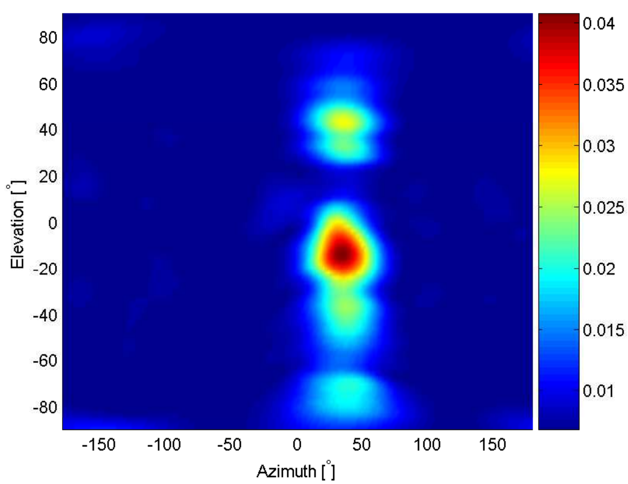
$\mathrm{Bartlett}(\hat{\mathbf{\Sigma}})$



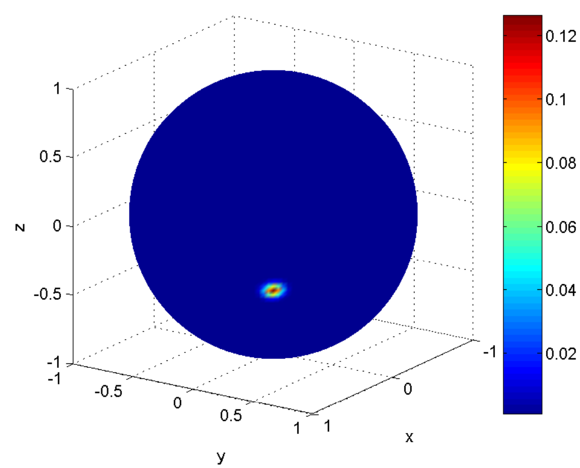
$\hat{P}(\phi, \theta)$



$\mathrm{Bartlett}(\mathbf{\Sigma}(\hat{\mathbf{\Theta}}))$



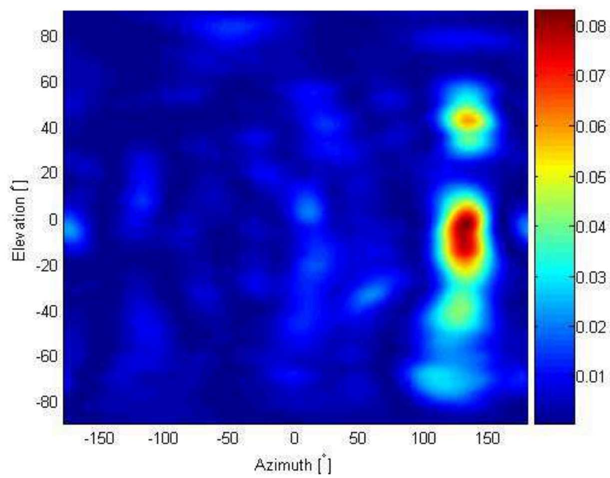
$\hat{P}(\mathbf{\Omega})$



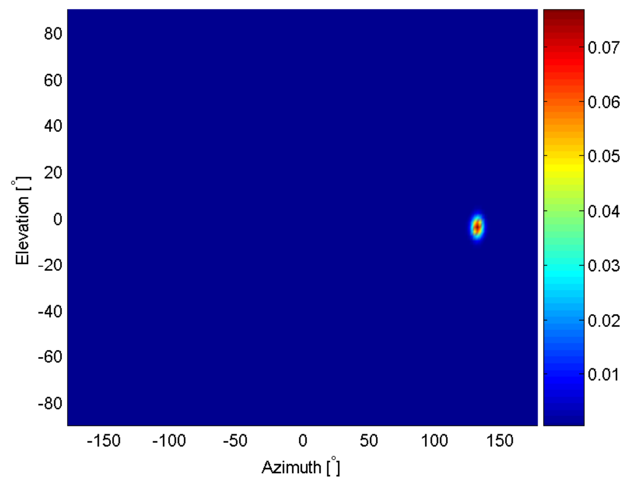
Estimate of Azimuth-Elevation Power Spectrum

$\tau = 115$ ns

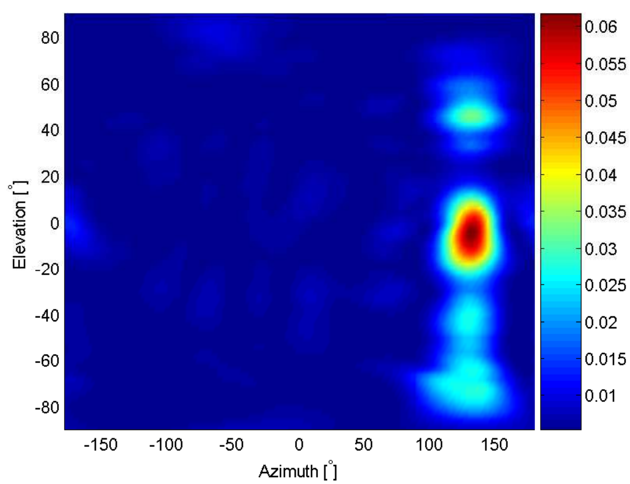
$\text{Bartlett}(\hat{\mathbf{\Sigma}})$



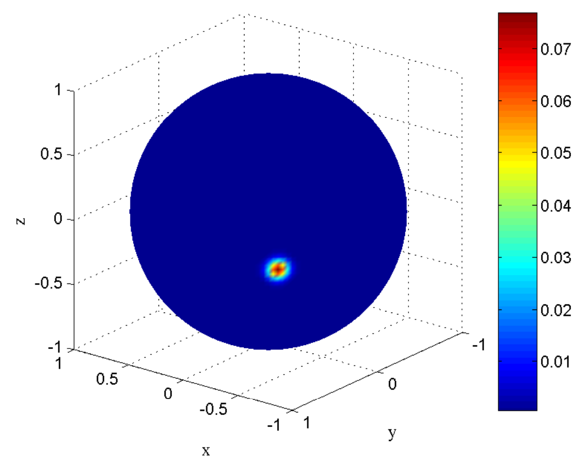
$\hat{P}(\phi, \theta)$



$\text{Bartlett}(\mathbf{\Sigma}(\hat{\mathbf{\Theta}}))$

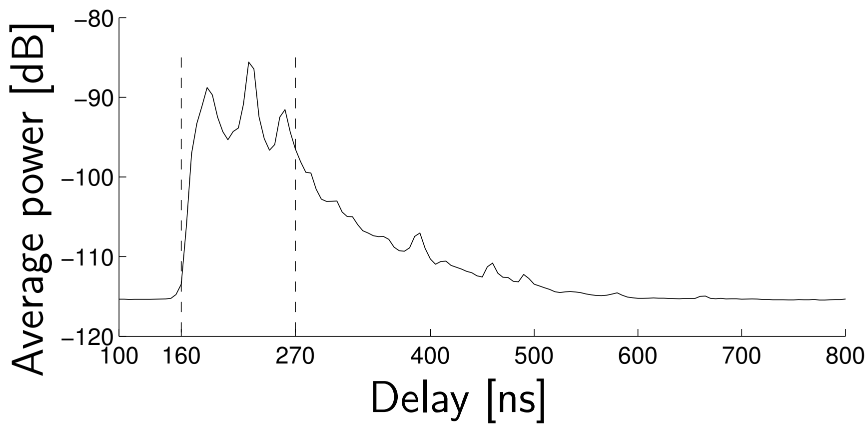


$\hat{P}(\mathbf{\Omega})$



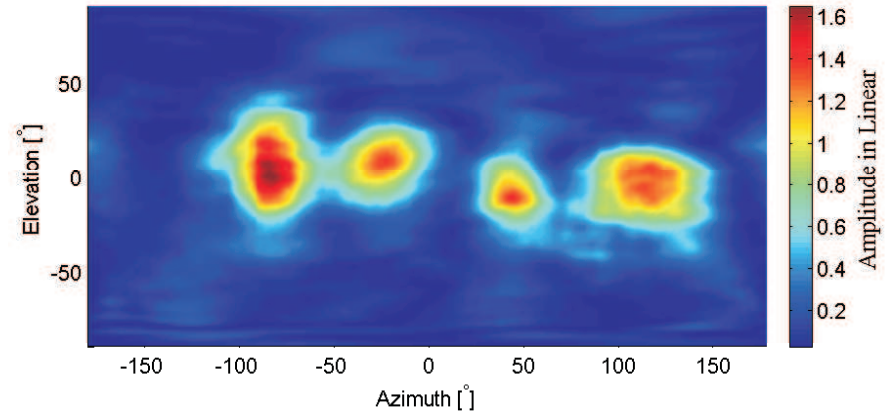
Experimental Investigations (Cont.)

- 50×1 MISO system
- Estimated delay power spectrum

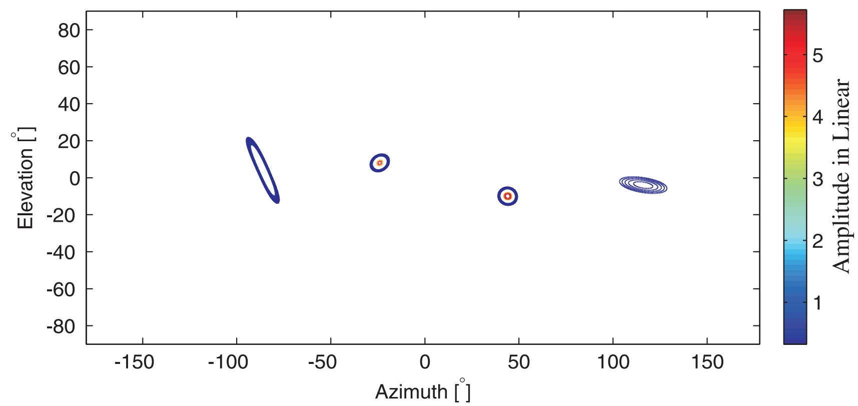


Estimate of Azimuth-Elevation Power Spectrum

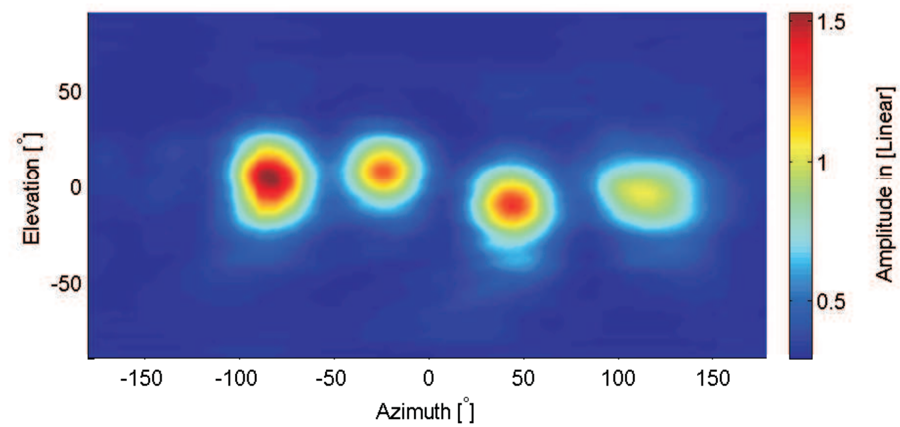
- Bartlett spectrum of the signal received at delay 160 ns



- Power spectrum estimate
  using the proposed characterization method



- Bartlett spectrum of the reconstructed signal



Experimental Result II: Estimate of Direction-Delay Dispersion

Signal Model for SIMO Channel Sounding (Wideband)

Received signal in a $1 \times M$ SIMO scenario

$$\boldsymbol{Y}(t) = \int_{-\infty}^{+\infty} \int_{\mathbb{S}_2} \boldsymbol{c}(\boldsymbol{\Omega}) u(t-\tau) H(\boldsymbol{\Omega}, \tau) \mathrm{d}\boldsymbol{\Omega} \mathrm{d}\tau + \boldsymbol{W}(t),$$

where

- $\boldsymbol{Y}(t) \in \mathbb{C}^{M \times 1}$: output signals of the Rx array
- $\boldsymbol{c}(\boldsymbol{\Omega}) \in \mathbb{C}^{M \times 1}$: Rx antenna array response
- $u(t)$: complex envelope of the transmitted signal
- $H(\boldsymbol{\Omega}, \tau)$: direction-delay spread function of the propagation channel
- $\boldsymbol{W}(t) \in \mathbb{C}^{M \times 1}$: complex white Gaussian noise.

Signal Model for SIMO Channel Sounding (Wideband)

In a scenario with D components

$$H(\mathbf{\Omega}, \tau) = \sum_{d=1}^{D} H_d(\mathbf{\Omega}, \tau).$$

The component spread function $H_d(\mathbf{\Omega}, \tau)$: $d \in \{1, \ldots, D\}$ satisfy

$$\mathrm{E}[H_d(\mathbf{\Omega}, \tau)^* H_{d'}(\mathbf{\Omega}', \tau')] = P_d(\mathbf{\Omega}, \tau)\delta_{dd'}\delta(\mathbf{\Omega} - \mathbf{\Omega}')\delta(\tau - \tau').$$

Thus,

$$\mathrm{E}[H(\mathbf{\Omega}, \tau)^* H(\mathbf{\Omega}', \tau')] = P(\mathbf{\Omega}, \tau)\delta(\mathbf{\Omega} - \mathbf{\Omega}')\delta(\tau - \tau')$$

and

$$P(\mathbf{\Omega}, \tau) = \sum_{d=1}^{D} P_d(\mathbf{\Omega}, \tau).$$

Signal Model for SIMO Channel Sounding (Wideband)

The component power spectrum $P_d(\mathbf{\Omega}, \tau)$ is written as

$$P_d(\mathbf{\Omega}, \tau) = P_d \cdot f_d(\mathbf{\Omega}, \tau).$$

We assume

$$f_d(\mathbf{\Omega}, \tau) = f(\mathbf{\Omega}, \tau; \boldsymbol{\theta}_d),$$

- $f(\mathbf{\Omega}, \tau; \boldsymbol{\theta}_d)$: the derived entropy-maximizing direction-delay pdf
- $\boldsymbol{\theta}_d \doteq [\bar{\phi}_d, \bar{\theta}_d, \bar{\tau}_d, \kappa_d, \zeta_d, \alpha_d, \beta_d, \eta_d, b_d]$: component-specific parameters.

Parameter vector for the D-component scenario:

$$\mathbf{\Theta} \doteq [P_d, \boldsymbol{\theta}_d; d = 1, \ldots, D].$$

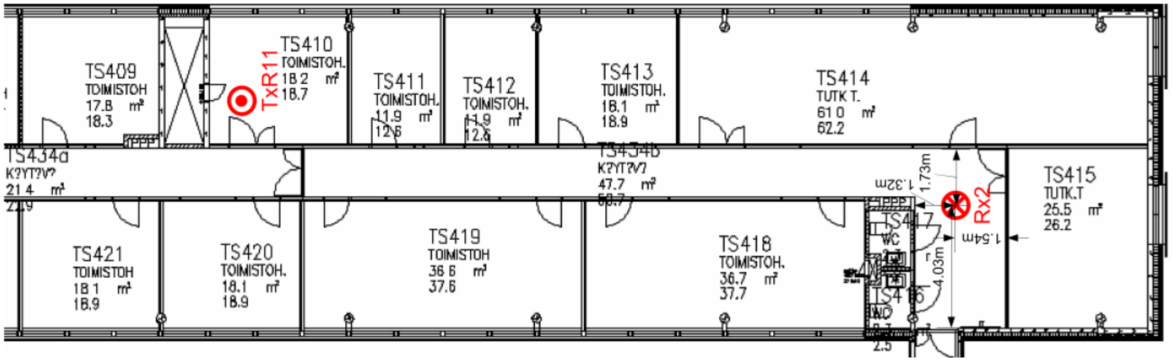
The SAGE algorithm can be used as an approximation of the maximum-likelihood estimate of $\mathbf{\Theta}$.

Experimental Investigations

(a) Surroundings of the Tx.

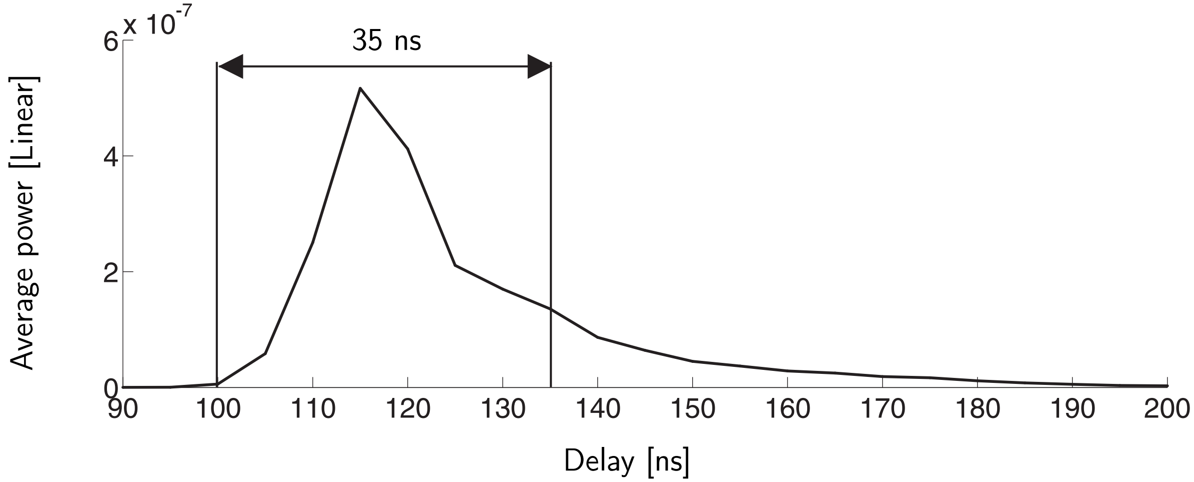
(b) Surroundings of the Rx.

(c) Map of the premises.



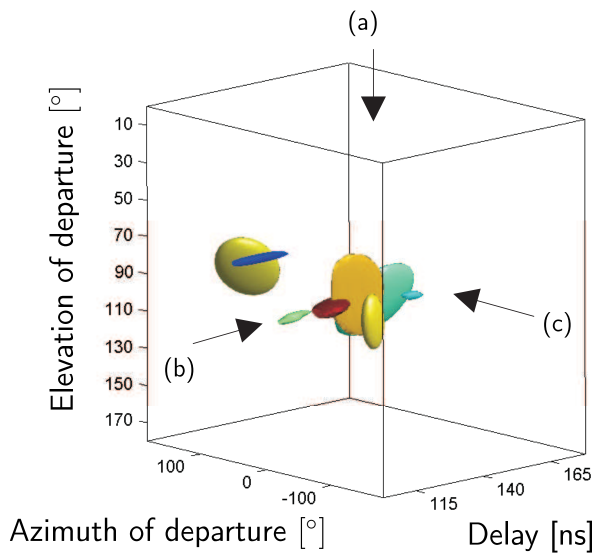
Experimental Investigations

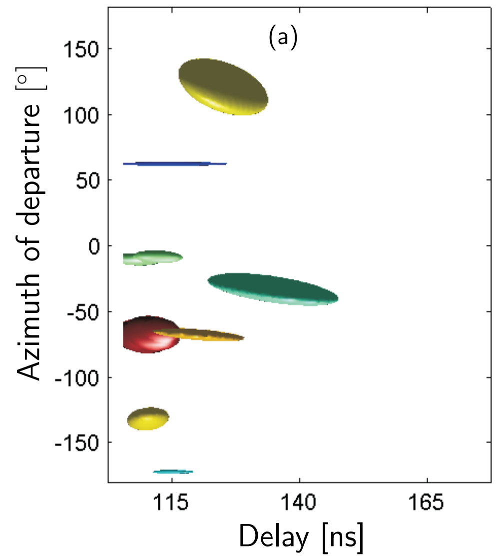
- 50×1 Multiple-Input-Single-Output (MISO) system.
- Delay range considered: $[100 \text{ ns}, 135 \text{ ns}]$.
  Average power delay profile:

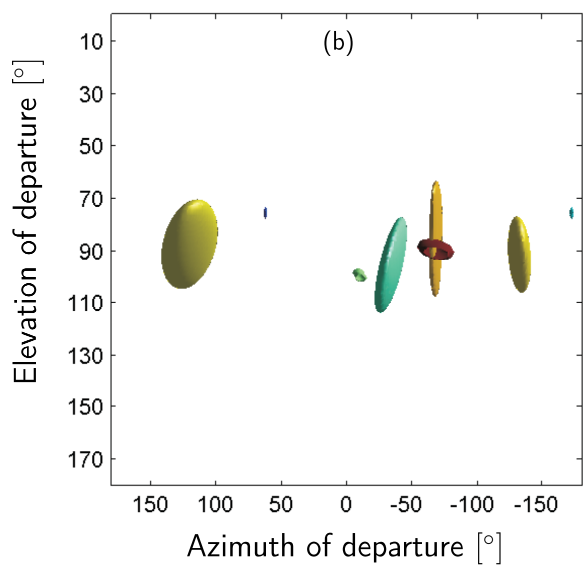


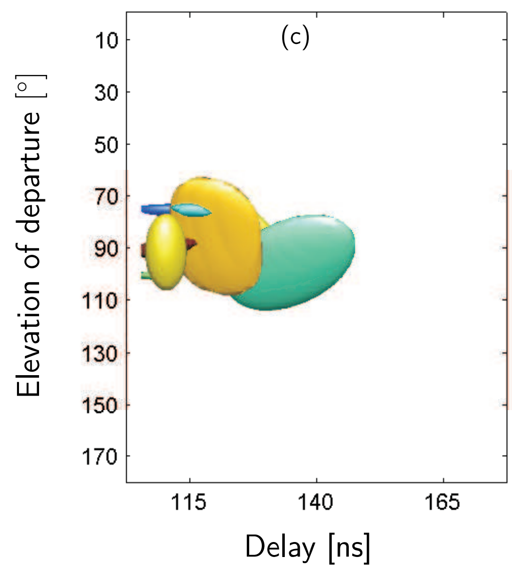
- Number of components: 10, equal to the observed dominant local maxima of the direction-delay Bartlett spectrum.

3 dB-Spread Surfaces of Estimated Component Azimuth–Elevation–Delay Power Spectra



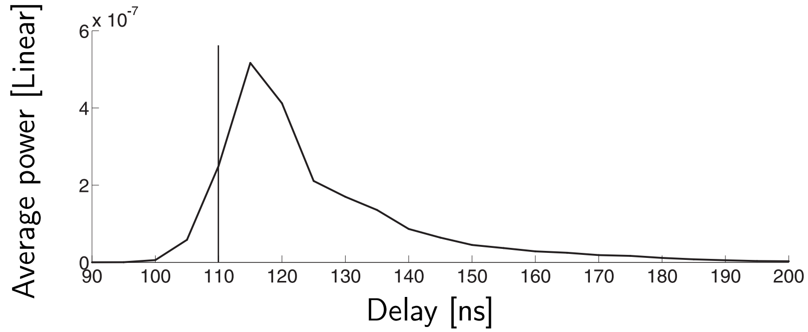


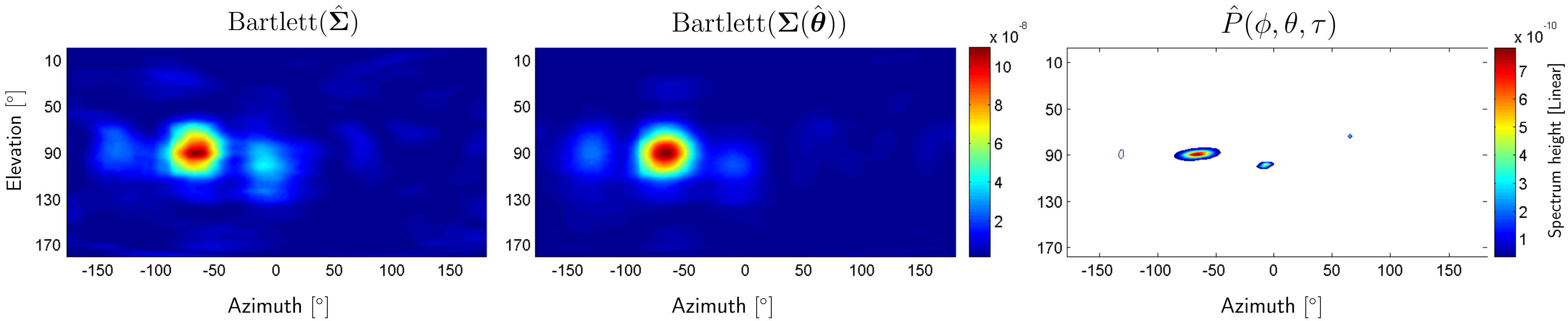




Estimates of Azimuth–Elevation–Delay Spectrum

Delay $\tau = 110$ ns:



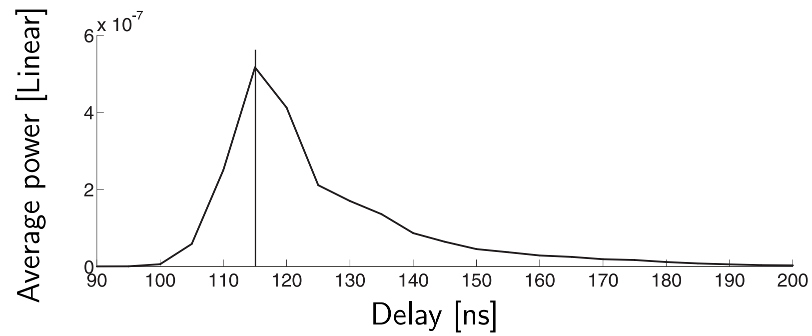


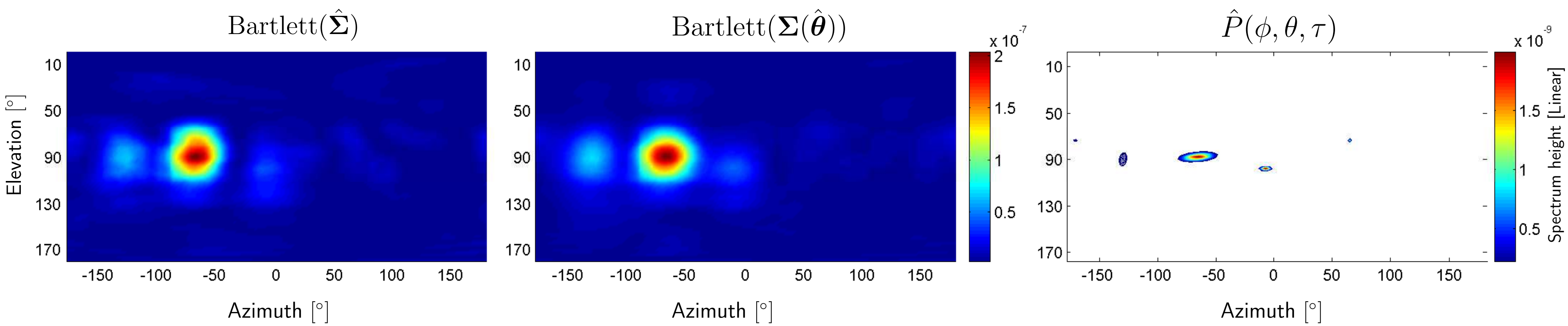
- $\mathrm{Bartlett}(\mathbf{\Sigma})$: Power spectrum estimate calculated using the Bartlett beamformer
- $\hat{\mathbf{\Sigma}}$: Sample covariance matrix
- $\mathbf{\Sigma}(\hat{\boldsymbol{\theta}})$: Covariance matrix computed based on $\hat{P}(\mathbf{\Omega}, \tau)$
- $\hat{P}(\phi, \theta, \tau)$: Azimuth–elevation–delay power spectrum estimate

$$\hat{P}(\phi, \theta, \tau) = \sin(\theta) \cdot \hat{P}(\mathbf{\Omega}, \tau)|_{\mathbf{\Omega}=\boldsymbol{e}(\phi,\theta)}$$

Estimates of Azimuth–Elevation–Delay Spectrum

Delay $\tau = 115$ ns:



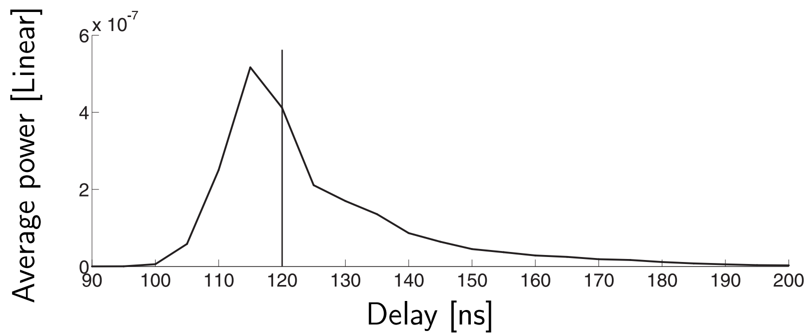


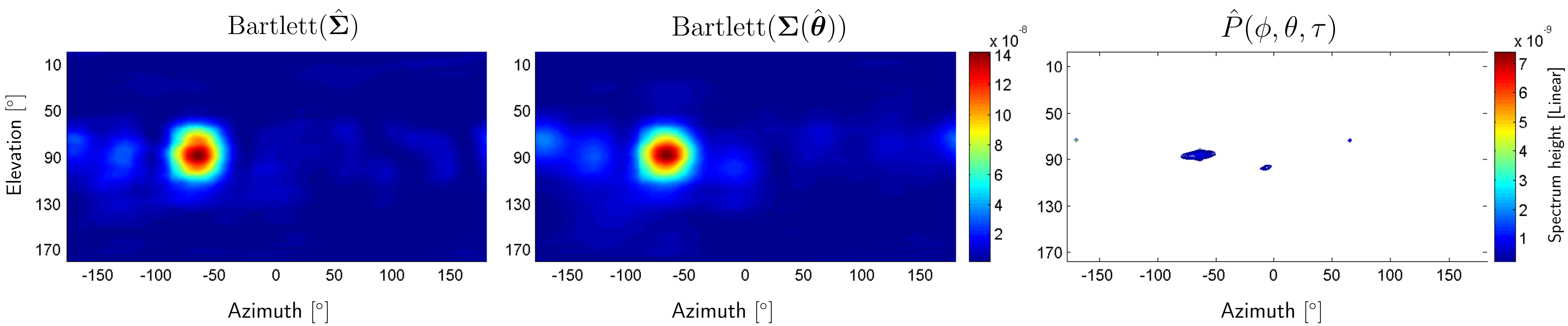
- $\text{Bartlett}(\boldsymbol{\Sigma})$: Power spectrum estimate calculated using the Bartlett beamformer
- $\hat{\boldsymbol{\Sigma}}$: Sample covariance matrix
- $\boldsymbol{\Sigma}(\hat{\boldsymbol{\theta}})$: Covariance matrix computed based on $\hat{P}(\boldsymbol{\Omega}, \tau)$
- $\hat{P}(\phi, \theta, \tau)$: Azimuth–elevation–delay power spectrum estimate

$$\hat{P}(\phi, \theta, \tau) = \sin(\theta) \cdot \hat{P}(\boldsymbol{\Omega}, \tau)|_{\boldsymbol{\Omega}=\boldsymbol{e}(\phi,\theta)}$$

Estimates of Azimuth–Elevation–Delay Spectrum

Delay $\tau = 120$ ns:



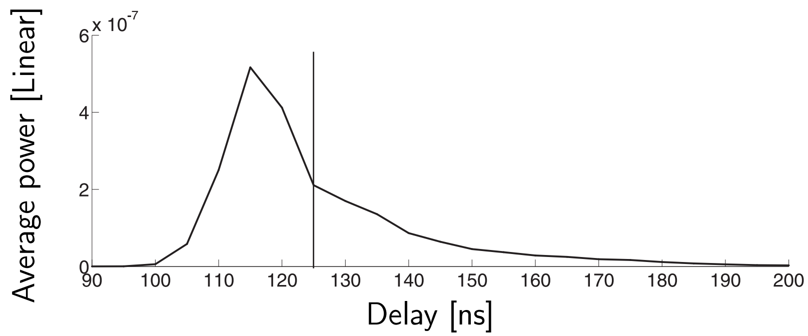


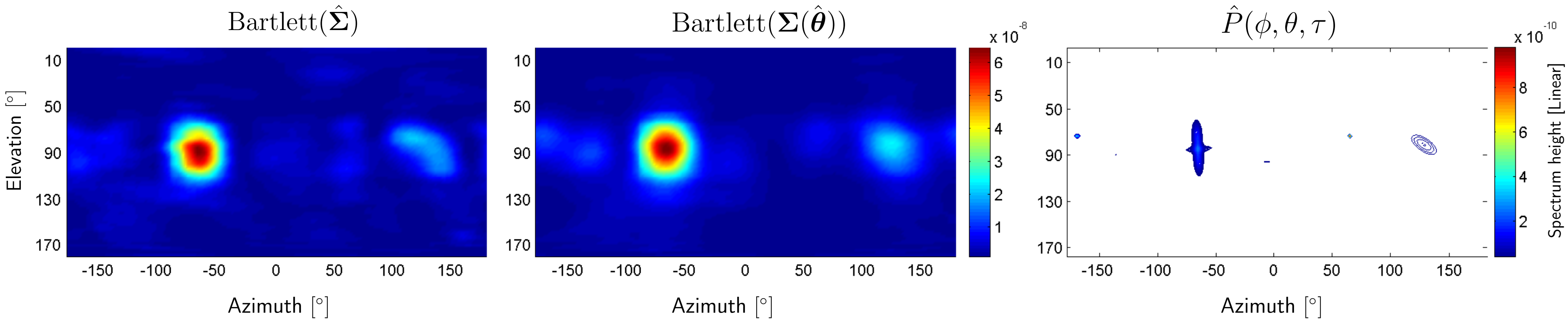
- $\mathrm{Bartlett}(\mathbf{\Sigma})$: Power spectrum estimate calculated using the Bartlett beamformer
- $\hat{\mathbf{\Sigma}}$: Sample covariance matrix
- $\mathbf{\Sigma}(\hat{\boldsymbol{\theta}})$: Covariance matrix computed based on $\hat{P}(\mathbf{\Omega}, \tau)$
- $\hat{P}(\phi, \theta, \tau)$: Azimuth–elevation–delay power spectrum estimate

$$\hat{P}(\phi, \theta, \tau) = \sin(\theta) \cdot \hat{P}(\mathbf{\Omega}, \tau)|_{\mathbf{\Omega}=\boldsymbol{e}(\phi,\theta)}$$

Estimates of Azimuth–Elevation–Delay Spectrum

Delay $\tau = 125$ ns:



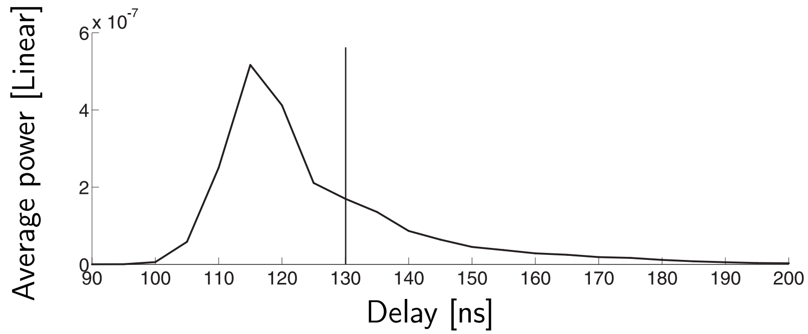


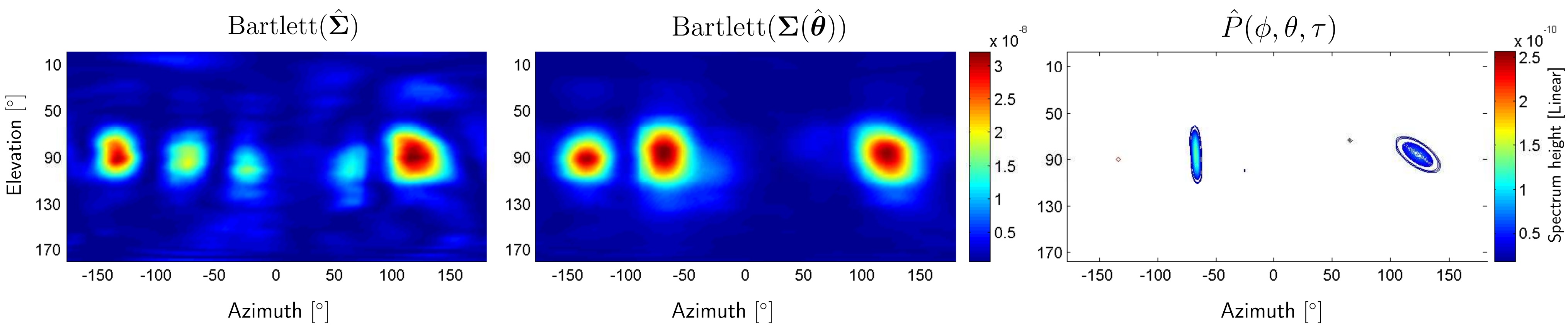
- $\mathrm{Bartlett}(\mathbf{\Sigma})$: Power spectrum estimate calculated using the Bartlett beamformer
- $\hat{\mathbf{\Sigma}}$: Sample covariance matrix
- $\mathbf{\Sigma}(\hat{\boldsymbol{\theta}})$: Covariance matrix computed based on $\hat{P}(\mathbf{\Omega}, \tau)$
- $\hat{P}(\phi, \theta, \tau)$: Azimuth–elevation–delay power spectrum estimate

$$\hat{P}(\phi, \theta, \tau) = \sin(\theta) \cdot \hat{P}(\mathbf{\Omega}, \tau)|_{\mathbf{\Omega} = \boldsymbol{e}(\phi, \theta)}$$

Estimates of Azimuth–Elevation–Delay Spectrum

Delay $\tau = 130$ ns:



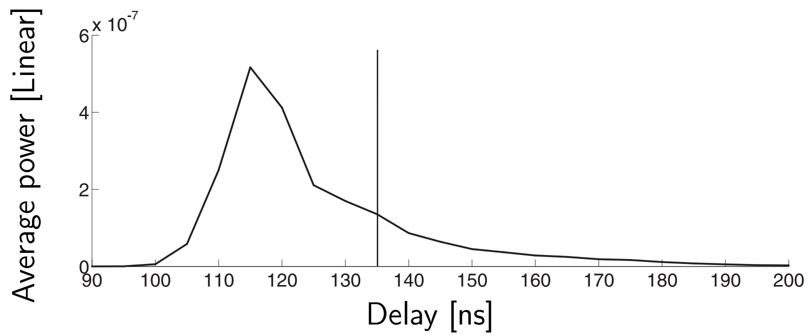


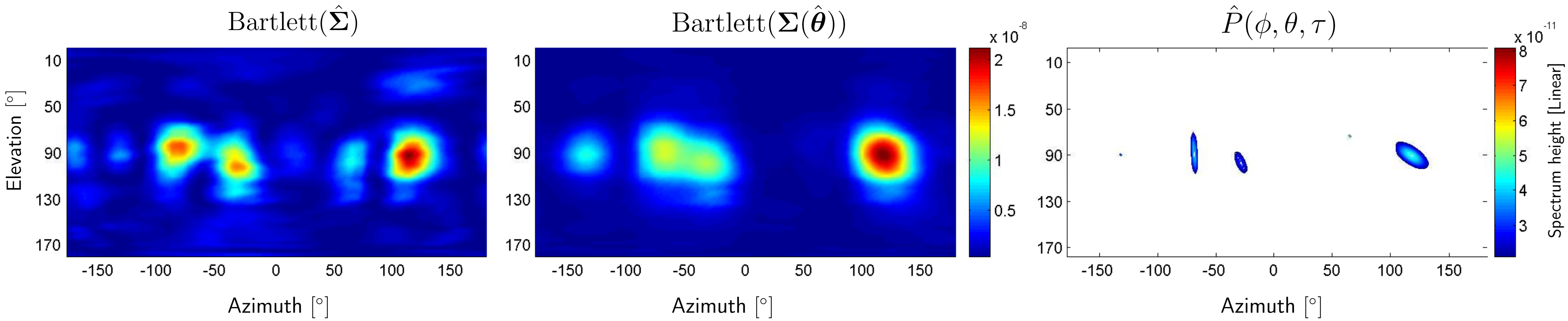
- $\mathrm{Bartlett}(\mathbf{\Sigma})$: Power spectrum estimate calculated using the Bartlett beamformer
- $\hat{\mathbf{\Sigma}}$: Sample covariance matrix
- $\mathbf{\Sigma}(\hat{\boldsymbol{\theta}})$: Covariance matrix computed based on $\hat{P}(\mathbf{\Omega}, \tau)$
- $\hat{P}(\phi, \theta, \tau)$: Azimuth–elevation–delay power spectrum estimate

$$\hat{P}(\phi, \theta, \tau) = \sin(\theta) \cdot \hat{P}(\mathbf{\Omega}, \tau)|_{\mathbf{\Omega}=\boldsymbol{e}(\phi,\theta)}$$

Estimates of Azimuth–Elevation–Delay Spectrum

Delay $\tau = 135$ ns:





- $\mathrm{Bartlett}(\boldsymbol{\Sigma})$: Power spectrum estimate calculated using the Bartlett beamformer
- $\hat{\boldsymbol{\Sigma}}$: Sample covariance matrix
- $\boldsymbol{\Sigma}(\hat{\boldsymbol{\theta}})$: Covariance matrix computed based on $\hat{P}(\boldsymbol{\Omega}, \tau)$
- $\hat{P}(\phi, \theta, \tau)$: Azimuth–elevation–delay power spectrum estimate

$$\hat{P}(\phi, \theta, \tau) = \sin(\theta) \cdot \hat{P}(\boldsymbol{\Omega}, \tau)|_{\boldsymbol{\Omega}=\boldsymbol{e}(\phi,\theta)}$$

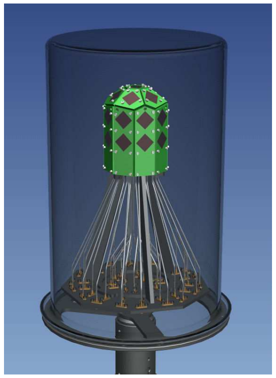
Experimental Result III: Estimate of Biazimuth-Delay Dispersion

Experimental Investigations

Tx environment

Rx environment

Antenna array

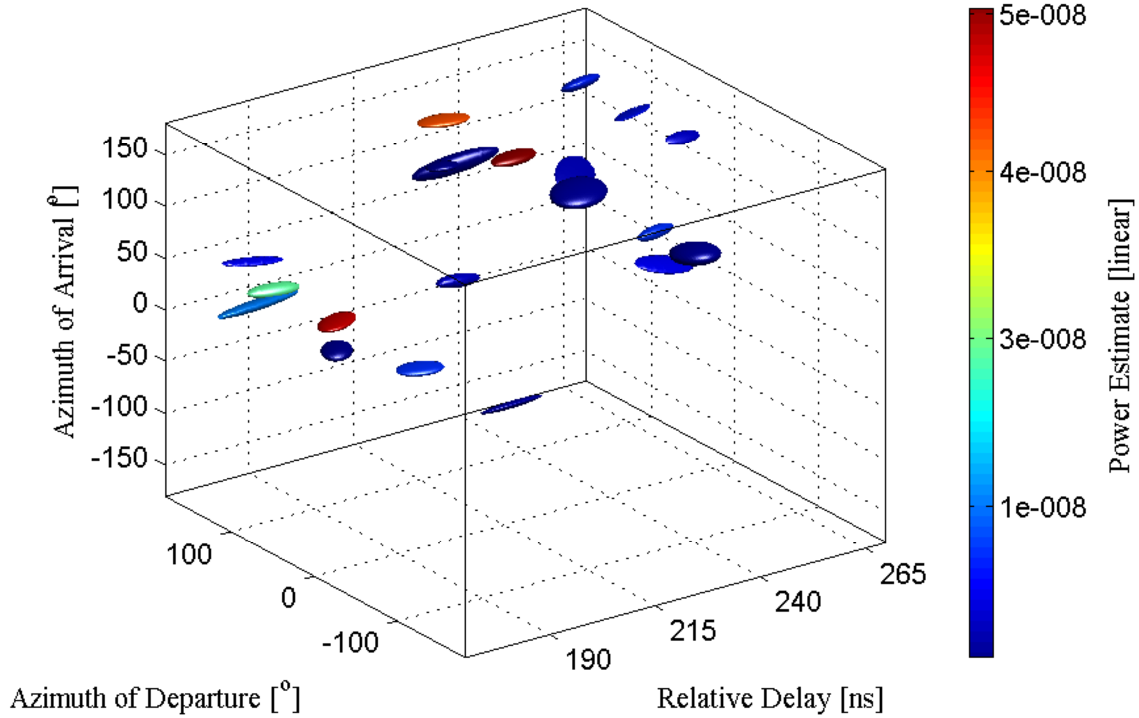


Measurement set-up:

- Carrier frequency: 5.2 GHz
- Bandwidth: 200 MHz
- 50-element Tx and Rx Patch arrays with $\pm 45^\circ$ slanted antennas
- Tx array height: 1.53 m; Rx array height: 0.82 m
- Big hall, time-variant
- 900 observations samples (60 s)

Estimate of Biazimuth-Delay Power Spectrum

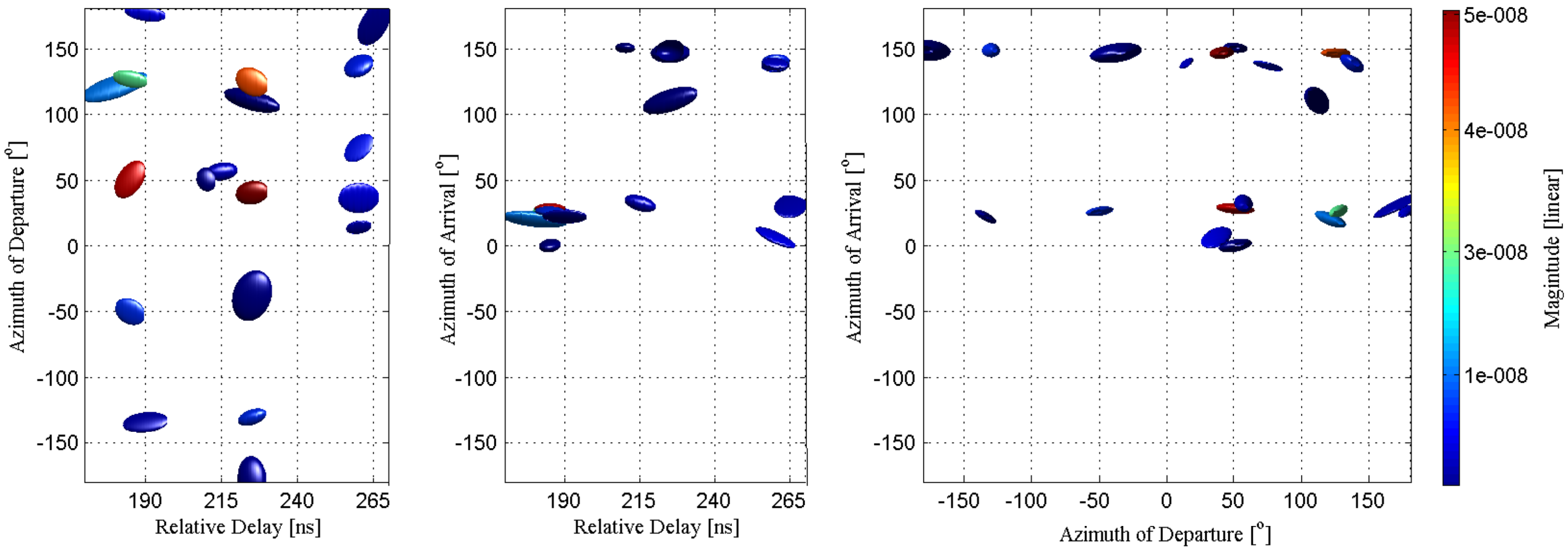
Estimated 3dB-spread surfaces of individual dispersed path components:



The color of the surfaces codes the path power estimates according to the color scale reported on the right.

Estimate of Biazimuth-Delay Power Spectrum

Estimated 3dB-spread surfaces of individual dispersed path components:

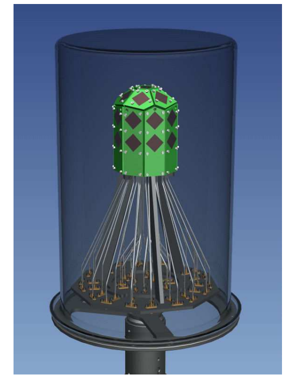


Experimental Investigations

Tx environment

Rx environment

Antenna array

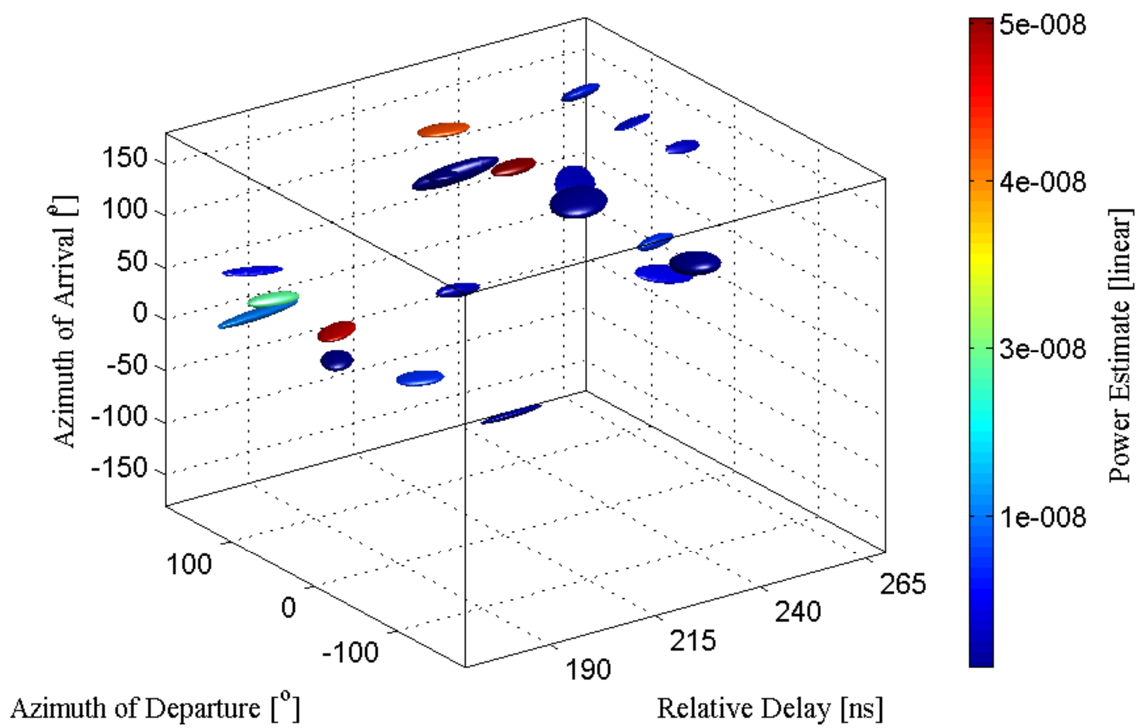


Measurement set-up:

- Carrier frequency: 5.2 GHz
- Bandwidth: 200 MHz
- 50-element Tx and Rx Patch arrays with $\pm 45^{\circ}$ slanted antennas
- Tx array height: 1.53 m; Rx array height: 0.82 m
- Big hall, time-variant
- 900 observations samples (60 s)

Estimate of Biazimuth-Delay Power Spectrum

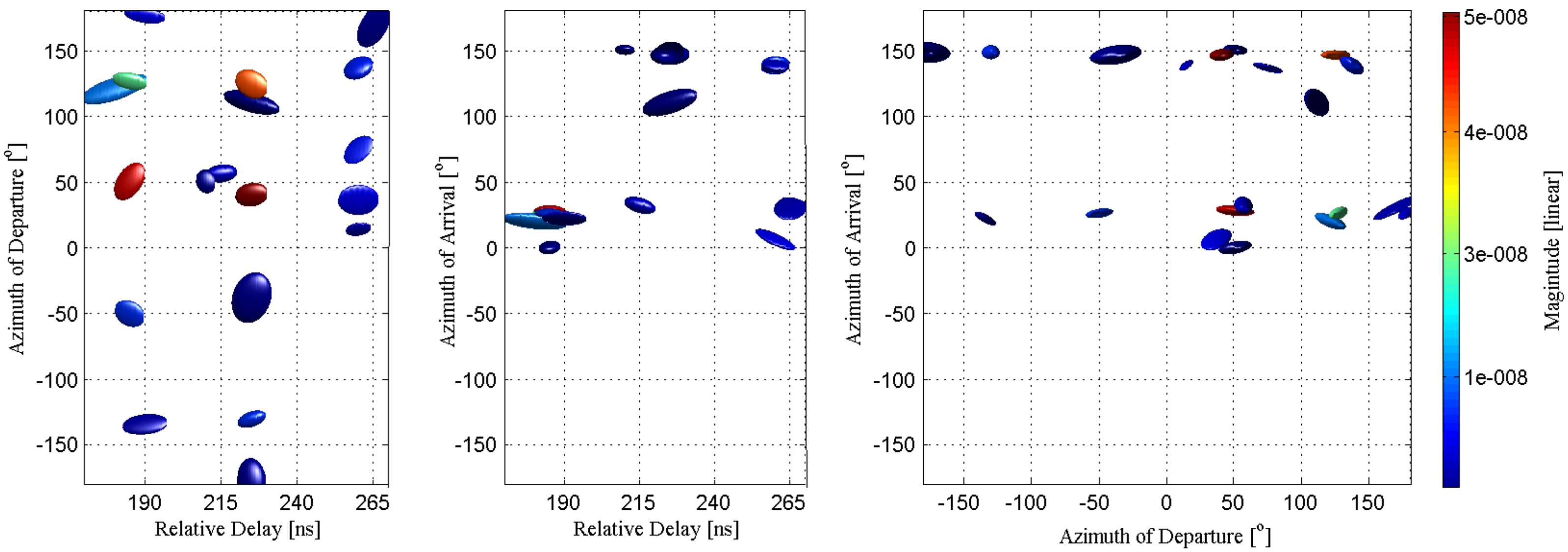
Estimated 3dB-spread surfaces of individual dispersed path components:



The color of the surfaces codes the path power estimates according to the color scale reported on the right.

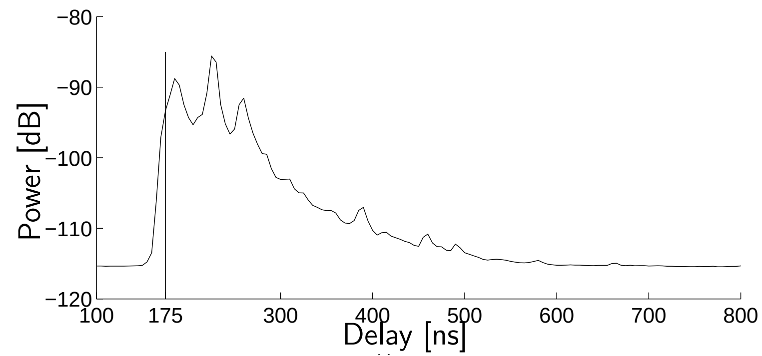
Estimate of Biazimuth-Delay Power Spectrum

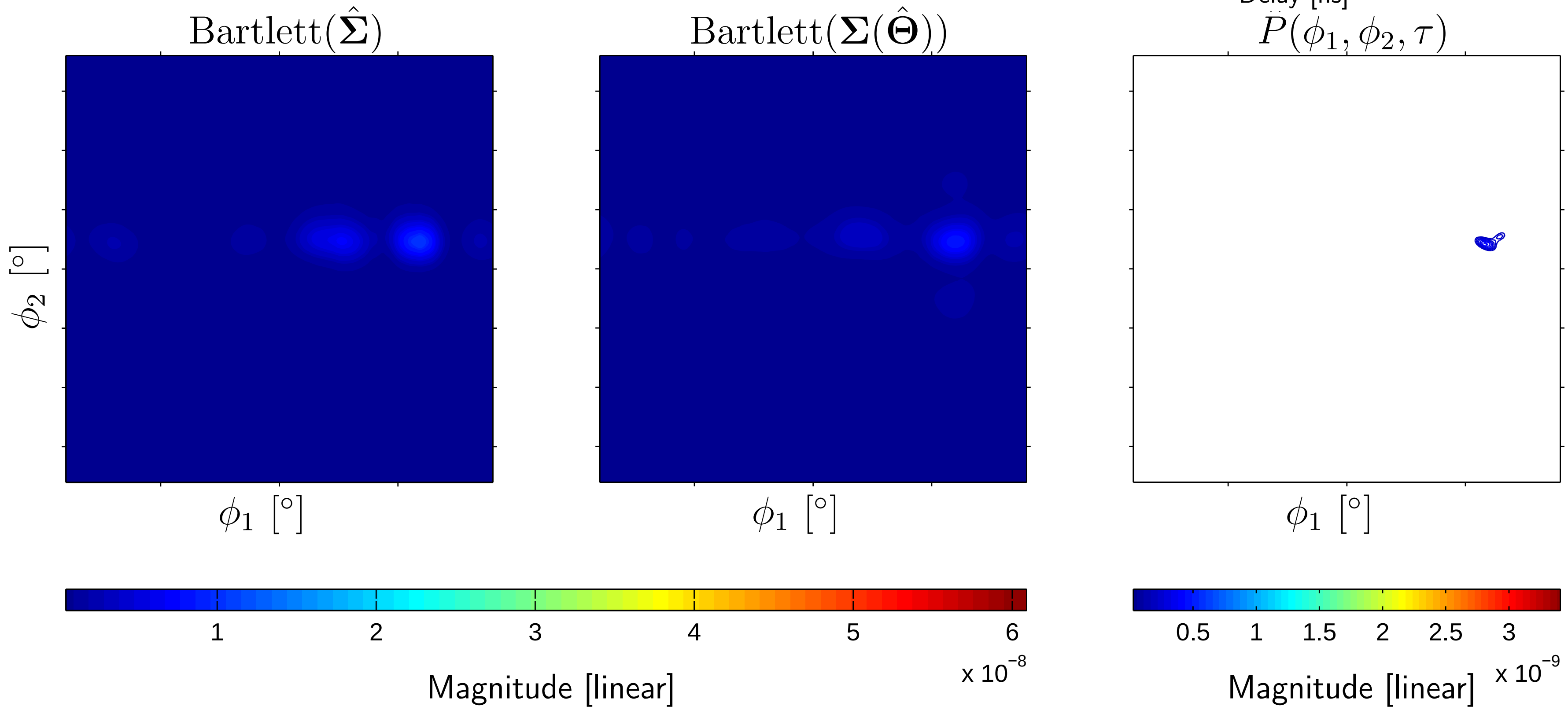
Estimated 3dB-spread surfaces of individual dispersed path components:



Estimate of biazimuth-delay power spectrum

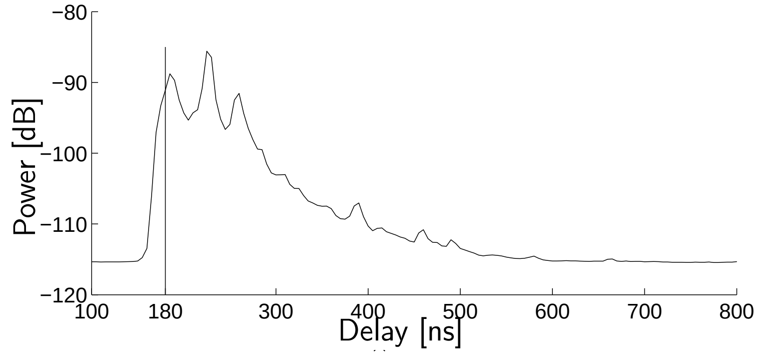
$\tau = 175$ ns

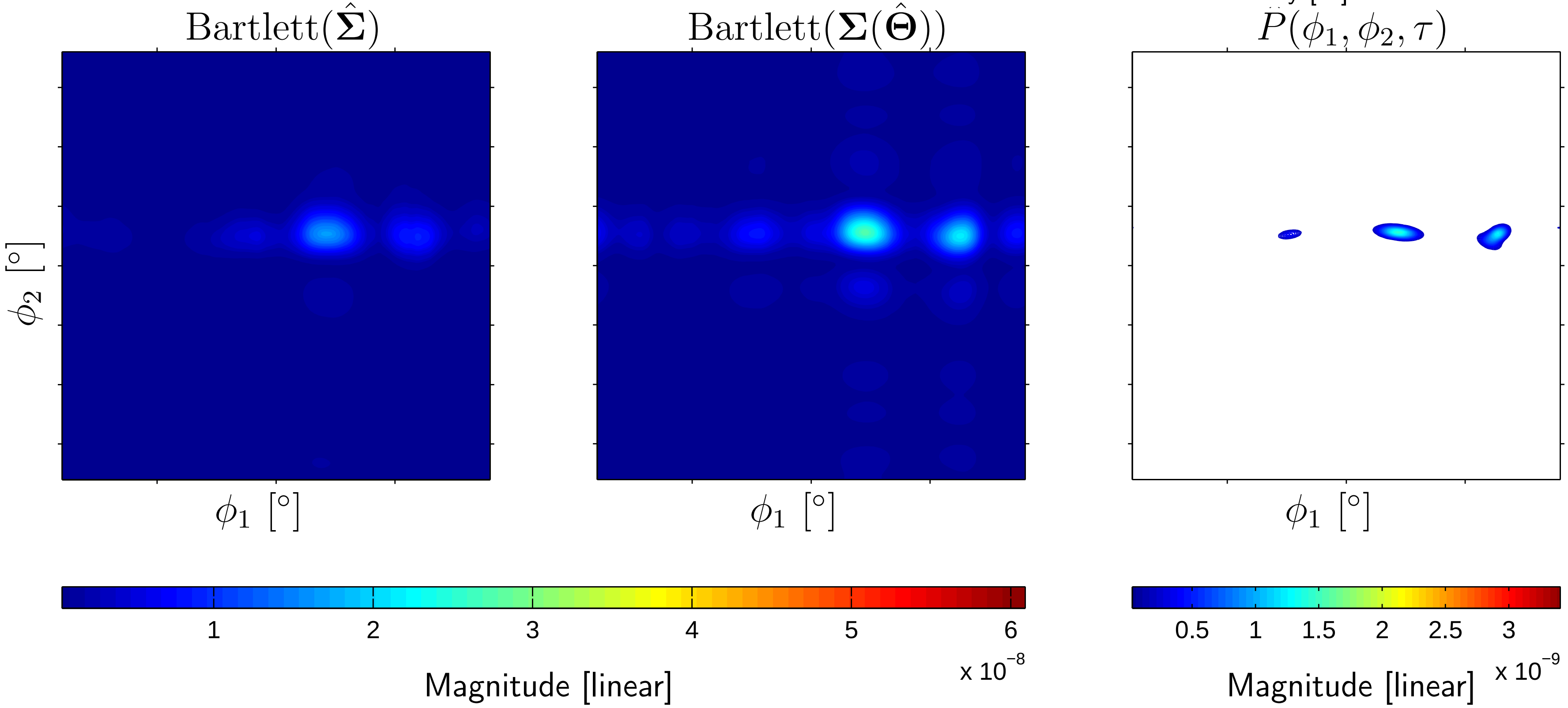




Estimate of biazimuth-delay power spectrum

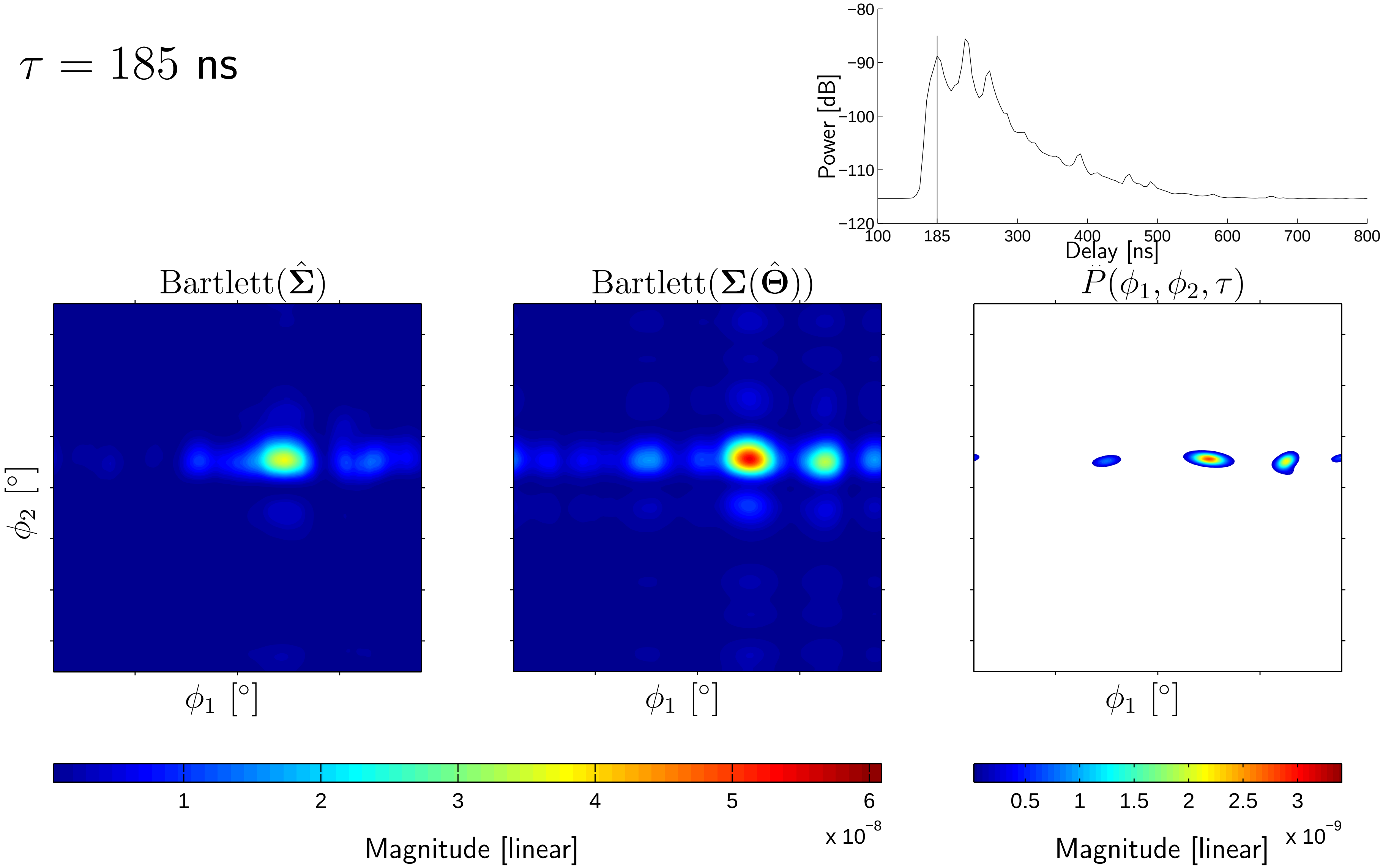
$\tau = 180$ ns





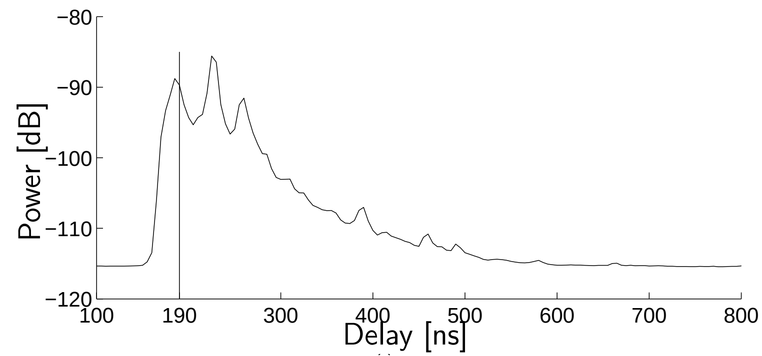
Estimate of biazimuth-delay power spectrum

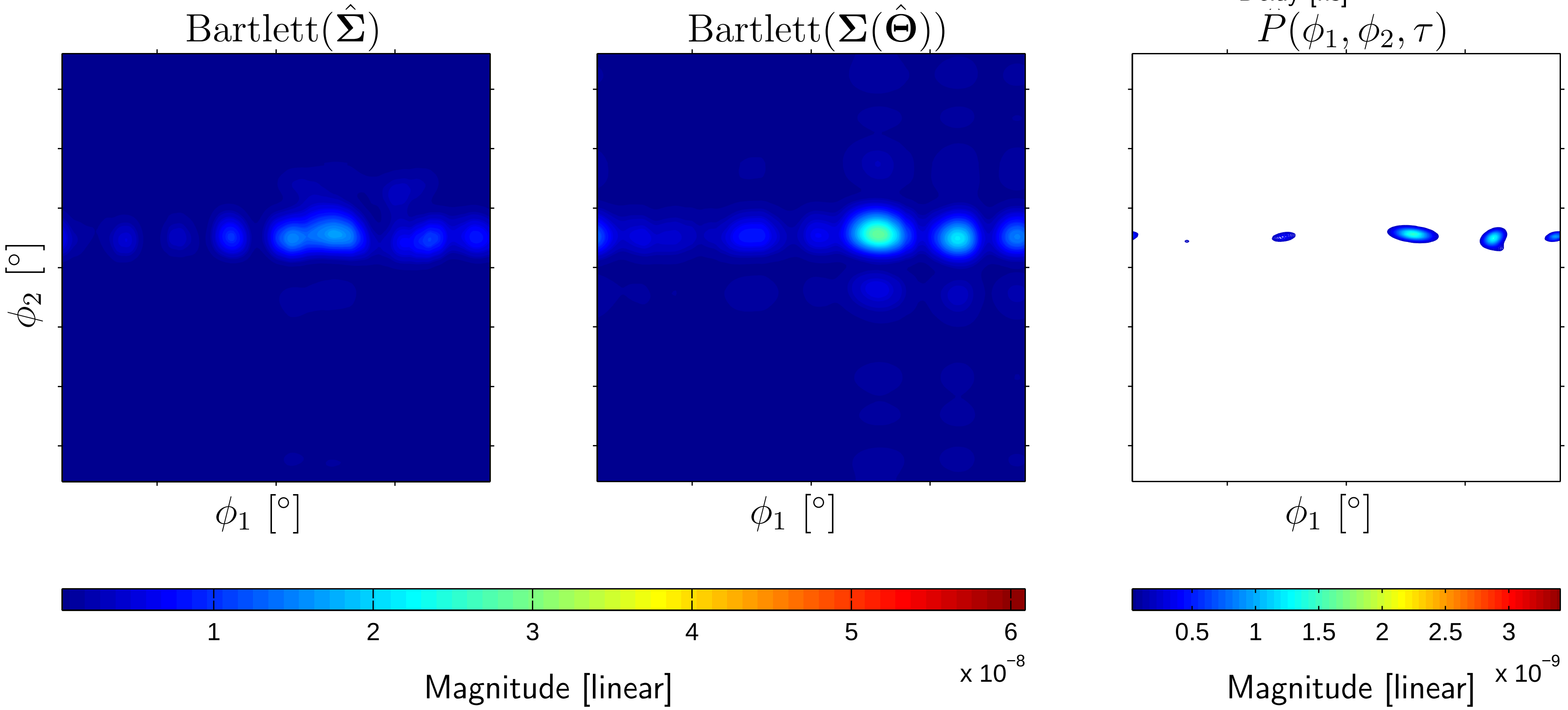
$\tau = 185$ ns



Estimate of biazimuth-delay power spectrum

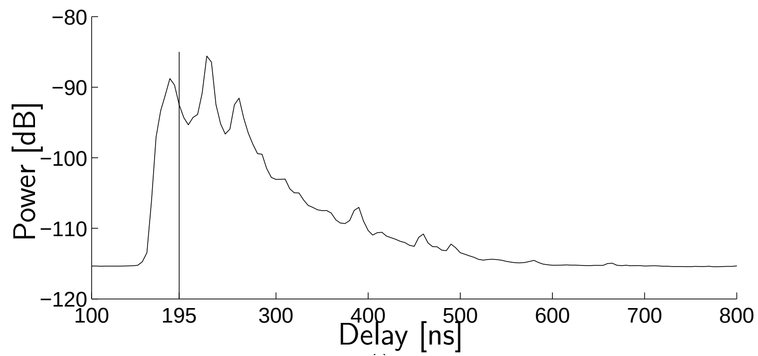
$\tau = 190$ ns

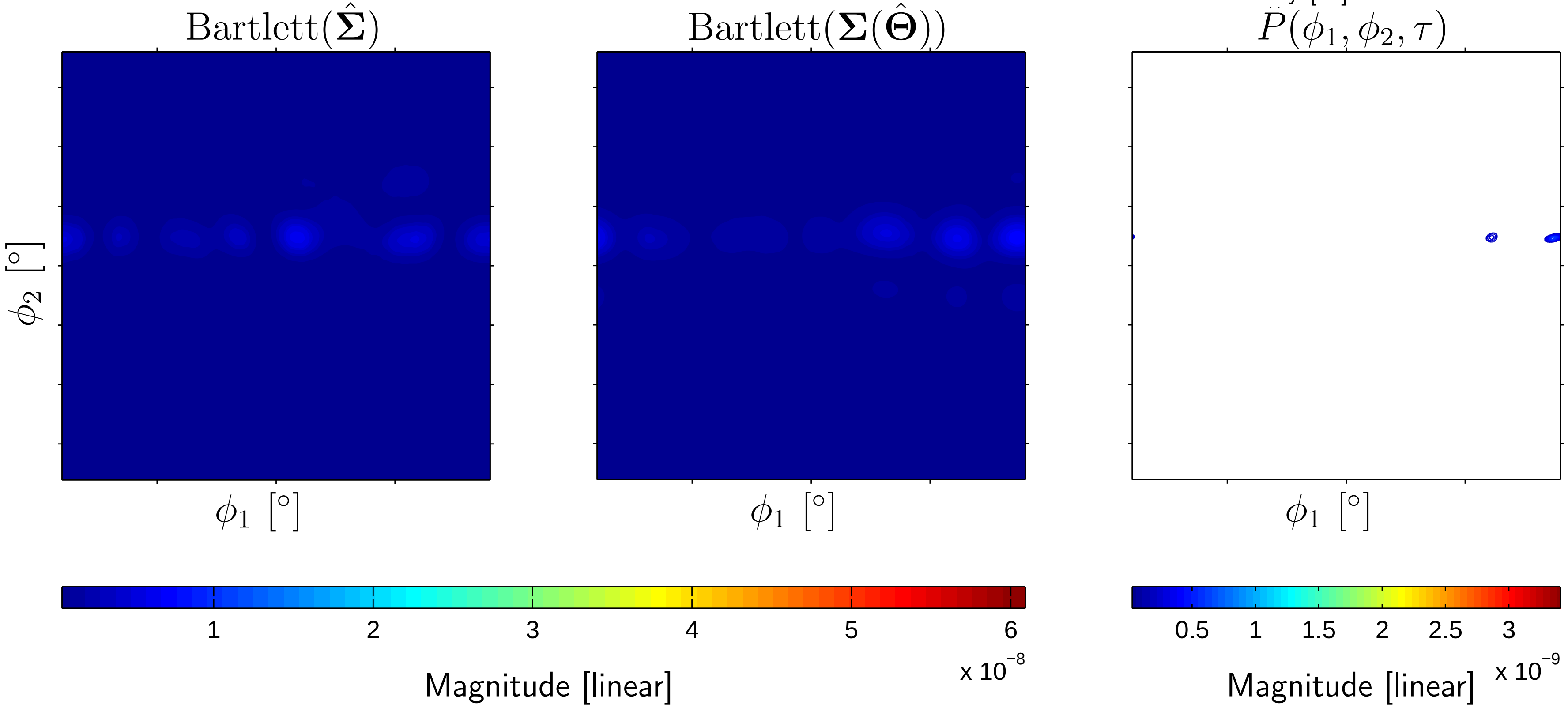




Estimate of biazimuth-delay power spectrum

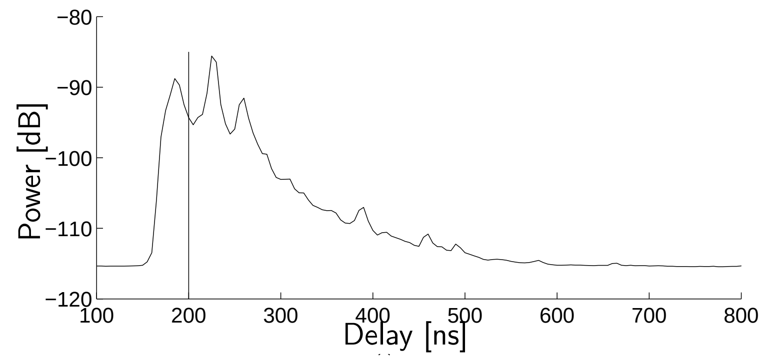
$\tau = 195$ ns

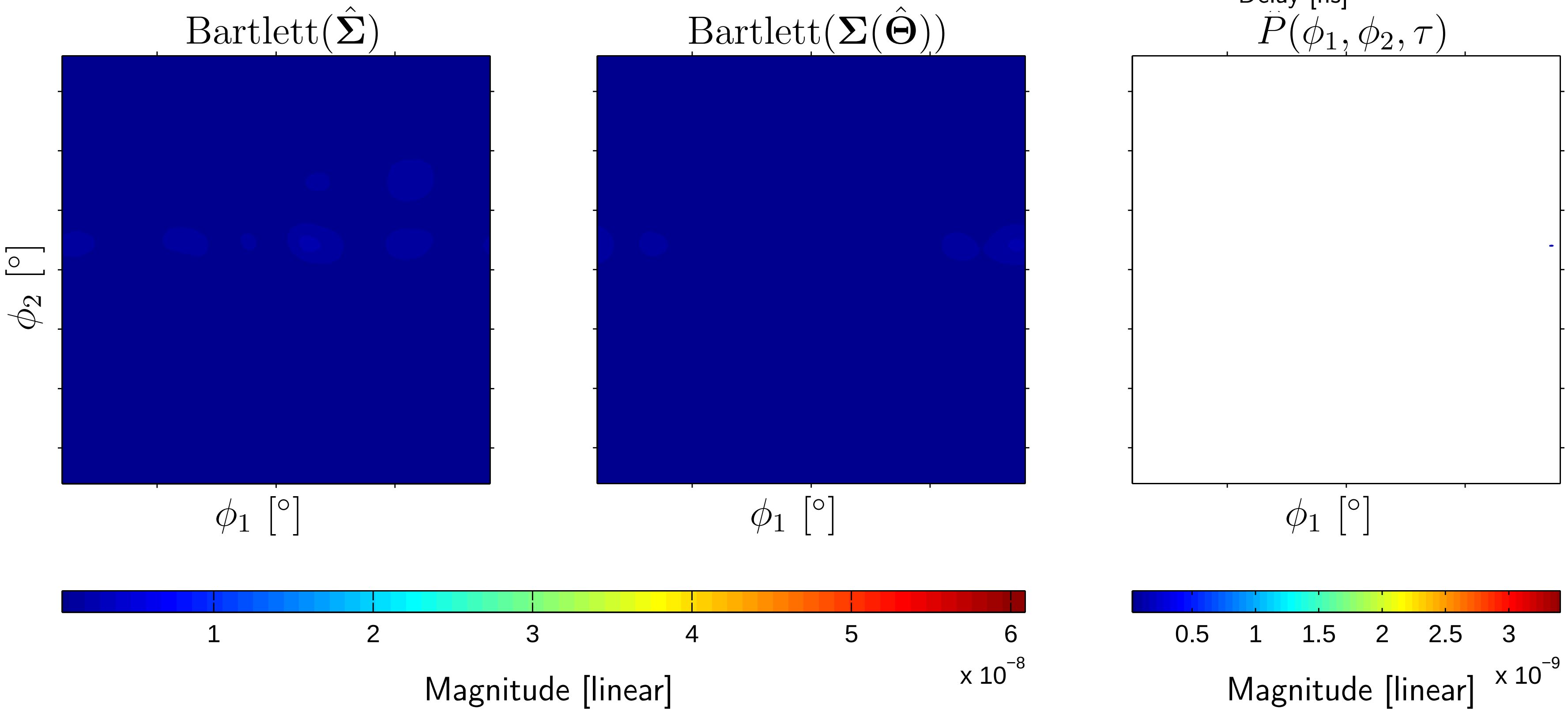




Estimate of biazimuth-delay power spectrum

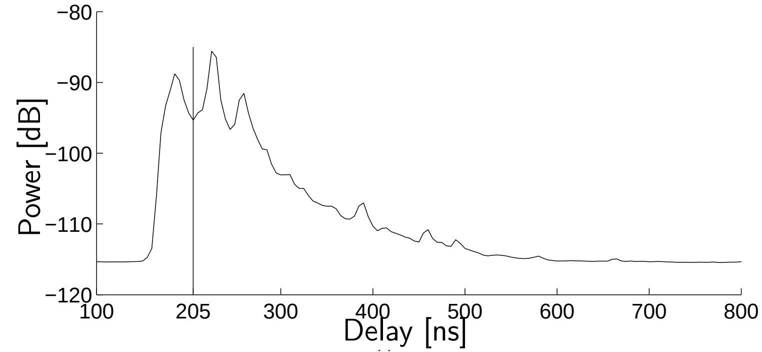
$\tau = 200$ ns

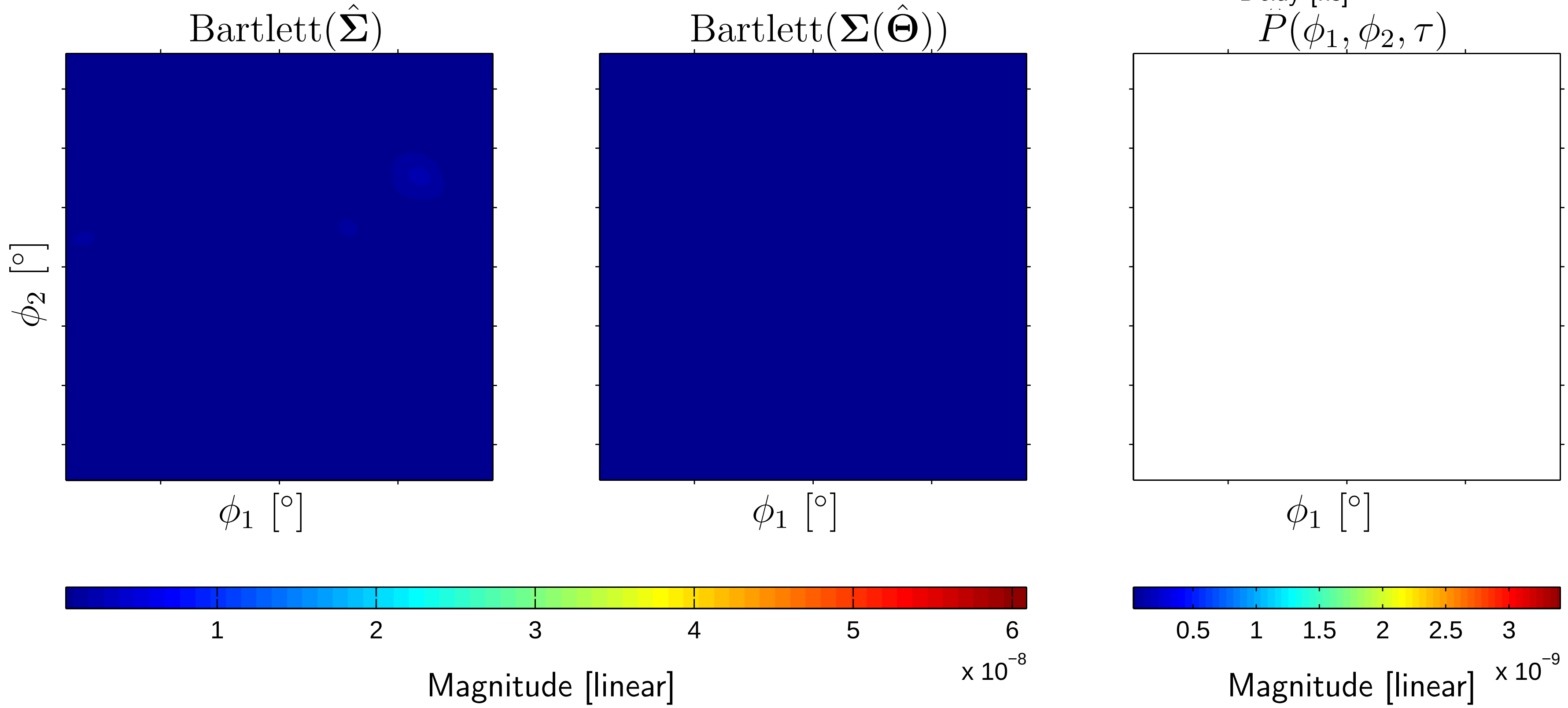




Estimate of biazimuth-delay power spectrum

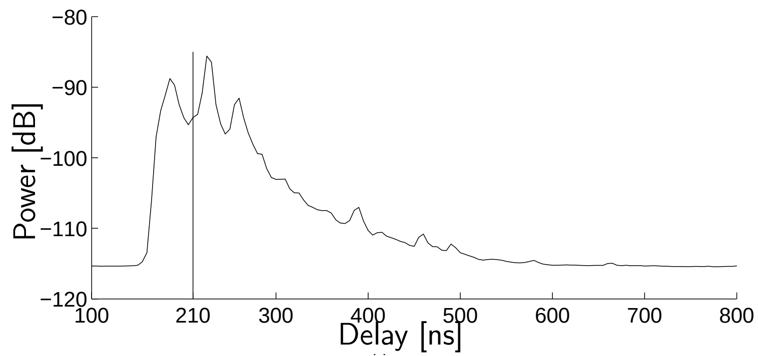
$\tau = 205$ ns

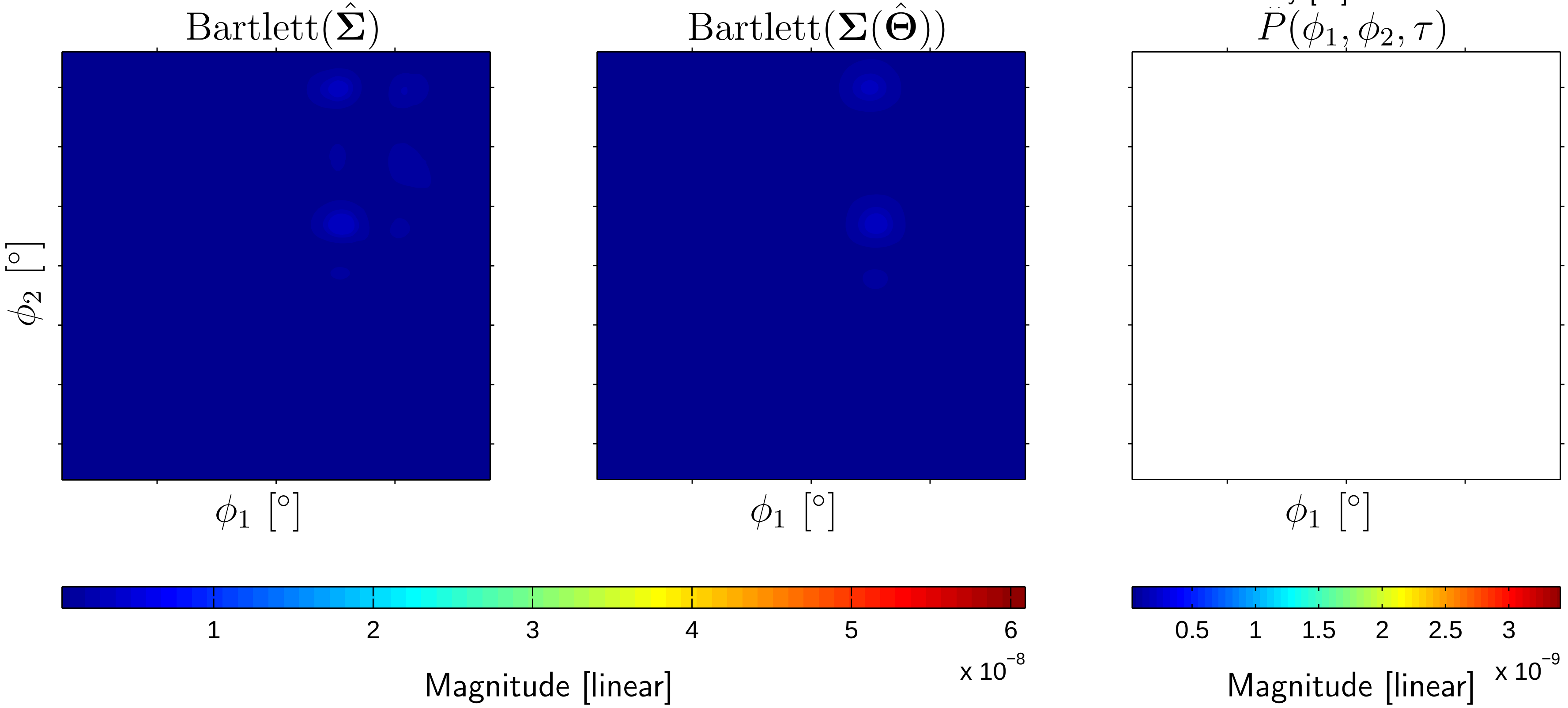




Estimate of biazimuth-delay power spectrum

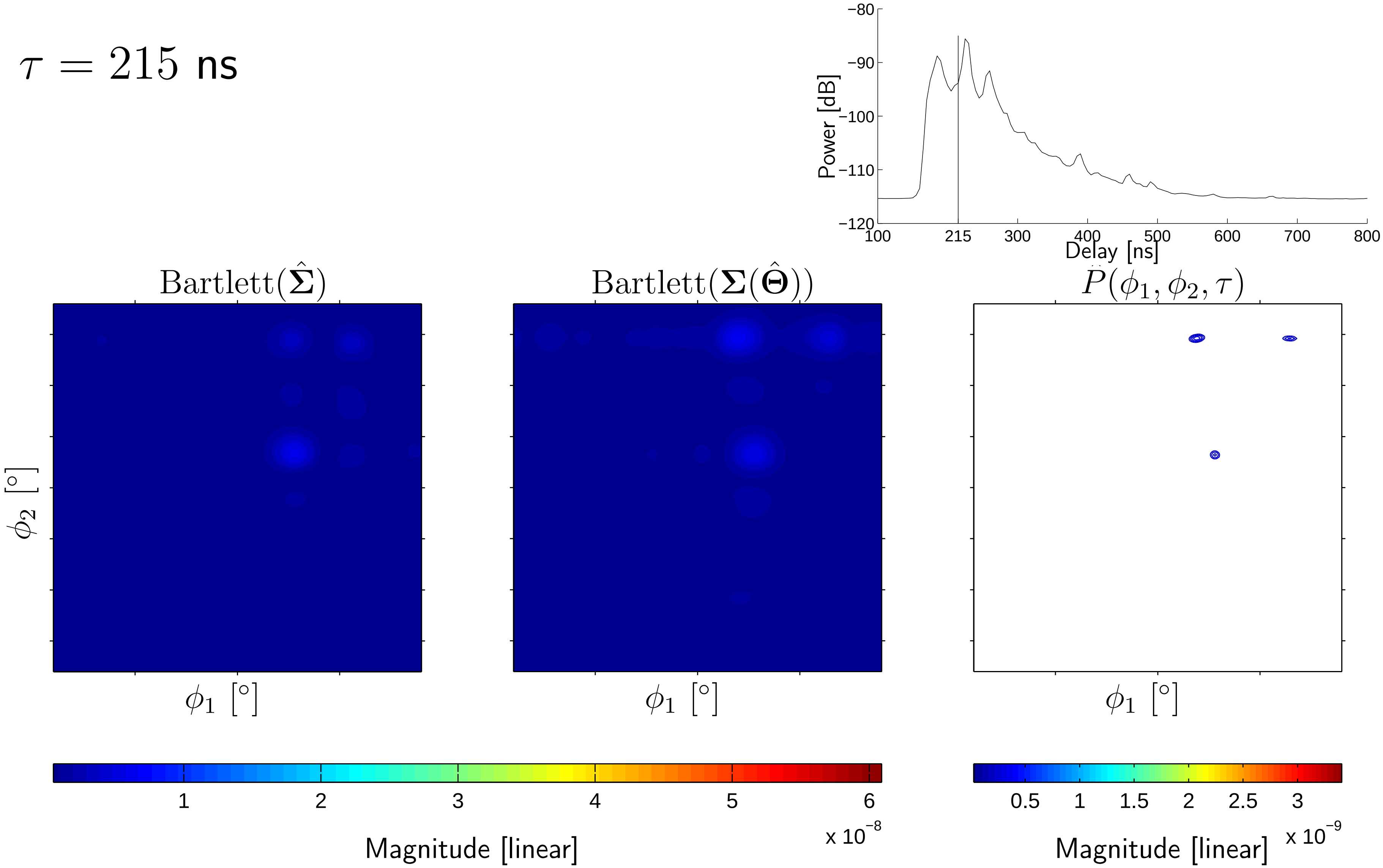
$\tau = 210$ ns





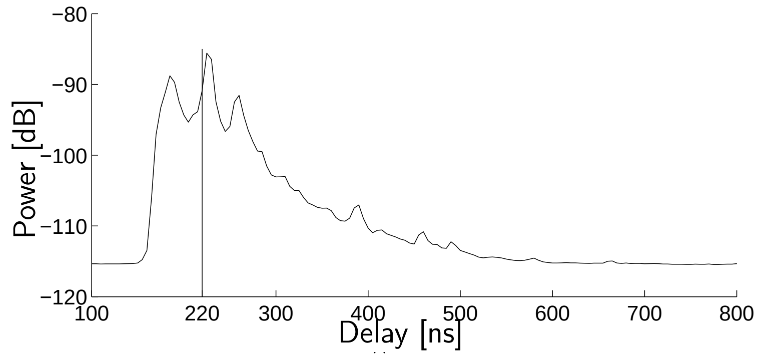
Estimate of biazimuth-delay power spectrum

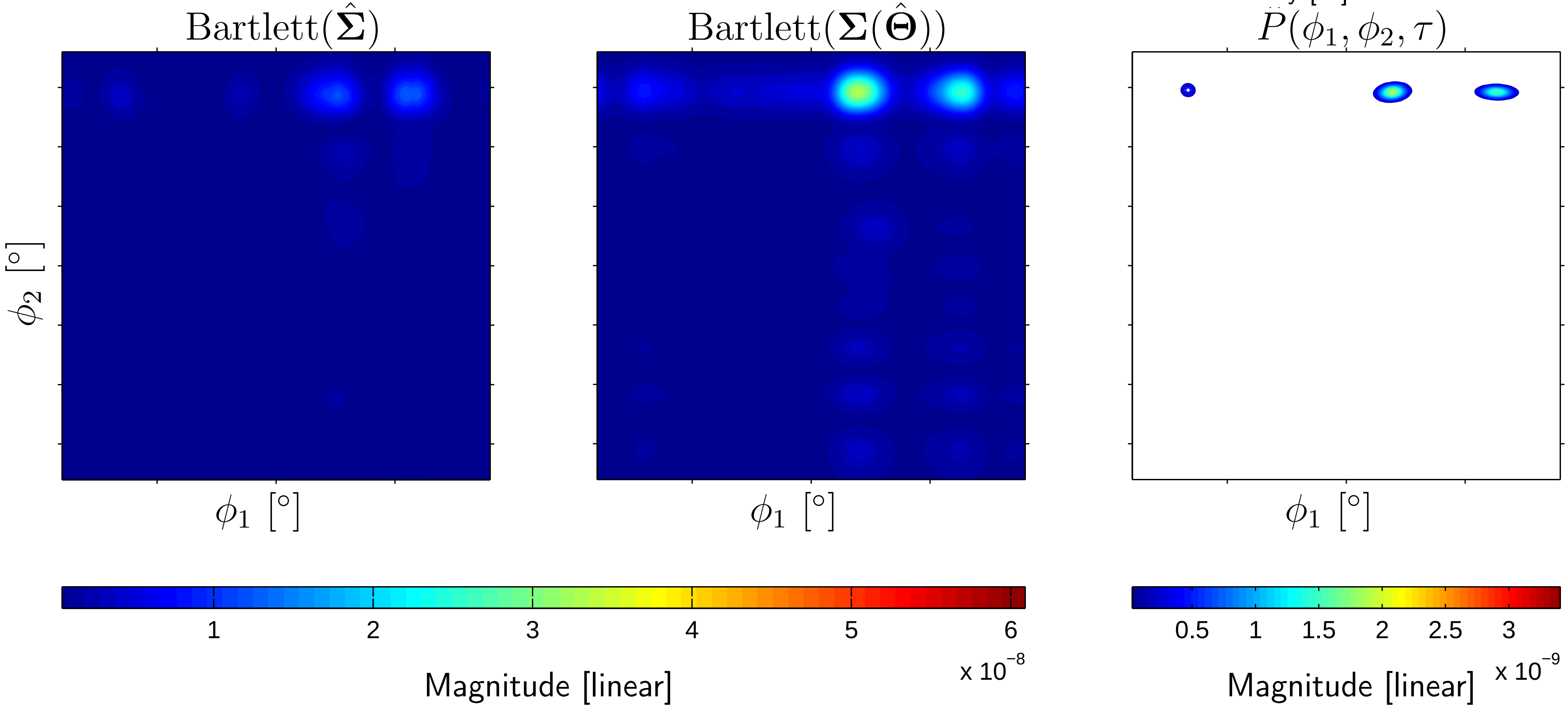
$\tau = 215$ ns



Estimate of biazimuth-delay power spectrum

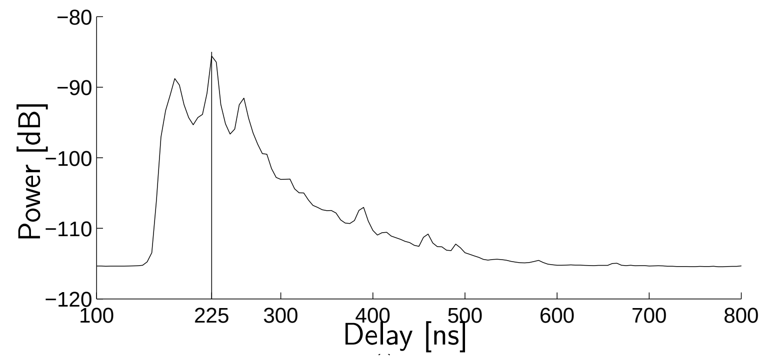
$\tau = 220$ ns

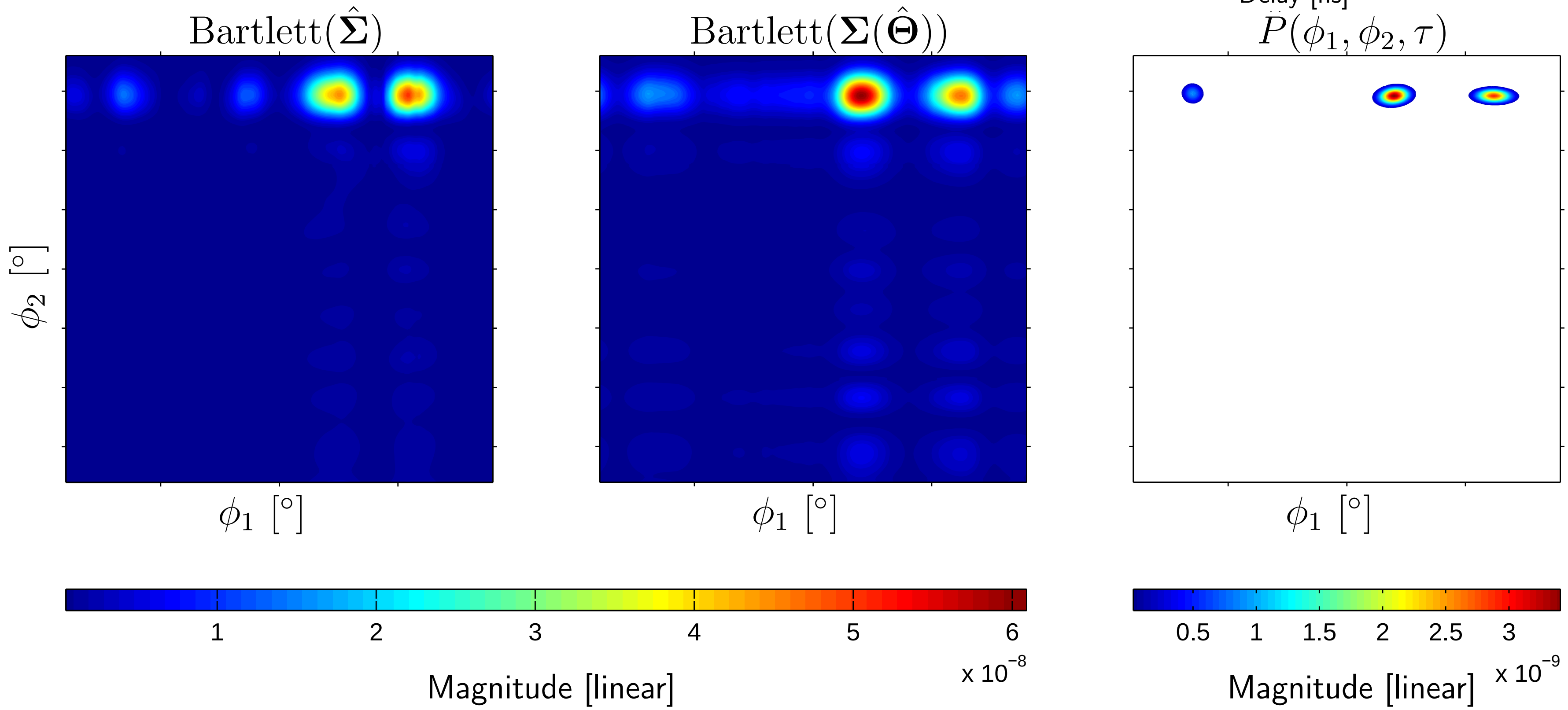




Estimate of biazimuth-delay power spectrum

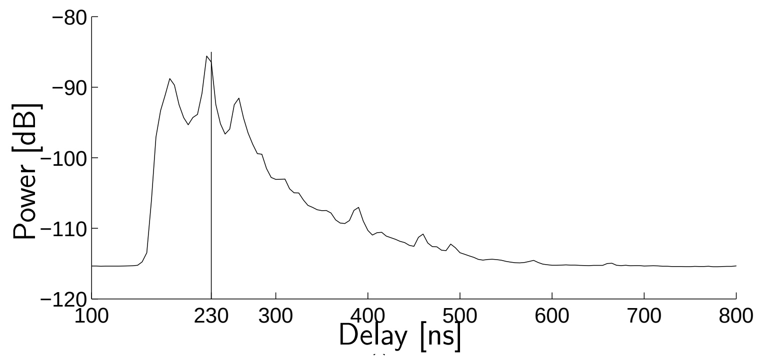
$\tau = 225$ ns

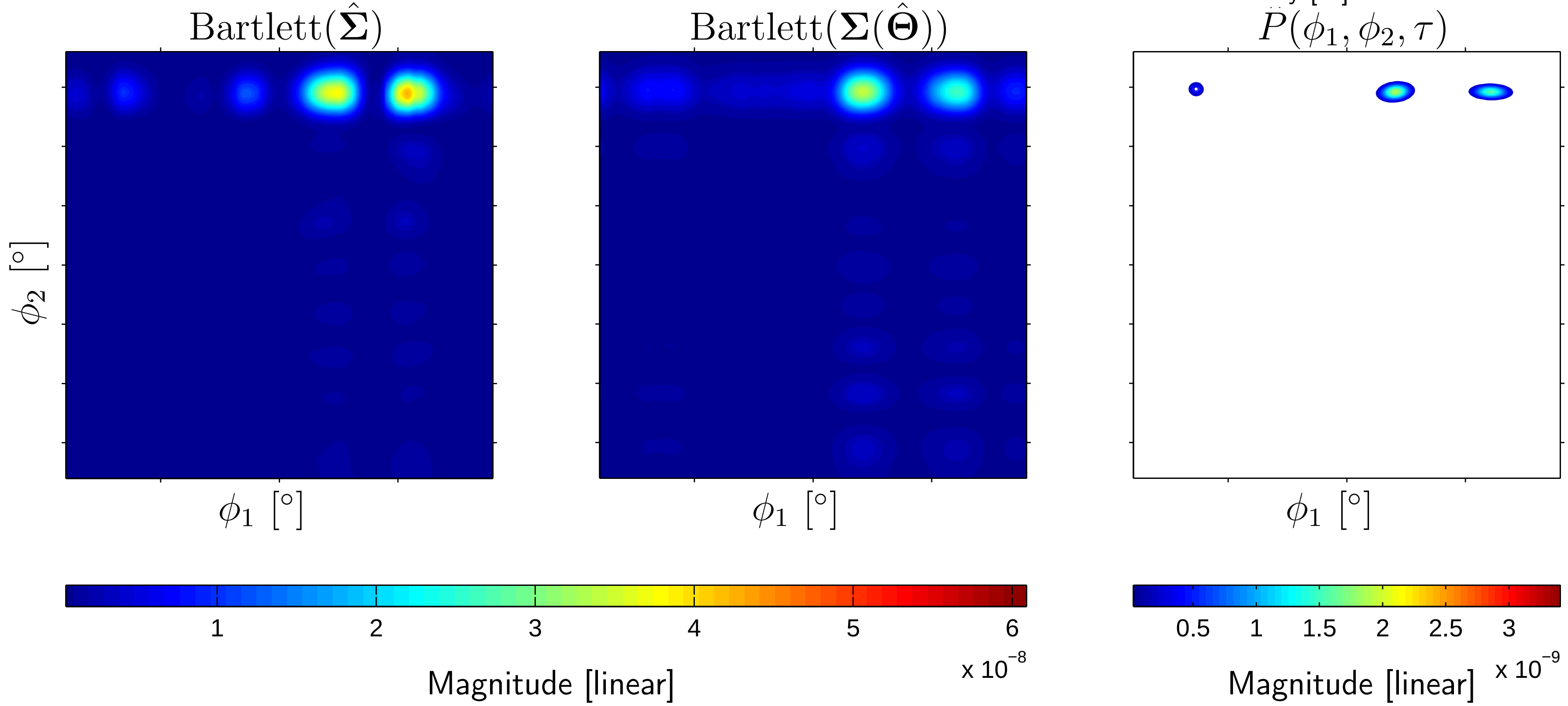




Estimate of biazimuth-delay power spectrum

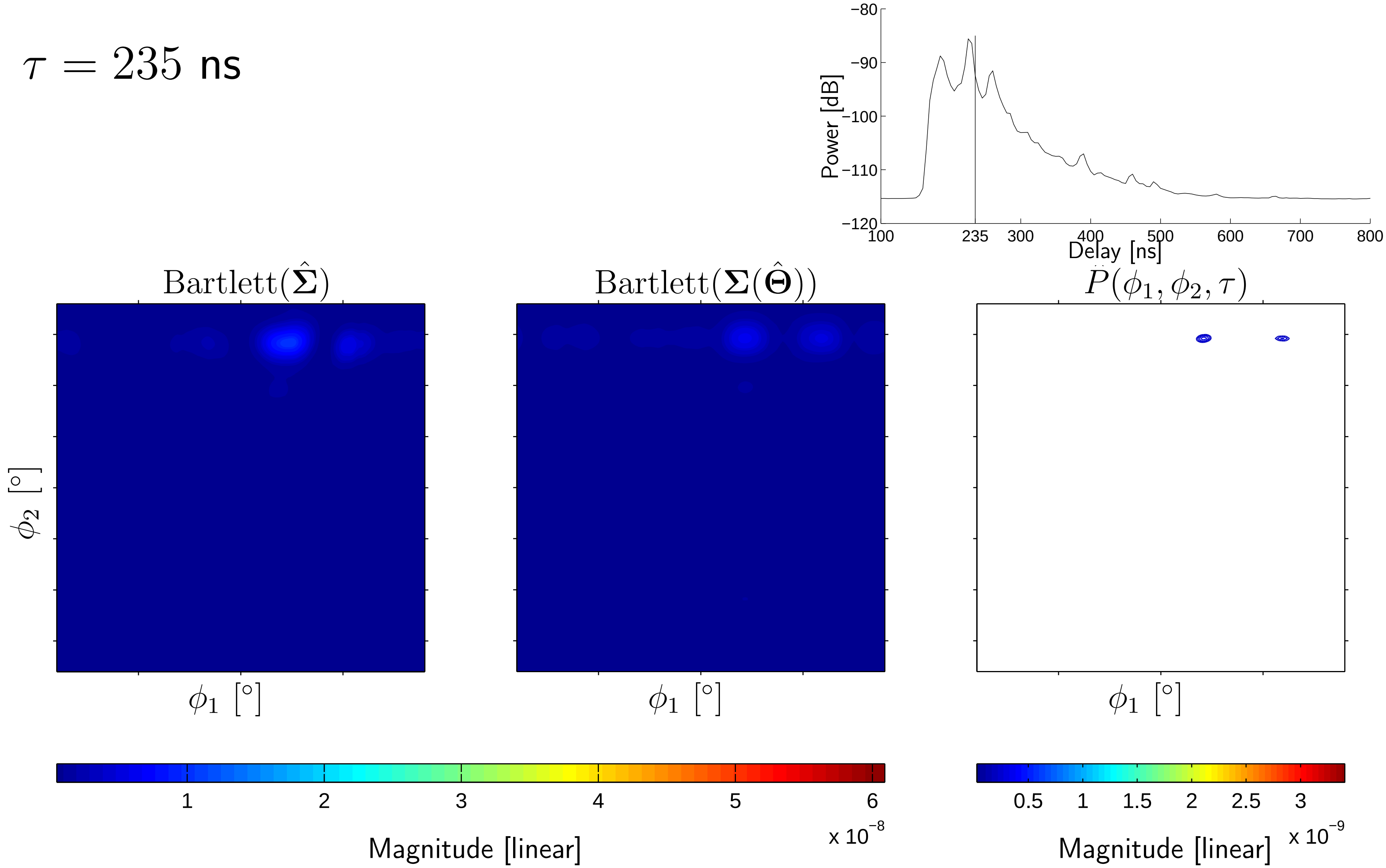
$\tau = 230$ ns





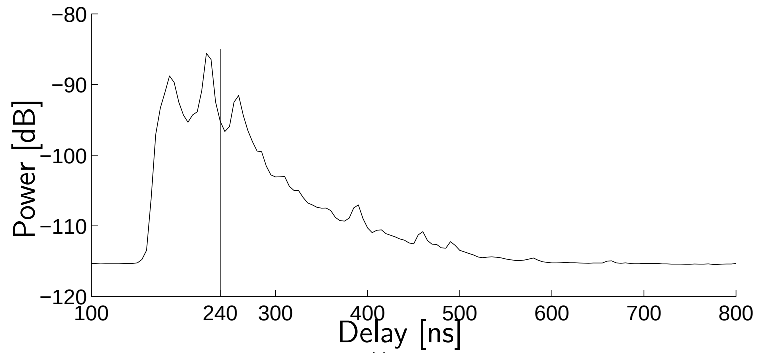
Estimate of biazimuth-delay power spectrum

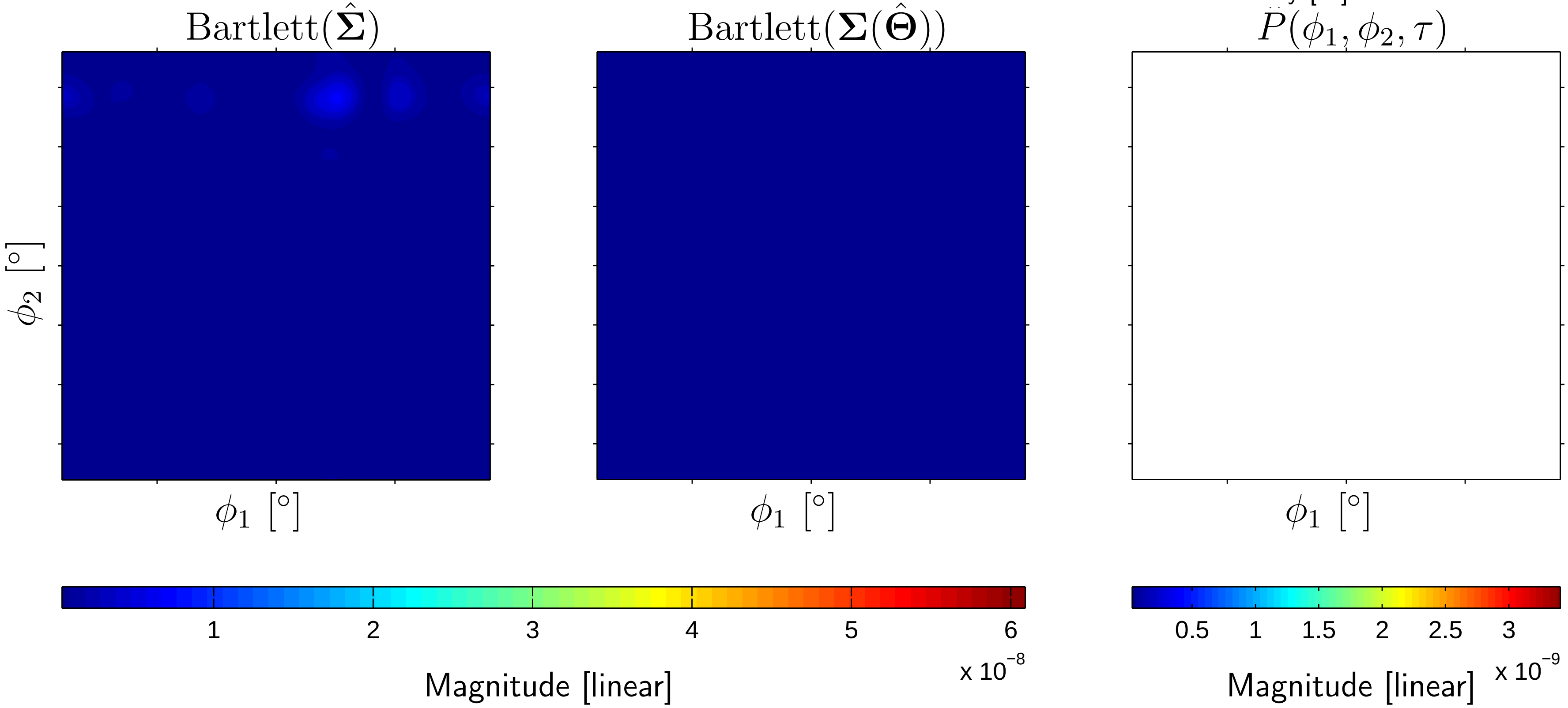
$\tau = 235$ ns



Estimate of biazimuth-delay power spectrum

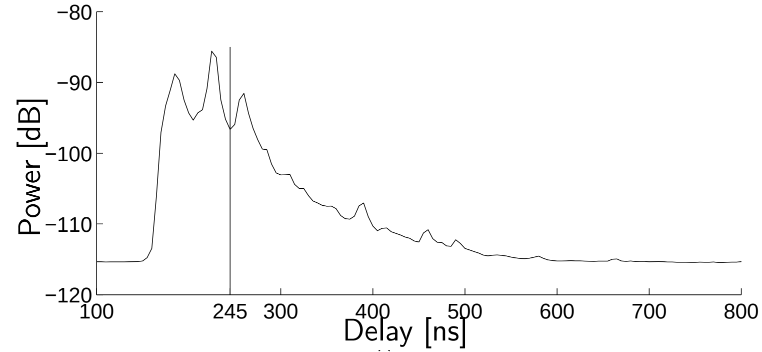
$\tau = 240$ ns

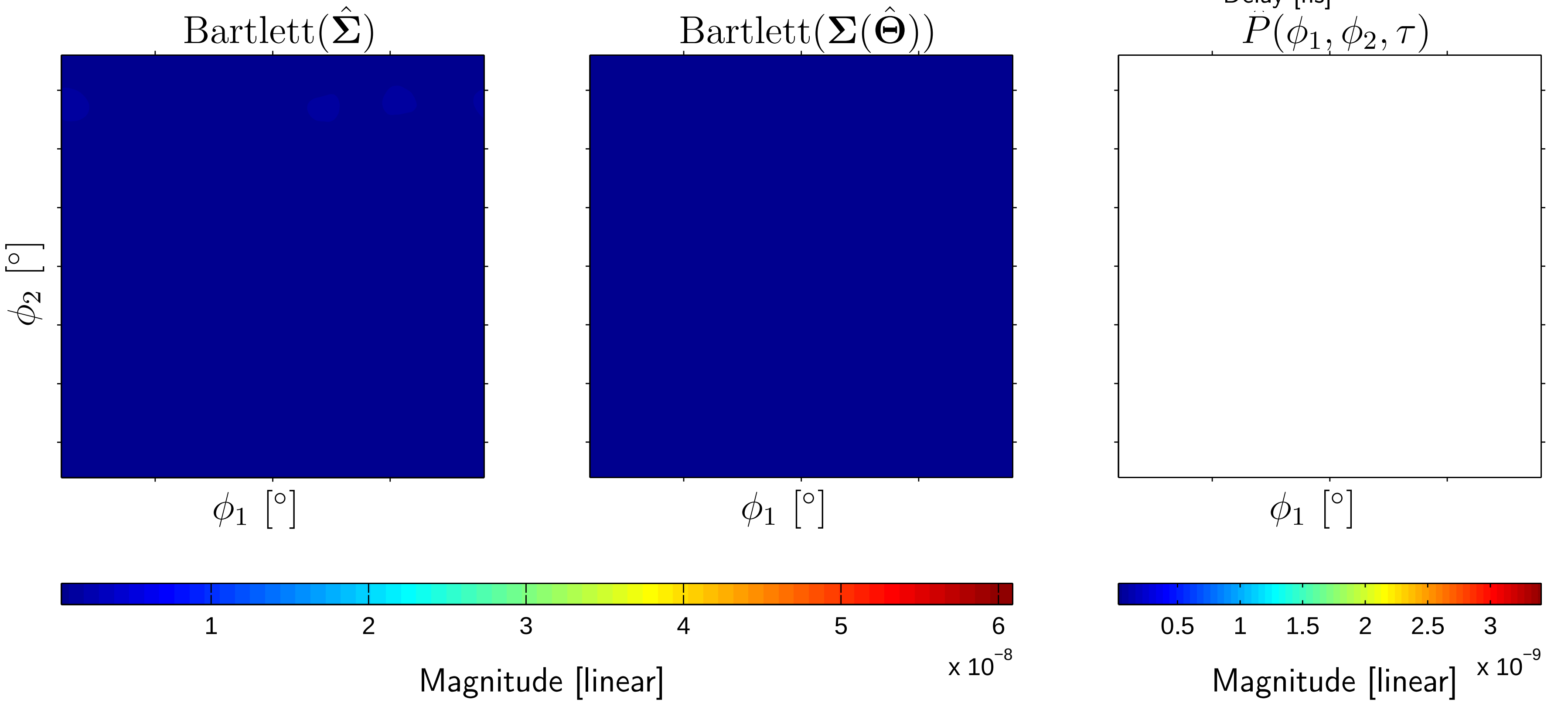




Estimate of biazimuth-delay power spectrum

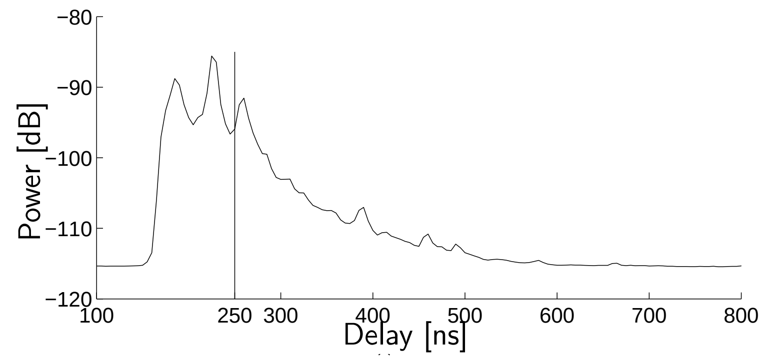
$\tau = 245$ ns

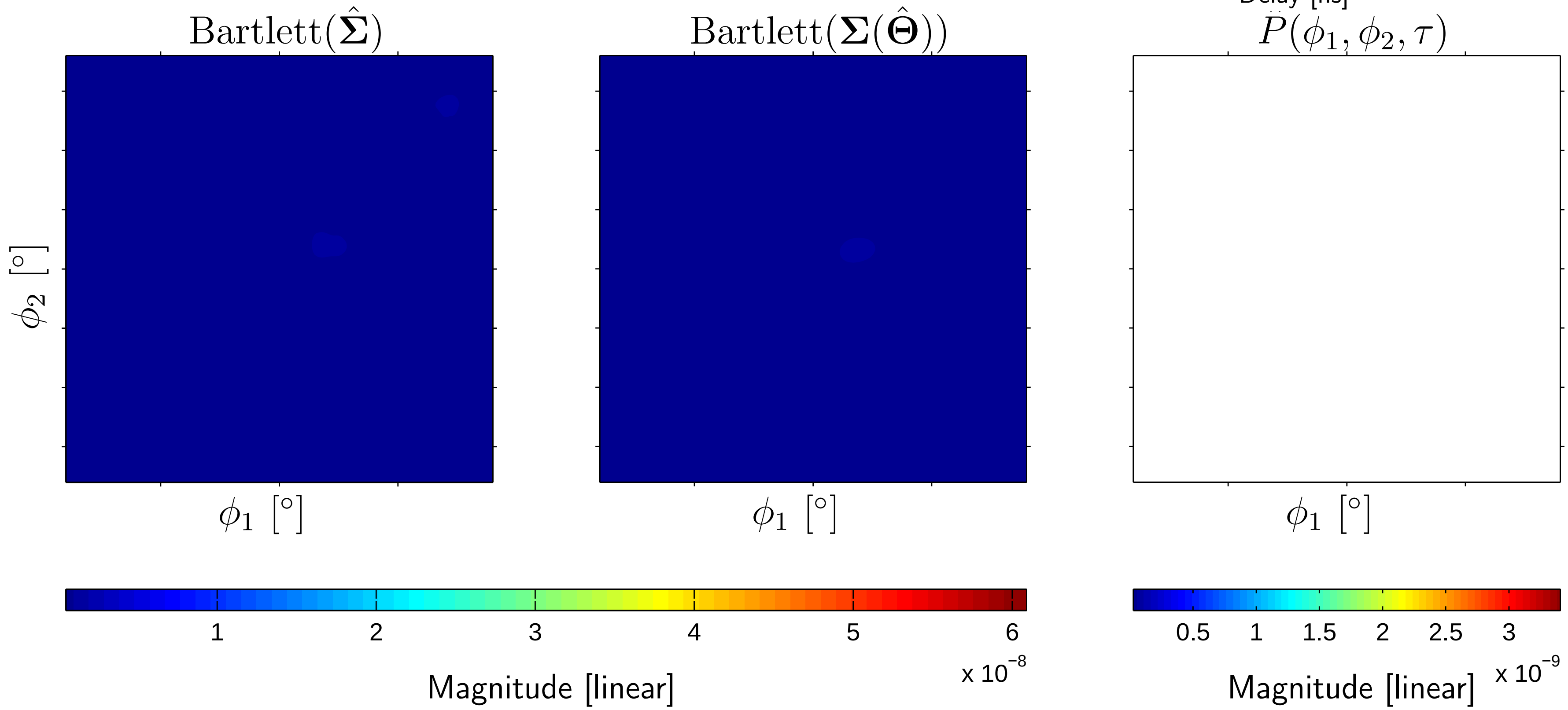




Estimate of biazimuth-delay power spectrum

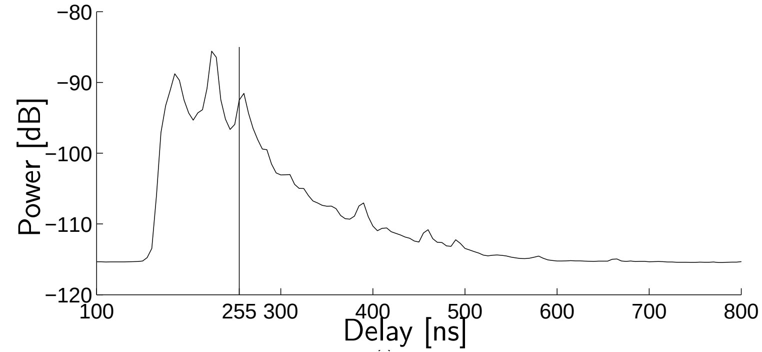
$\tau = 250$ ns

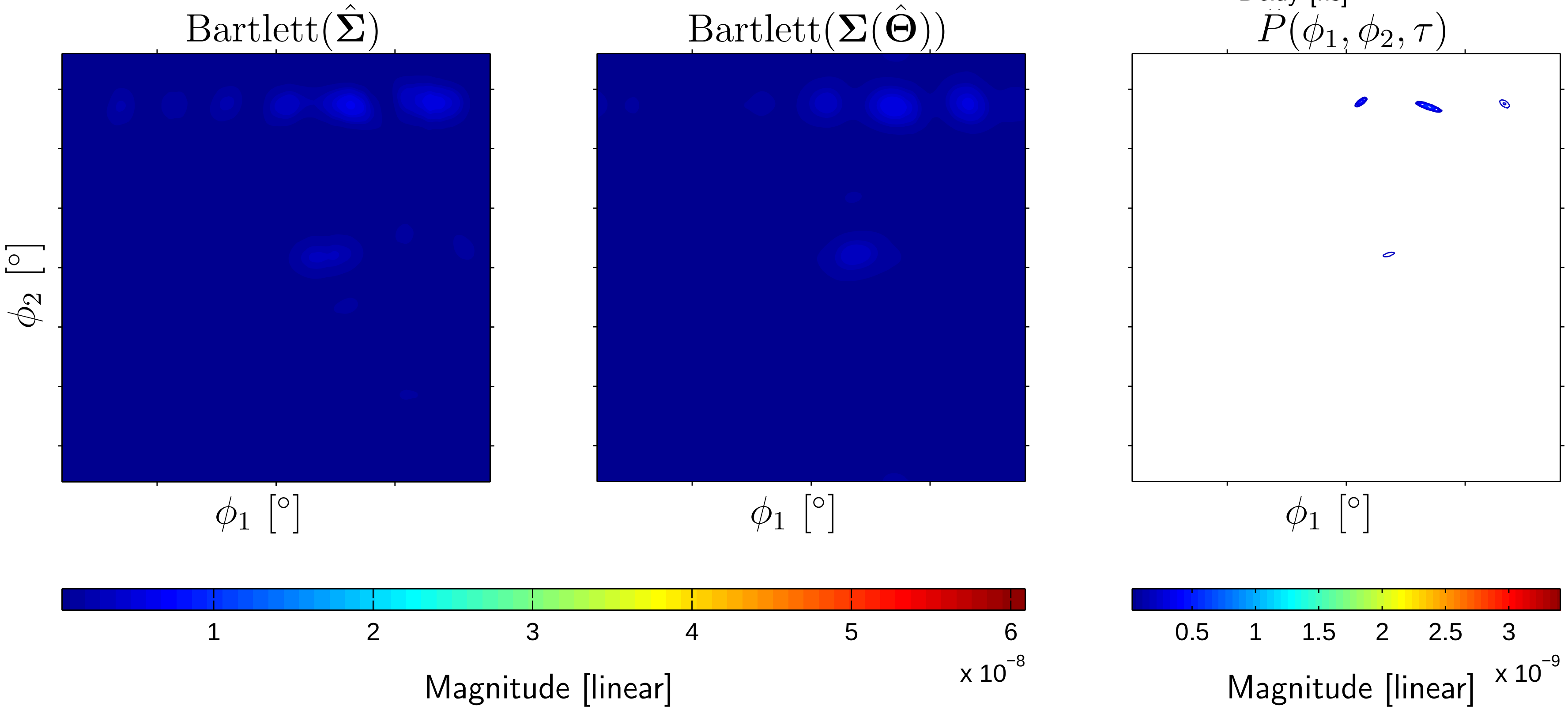




Estimate of biazimuth-delay power spectrum

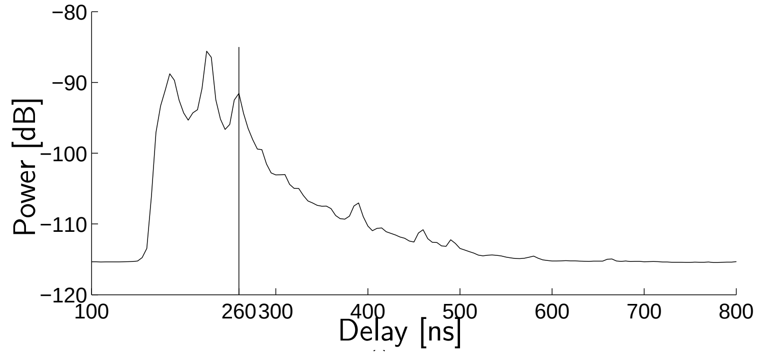
$\tau = 255$ ns

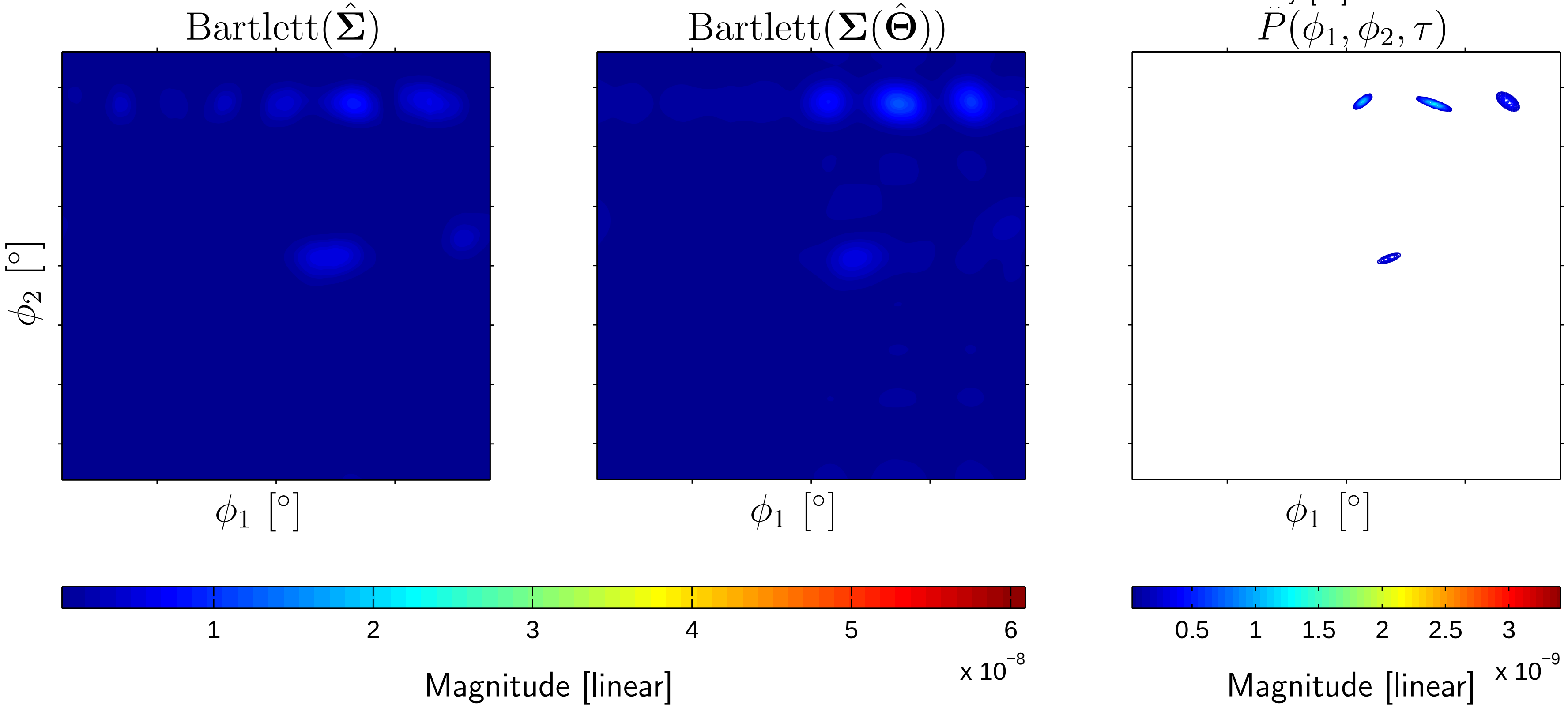




Estimate of biazimuth-delay power spectrum

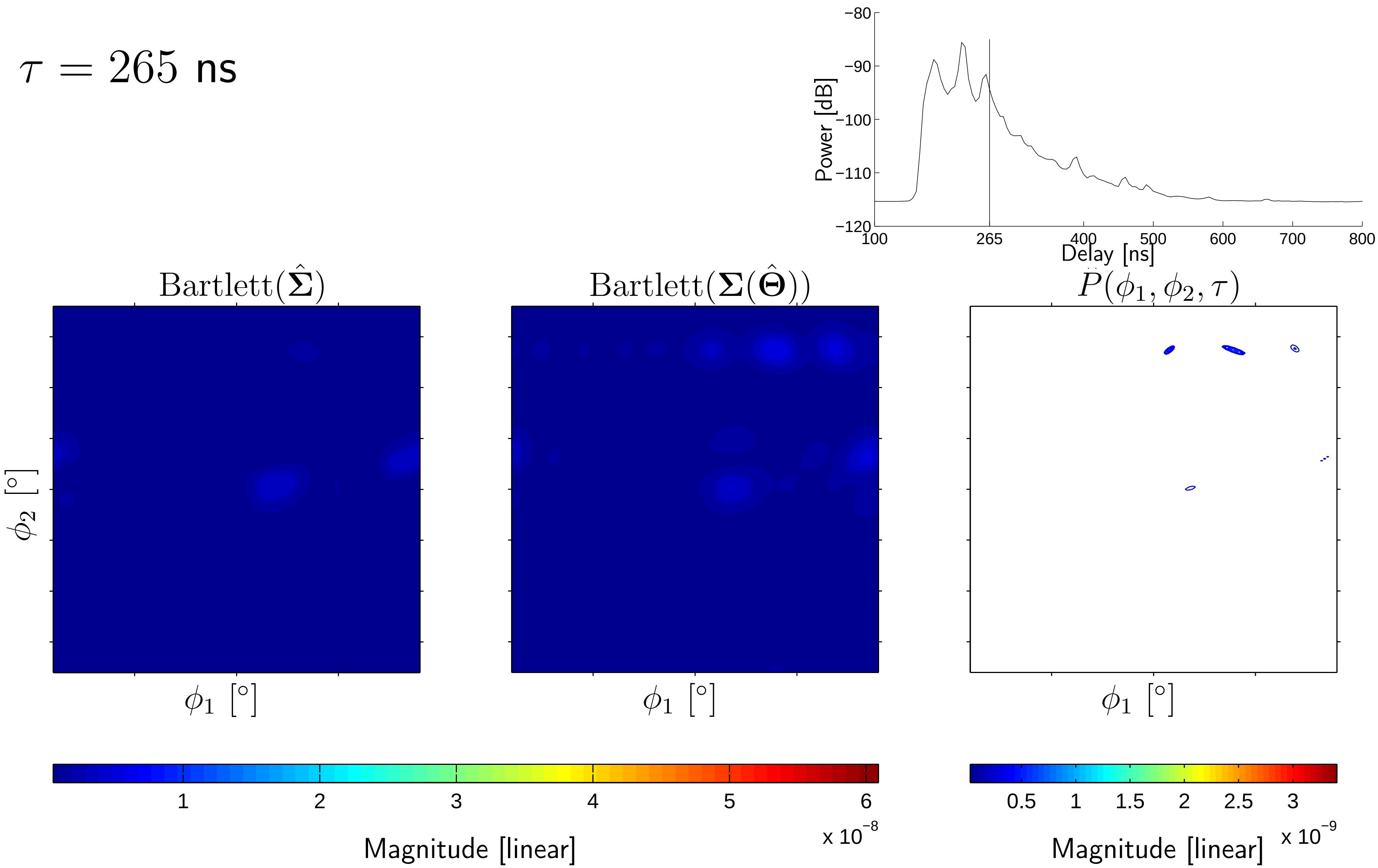
$\tau = 260$ ns





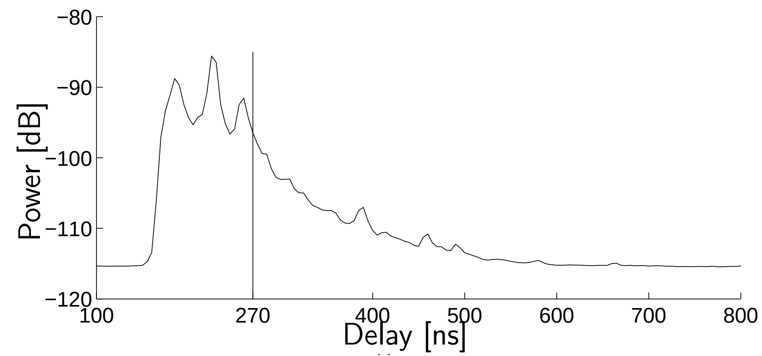
Estimate of biazimuth-delay power spectrum

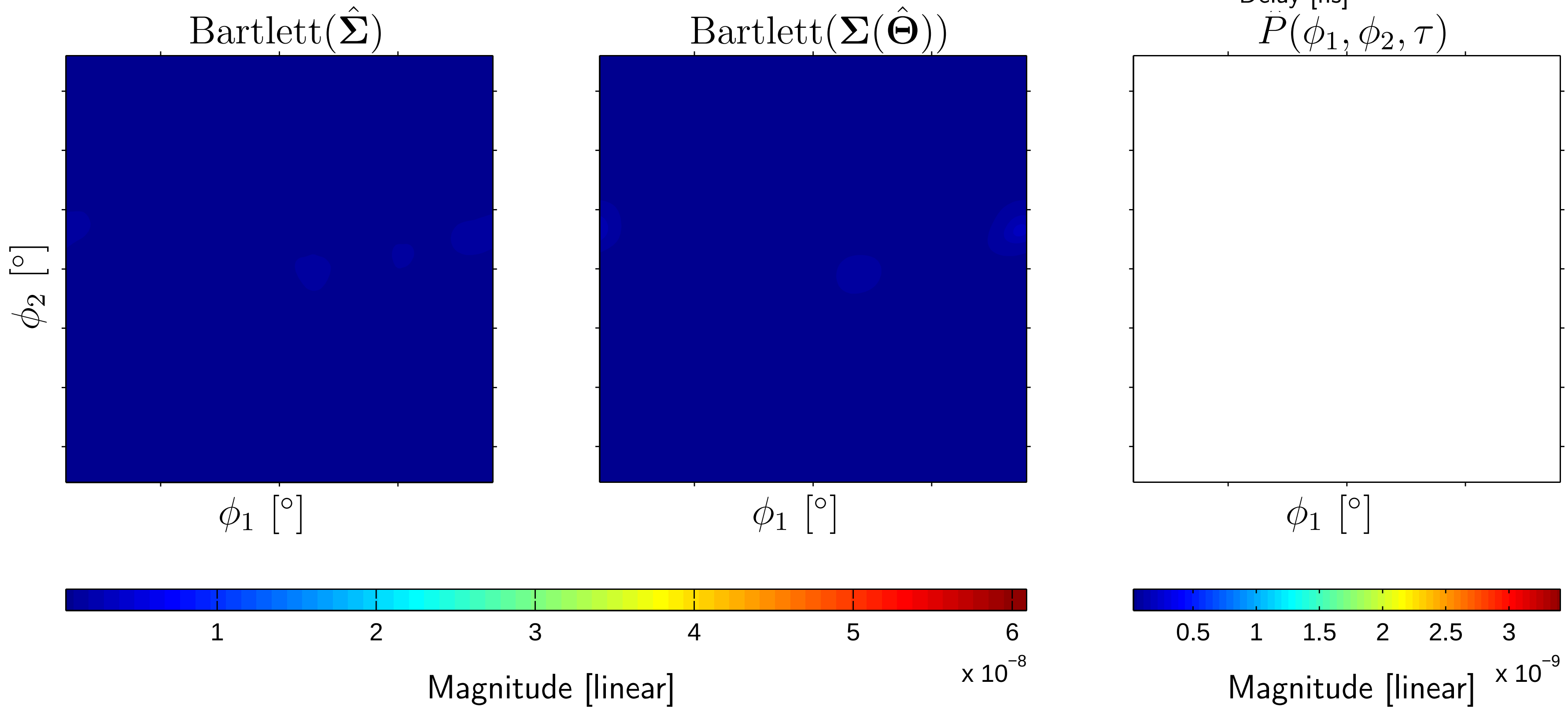
$\tau = 265$ ns



Estimate of biazimuth-delay power spectrum

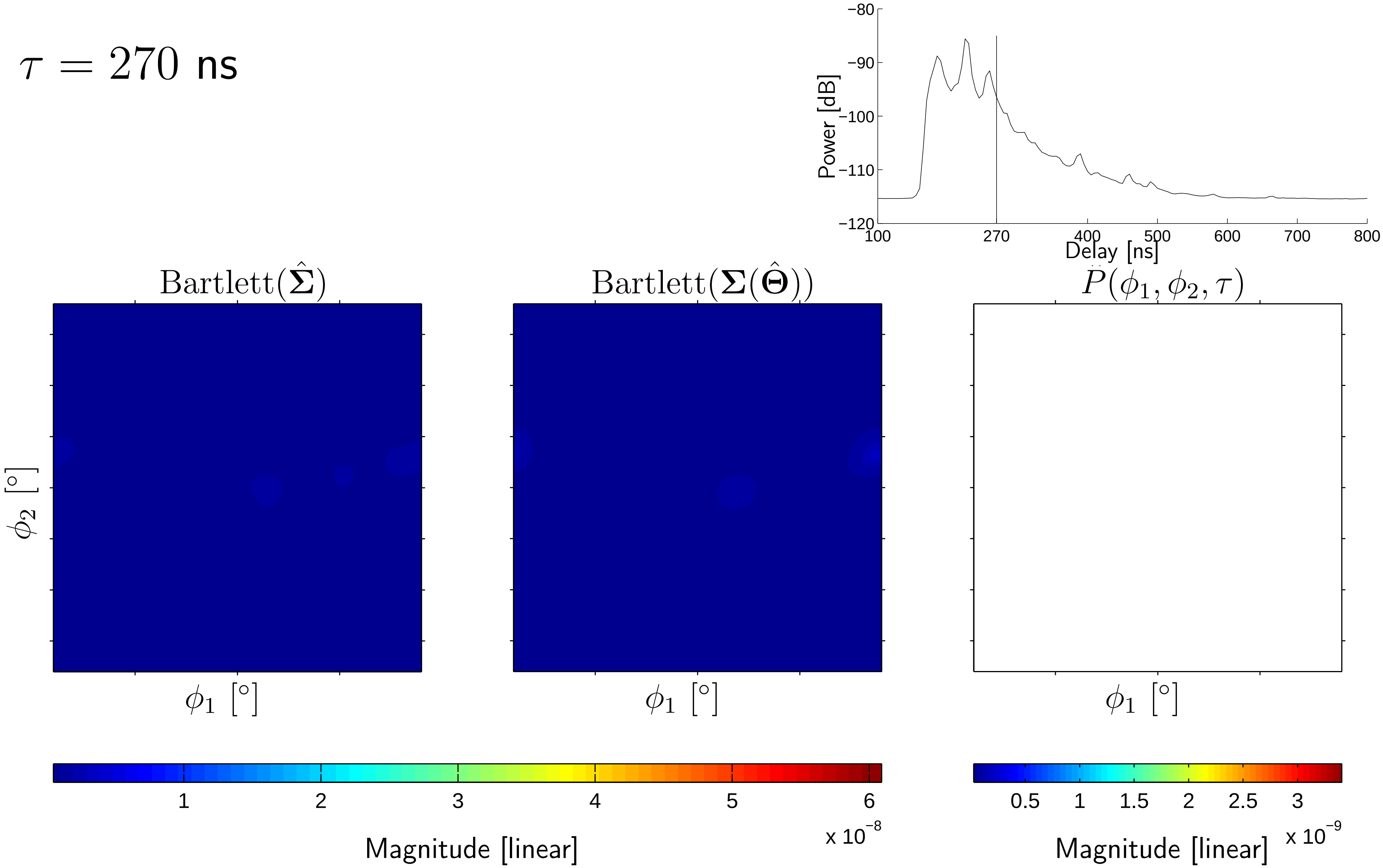
$\tau = 270$ ns





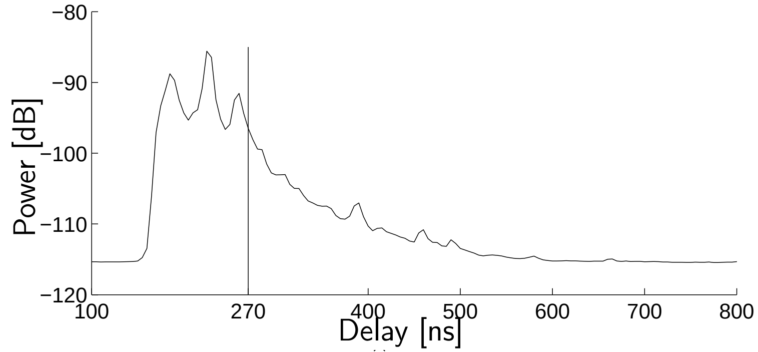
Estimate of biazimuth-delay power spectrum

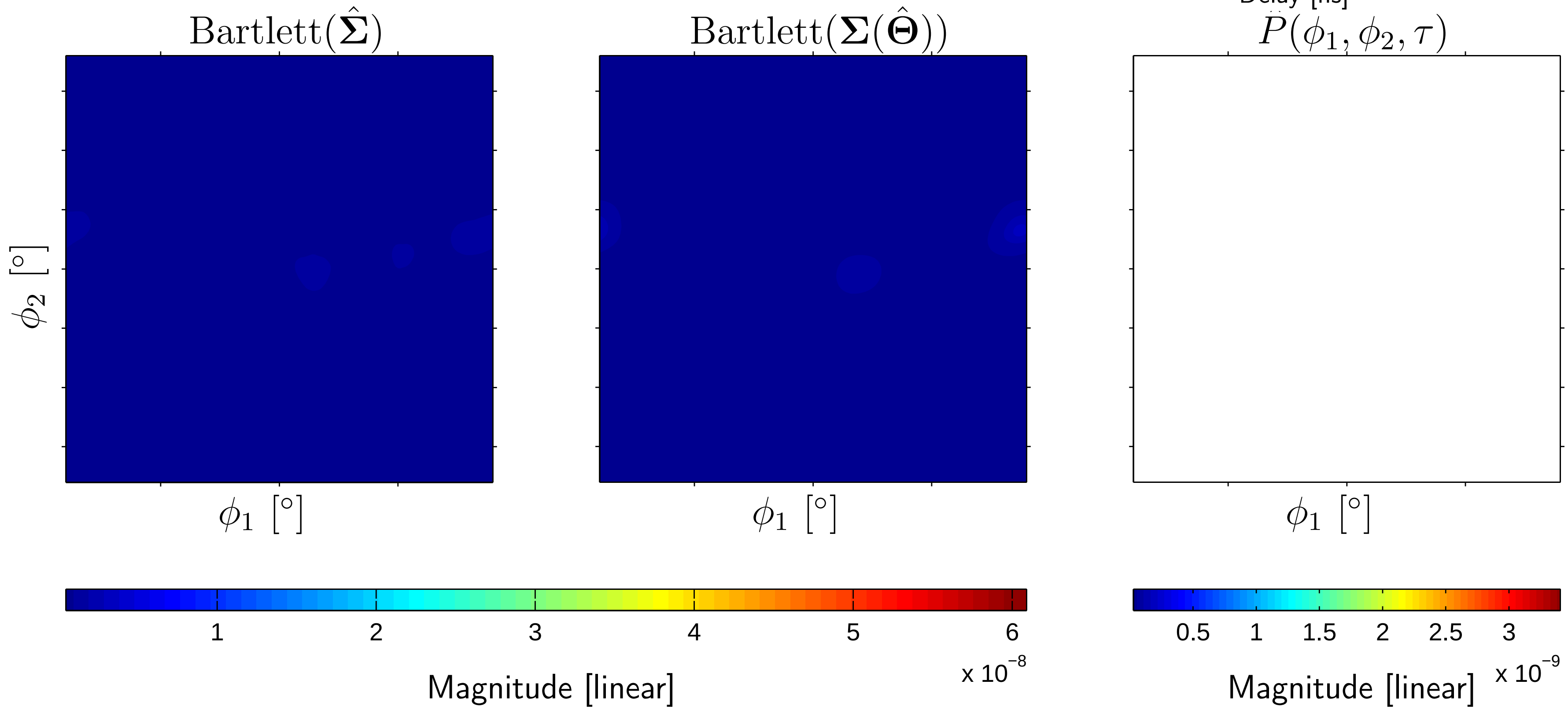
$\tau = 270$ ns



Estimate of biazimuth-delay power spectrum

$\tau = 270$ ns





Conclusions

- Pdfs are derived and used to characterize the normalized multi-dimensional power spectral density of individual components in propagation channels.
- The proposed pdfs maximize the entropy under the constraint that its first and second moments are specified.
- Experimental investigations show that the SAGE algorithm together with the characterization method provide an efficient tool for extracting multi-dimensional dispersion of the propagation channel.
- The estimated components are noticeably more concentrated compared to their corresponding footprints in the Bartlett spectrum.
- The results demonstrate the dependence of the spread across the dispersion dimensions for individual components.