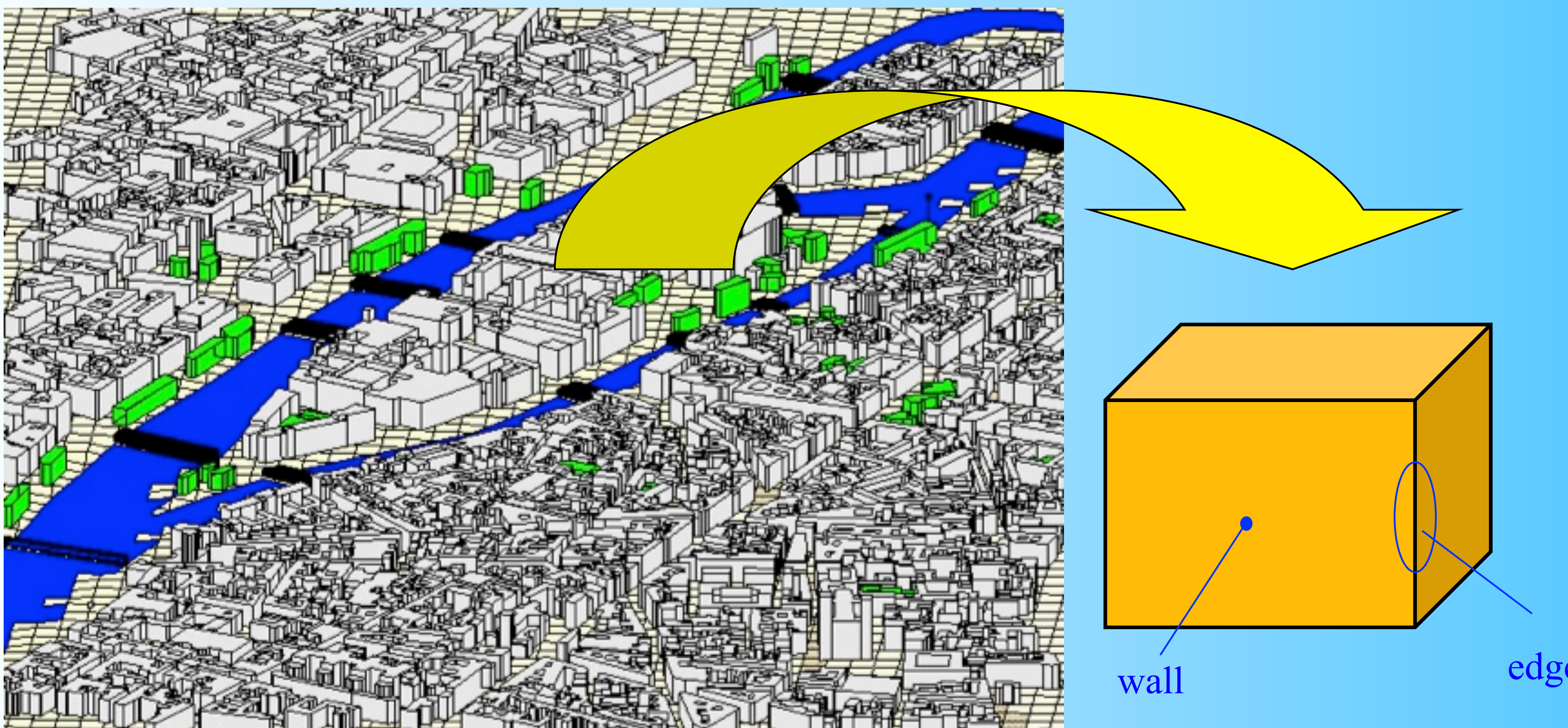
Deterministic radio propagation modeling and ray tracing

1) Introduction to deterministic propagation modelling
2) Geometrical Theory of Propagation I - The ray concept – Reflection and transmission
3) Geometrical Theory of Propagation II - Diffraction, multipath
4) Ray Tracing I
5) Ray Tracing II – Diffuse scattering modelling
6) Deterministic channel modelling I
7) Deterministic channel modelling II – Examples
8) Project - discussion

Geometrical theory of propagation (I)

It is useful when propagation takes place in a region with concentrated obstacles. Obstacles are here represented as plane walls and rectilinear edges (*canonical obstacles*)



Geometrical theory of propagation (II)

electromagnetic constants:

air	wall (generic medium)	
$\varepsilon_o = \frac{1}{36\pi} 10^{-9}\ Farad/m$	ε	electric permittivity
$\mu_o = 4\pi 10^{-7}\ Henry/m$	$\mu = \mu_o$	magnetic permeability
$\sigma = 0$	σ (if lossy)	electric conductivity
	$\varepsilon_c = \varepsilon + \frac{\sigma}{j\omega} = \varepsilon - j\frac{\sigma}{\omega}$	complex permittivity
n=1	$n \triangleq \sqrt{\frac{\varepsilon_c}{\varepsilon_o}}$	refraction index
$\eta_o = 120\pi\ \Omega$	$\eta \triangleq \sqrt{\frac{\mu}{\varepsilon_c}} = \sqrt{\frac{\mu_o}{\varepsilon_c}}$	intrinsic impedance

Geometrical theory of propagation (II)

- Geometrical theory of propagation is an extension of Geometrical Optics, (GO) but is not limited to optical frequencies

- Like GO, it corresponds to an asymptotic, high-frequency approximation of basic electromagnetic theory, and is based on the concept of ***ray***

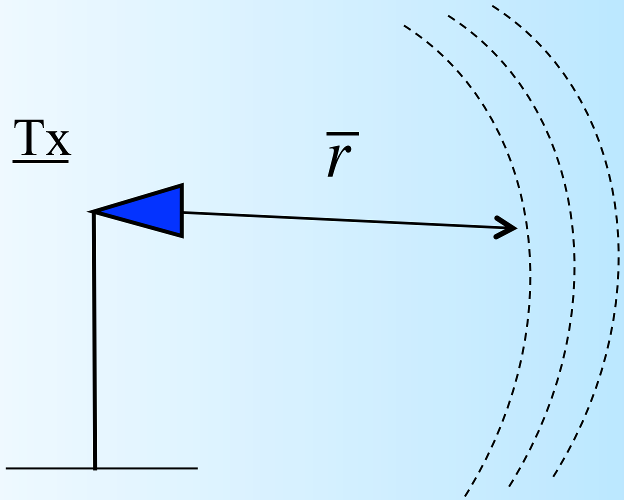
- Since GO does not account for diffraction, then diffraction is introduced through an extension called Geometrical Theory of Diffraction (GTD)

- The combination of GO and GTD, applied to radio wave propagation may be called Geometrical Theory of Propagation (GTP)

- GTP is the base of deterministic, ray-based propagation models (ray-tracing etc.)

Tx antenna in free space

Far field: (r >> λ   r >> D , antenna's dimension)



(Spherical wave)

$$\vec{E}(r,\theta,\varphi) \approx \vec{E}_o(\theta,\varphi)\frac{e^{-j\beta r}}{r}$$

vectorial amplitude

$$\bar{E}_o \perp \bar{r}$$

propagation factor

$$\beta = \frac{2\pi}{\lambda}$$

$\vec{E}$ Complex vector or *phasor*, so that

$$\vec{e}(r,\theta,\varphi,t) = \Re e\left\{\vec{E}(r,\theta,\varphi)e^{j\omega t}\right\}$$

Far field expressions

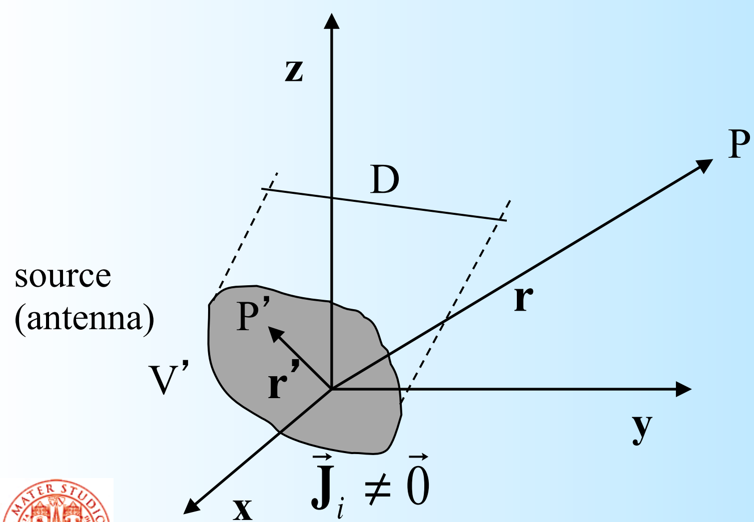
(r >> λ   r >> D)

$$\vec{E}(r,\theta,\varphi) \approx -j\frac{\eta}{2\lambda}\Big[\underbrace{\overbrace{\hat{\mathrm{i}}_r \times \vec{M}(\theta,\varphi) \times \hat{\mathrm{i}}_r}^{\vec{M}^t}\Big]}_{\vec{E}_o(\theta,\varphi)}\frac{e^{-j\beta r}}{r}$$

$$\vec{H}(r,\theta,\varphi) \approx \frac{1}{\eta}\hat{\mathrm{i}}_r \times \vec{E}$$

$$\vec{S} = \frac{1}{2}\vec{E} \times \vec{H}^* = \frac{|\vec{E}|^2}{2\eta}\hat{\mathrm{i}}_r$$

Poynting vector



Antenna' s *radiation vector*

$$\vec{M}(\theta,\varphi) \approx \int_{V'} \vec{\mathbf{J}}_i(P')\exp\left[j\beta\left(\vec{r}\,'\cdot\hat{\mathbf{i}}_r\right)\right]dV'$$

Polarization vector

$$\hat{\mathbf{p}} \triangleq \frac{\vec{E}}{\left|\vec{E}\right|} \cdot e^{j\chi}$$

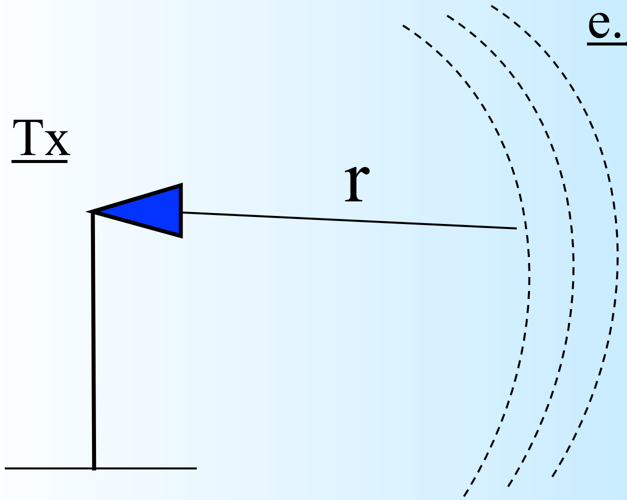
<u>The polarization vector defines the polarization of the field (and also of the antenna)</u>

It is strictly related to the radiation vector **M**

$$\hat{\mathbf{p}} = \frac{\vec{E}}{\left|\vec{E}\right|} \cdot e^{j\chi} = \frac{-j\dfrac{\eta}{2\lambda r}\,\vec{M}^{t}\,\dfrac{e^{-j\beta r}}{r}}{\dfrac{\eta}{2\lambda r}\left|\vec{M}^{t}\right|\dfrac{1}{r}} \cdot e^{j\chi} = \frac{\vec{M}^{t}}{\left|\vec{M}^{t}\right|} e^{-j\left(\beta r+\pi/2\right)} \cdot e^{j\chi} \;\Rightarrow\; \hat{\mathbf{p}} = \frac{\vec{M}^{t}}{\left|\vec{M}^{t}\right|} = \hat{M}^{t}$$

Wavefront

A *wavefront* is a *locus* where the field has constant phase



e.g:

$$E_x(r,\theta,\varphi) = E_{0x}(\theta,\varphi)\frac{e^{-j\beta r}}{r} = \frac{|E_{0x}|}{r} e^{j\left(\arg(E_{0x}) - \beta r\right)}$$

Therefore the wavefront is defined by:

$$\arg(E_{0x}) - \beta r = -\bar{\varphi}$$

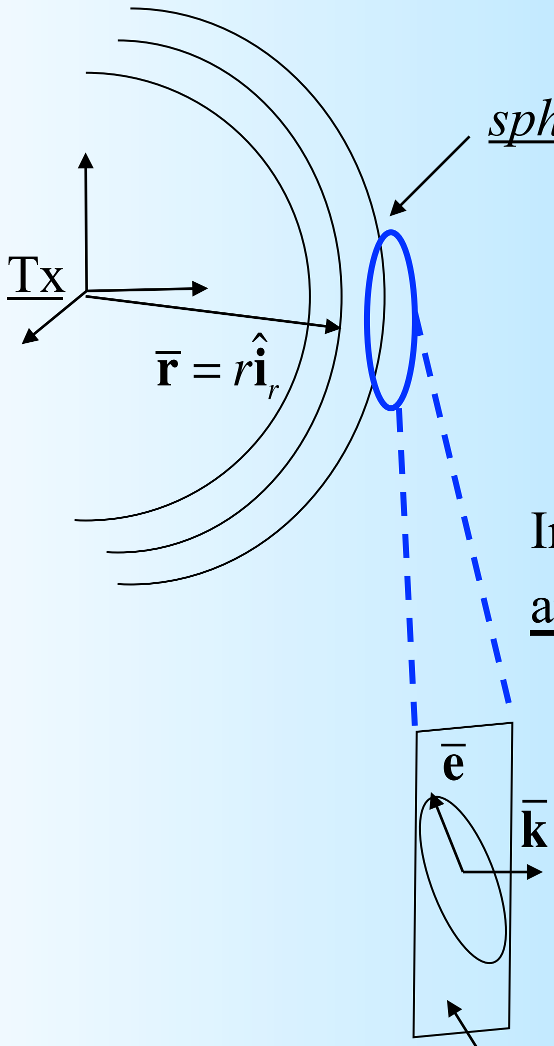
If r is large and $\arg(E_{0x})$ is limited we have

$$\arg(E_{0x}) - \beta r \approx -\beta r = -\bar{\varphi} \quad \Rightarrow \boxed{r = \bar{\varphi}/\beta}$$

Therefore the wavefront is a spherical surface in our case. That's why it's called "spherical wave".

Spherical and plane waves

spherical wavefront



Spherical wave

$$\vec{E}(r,\theta,\varphi) \approx \vec{E}_o(\theta,\varphi)\frac{e^{-j\beta r}}{r}$$

In far field a spherical wave can be locally approximated with a Plane wave

$$\vec{E}(r,\theta,\varphi) \approx \vec{E}_o^{'} e^{-j\,\bar{\mathbf{k}}\bullet\bar{\mathbf{r}}}$$

with

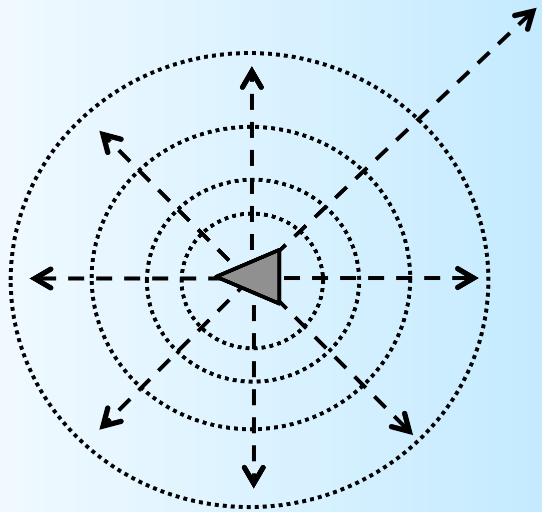
$$\vec{E}_o^{'} = \frac{\vec{E}_o(\theta,\varphi)}{r} \qquad \bar{\mathbf{k}} = \beta\hat{\mathbf{i}}_r$$

≈ plane wavefront

Definition of Ray (1/2)

- Given a propagating wave, every curve that is everywhere perpendicular to the wavefront is called ***electromagnetic ray***. A ray is *the path* of the wavefront. There is a mutual identification btw wave and ray

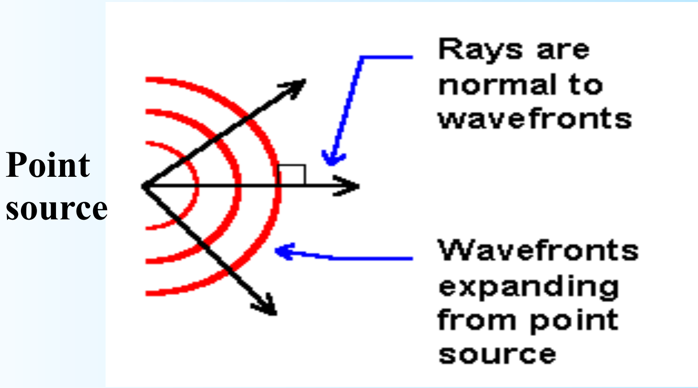
- Therefore we assume that the ray also has a field, the field of the wave at every point



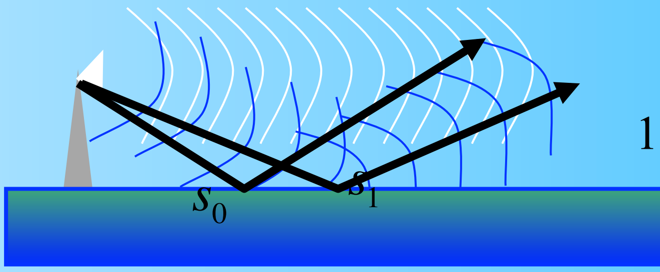
Definition of Ray (2/2)

- In free space, rays are rectilinear
- In presence of ***concentrated obstacles*** rays are piecewise-rectilinear and wavefronts can be of various kinds (see further on)
- In non-homogeneous media rays can be curved (not treated here)

Ex.1 Sperical wave and ***rectilinear*** *rays*

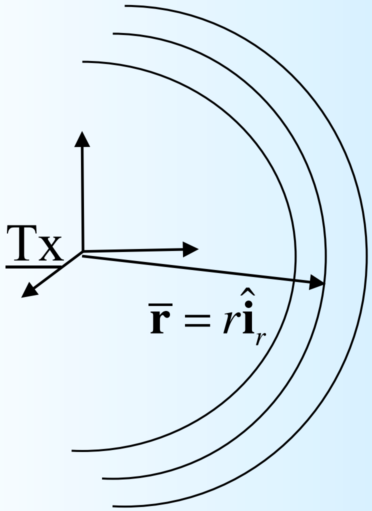


Ex.2 reflected spherical wave and ***piece-wise rectilinear*** *rays*



Poynting vector and intensity

It can be shown that the Poynting vector always has the same direction as the ray.
For a spherical wave this can be shown easily:



$$\bar{S} = \frac{\bar{E} \times \bar{H}^*}{2} = \frac{\left|\bar{E}\right|^2}{2\eta}\hat{i}_r$$

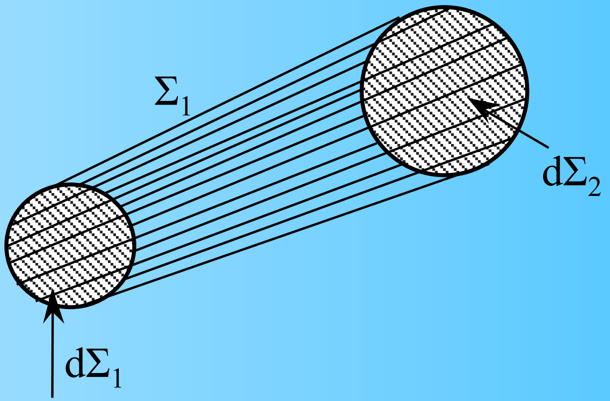
Therefore:
<u>e.m. power propagates along rays!</u>

radiation *Power-Density* or *Intensity* is defined as:

$$\left|\bar{S}\right| = S = \frac{\left|\bar{E}\right|^2}{2\eta}$$

Tube of flux concept and Power-density law

Def: a Ray tube (tube of flux) is a closed surface whose lateral surface is formed by a set of rays and the bases are two wavefront sections



By applying the Poynting's theorem (energy conservation) to a ray tube having bases $d\Sigma_1$, $d\Sigma_2$ small enough to assume the field constant on them, and considering a lossless medium we have:

$$\oint_{\Sigma} \vec{S}\cdot\hat{n}\ d\Sigma = \int_{d\Sigma_1} \vec{S}\cdot\hat{n}\ d\Sigma + \underbrace{\int_{\Sigma_\ell} \vec{S}\cdot\hat{n}\ d\Sigma}_{=0} + \int_{d\Sigma_2} \vec{S}\cdot\hat{n}\ d\Sigma =$$

$$= -\left|\vec{S}_1\right|\cdot d\Sigma_1 + \left|\vec{S}_2\right|\cdot d\Sigma_2 = 0 \quad \Rightarrow \quad S_1\cdot d\Sigma_1 = S_2\cdot d\Sigma_2 \Rightarrow \quad \frac{S_2}{S_1} = \frac{d\Sigma_1}{d\Sigma_2}$$

<u>Power-density (or Intensity) law of Geometrical Optics:</u>

"Power-density is inversely proportional to the cross-section of the ray tube"

Spreading Factor

We can therefore define the *spreading* or *divergence factor A:*

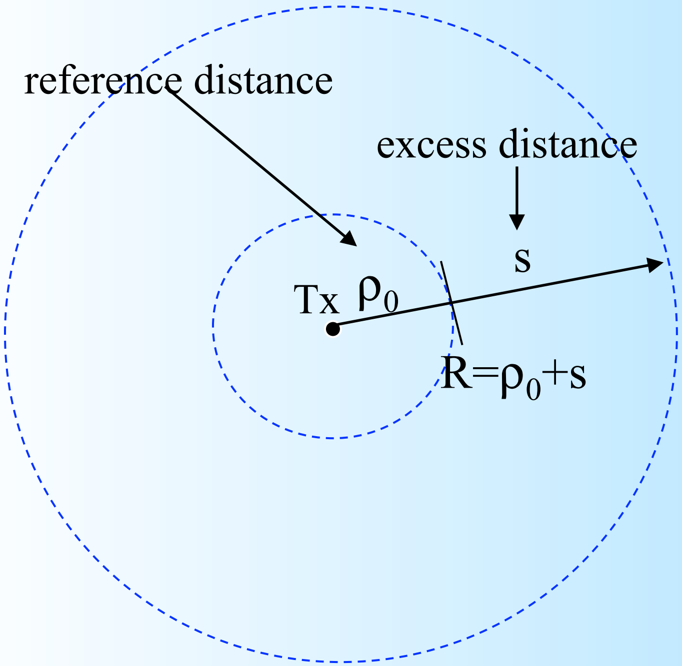
$$A = \sqrt{\frac{|\bar{S}_2|}{|\bar{S}_1|}} = \frac{|\bar{E}_2|}{|\bar{E}_1|} = \sqrt{\frac{d\Sigma_1}{d\Sigma_2}}$$

The spreading factor accounts for the attenuation due to the enlarging of the ray-tube cross-section.

Power carried by a ray decreases even in lossless media because the power density spreads on an enlarging wavefront surface as the wave propagates.

Re-writing the spherical wave

Using a reference distance ρ_o we have



field:

$$\mathbf{E}(R) = \mathbf{E}_0 \frac{e^{-j\beta R}}{R} = \mathbf{E}_0 \frac{e^{-j\beta\rho_0}}{\rho_0} \frac{\rho_0}{R} e^{-j\beta(R-\rho_0)} =$$

$$\mathbf{E}(\rho_0)\frac{\rho_0}{R} e^{-j\beta(R-\rho_0)} = \mathbf{E}(\rho_0)\frac{\rho_0}{\rho_0+s} e^{-j\beta(s)}$$

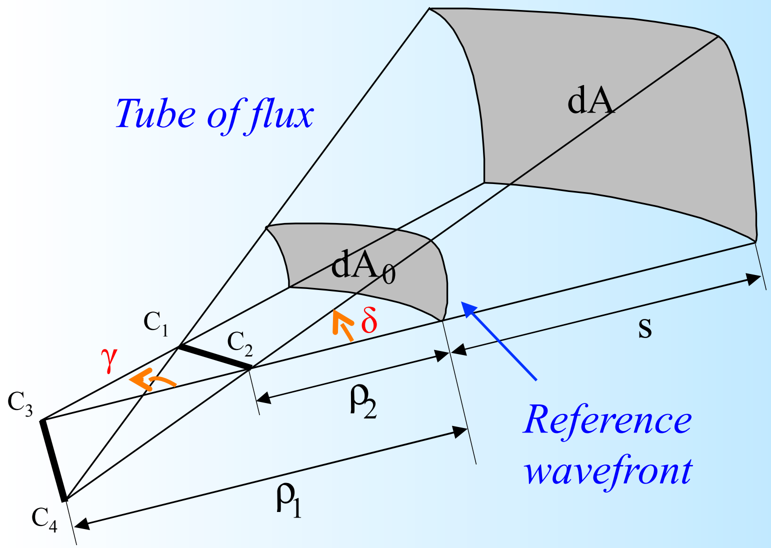
Power density:

$$S(R) = S(\rho_0)\left(\frac{\rho_0}{\rho_0+s}\right)^2$$

Spreading factor

Since received power P_r is proportional to power density, that's why P_r attenuates with the square of distance !

The generic, astigmatic wave



If the mean is homogeneous (➔ rectilinear rays) [2] the generic wave's divergence factor is:

$$A(\rho_1,\rho_2,s)=\sqrt{\frac{\rho_1\cdot\rho_2}{(\rho_1+s)\cdot(\rho_2+s)}}\left(=\sqrt{\frac{dA_0}{dA}}\right)$$

- ***A* : Divergence or Spreading factor**
- ρ_1, ρ_2 : curvature radii
- $\overline{C_1C_2}$, $\overline{C_3C_4}$: wave caustics

There are 3 main reference cases:

- Spherical wave: $\rho_1=\rho_2=\rho_0$ ➔ $A=\frac{\rho_0}{\rho_0+s}$
- Cylindrical wave: $\rho_1=\infty$, $\rho_2=\rho_0$ ➔ $A=\sqrt{\frac{\rho_0}{\rho_0+s}}$
- Plane wave: $\rho_1=\rho_2=\infty$ ➔ $A = 1$

notice that, for power conservation:

$$A=\sqrt{\frac{dA_0}{dA}}=\sqrt{\frac{|E|^2}{|E_0|^2}}=\frac{|E|}{|E_0|}=\sqrt{\frac{1}{L}}$$

L: Path Loss

The generic wave: field

The divergence factor gives the field- (and thus power-) attenuation law along the ray. But since the field is a complex vector, we also have polarization.
The generic (astigmatic) wave in free space has the electric field:

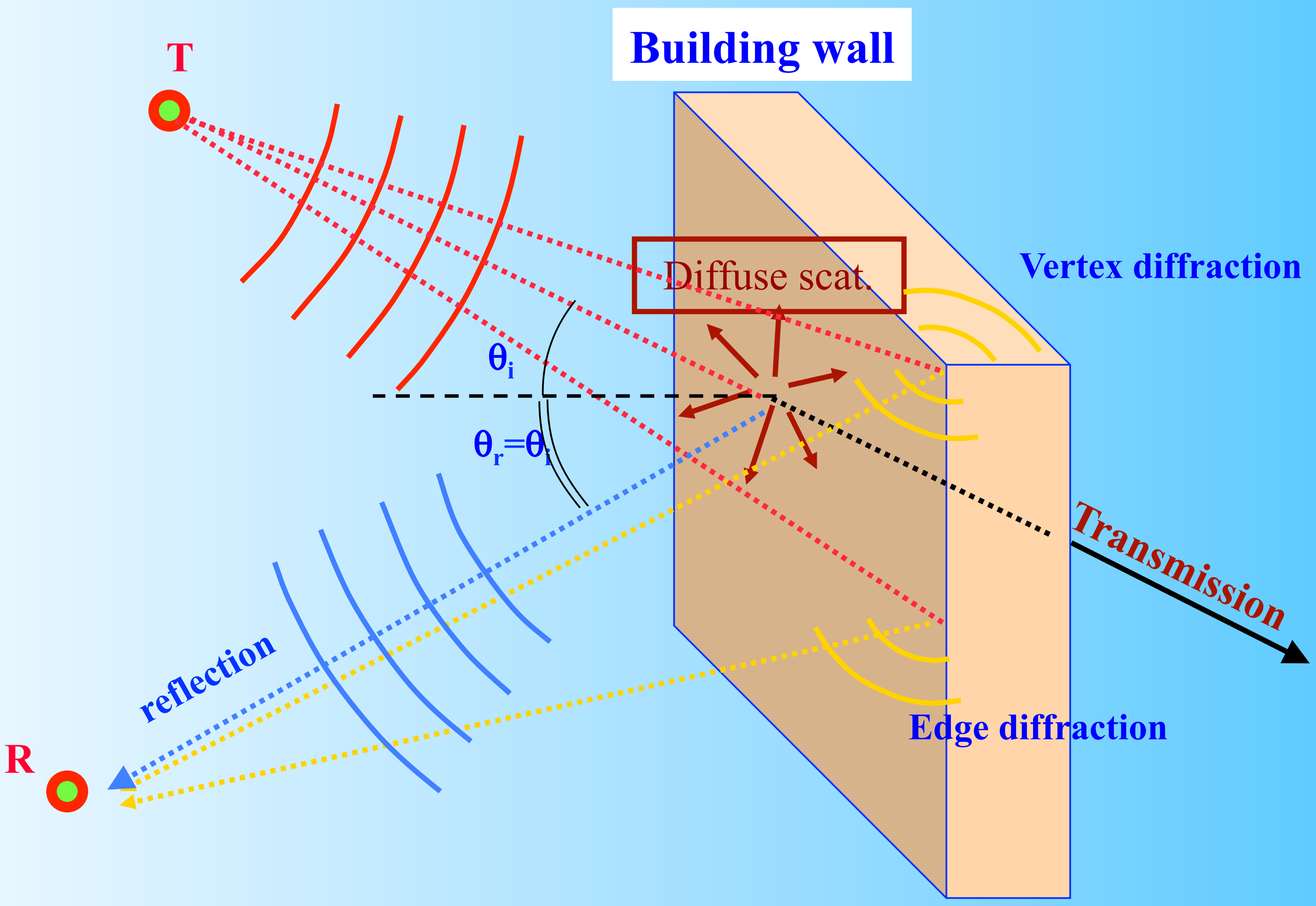
$$\vec{E}(s) = \underbrace{\vec{E}(0)}_{\substack{\textit{Field}\text{ at reference}\\ \text{point (s=0)}}} \cdot \underbrace{\underbrace{\sqrt{\frac{\rho_1 \cdot \rho_2}{(\rho_1 + s)\cdot(\rho_2 + s)}}}_{\textbf{Divergence factor}} \cdot \underbrace{e}_{\textit{Phase}\text{ factor}}{}^{-j\beta s}}_{\text{Pr}\textit{opagation factor}}$$

(Example: spherical wave)

$$\vec{E}(s) = \vec{E}(\rho_0) \cdot \frac{\rho_0}{\rho_0 + s} e^{-j\beta s}$$

$$= \hat{\mathbf{p}}\mathrm{K}\left(\frac{e^{-j\beta\rho_0}}{\rho_0}\right)\frac{\rho_0}{\rho_0 + s} e^{-j\beta s} = \hat{\mathbf{p}}\mathrm{K}\underbrace{\left(\frac{e^{-j\beta s_{tot}}}{s_{tot}}\right)}_{\text{Pr}\textit{opagation factor}}$$

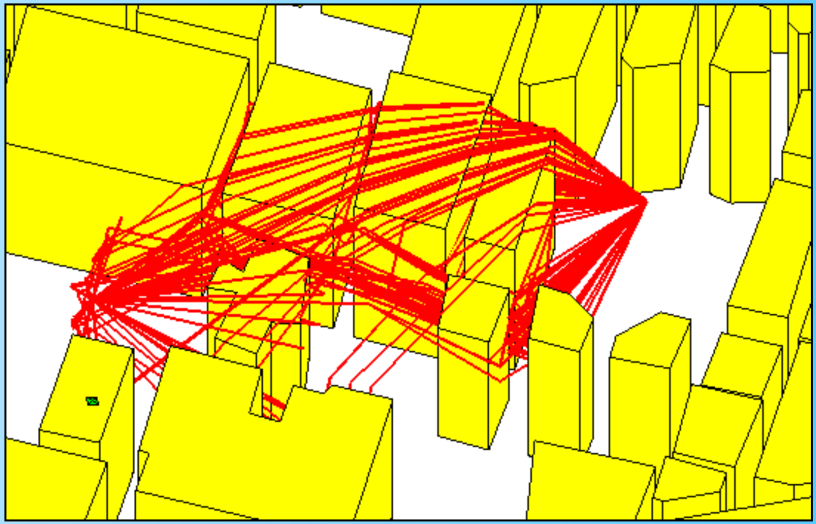
Interaction mechanisms



GTP basics

- GTP is based on the couple: (ray , field)

- The propagating field is computed as a set of rays



- Given a ray departing from an antenna we must “follow” the ray and predict both its geometry and its field at every point until it reaches the receiver

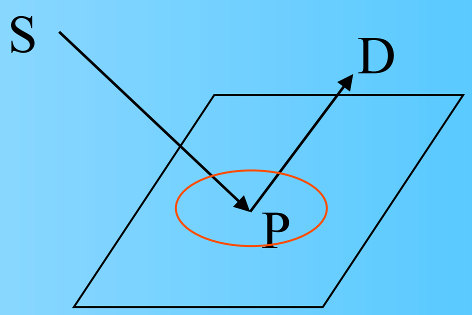
- It is therefore necessary to predict what happens at both the trajectory and the field at each interaction with an obstacle

- To this end we rely on the two GTP basic principles

GTP: basic principles

"Local field principle"

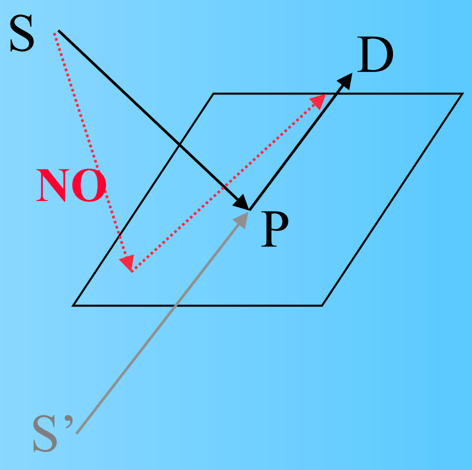
- The wave can be locally assumed plane (for the interaction coefficient computation)



- The field associated with the reflected/transmitted/ diffracted ray only depends on the electromagnetic and geometric properties of the obstacle in the vicinity of the interaction point

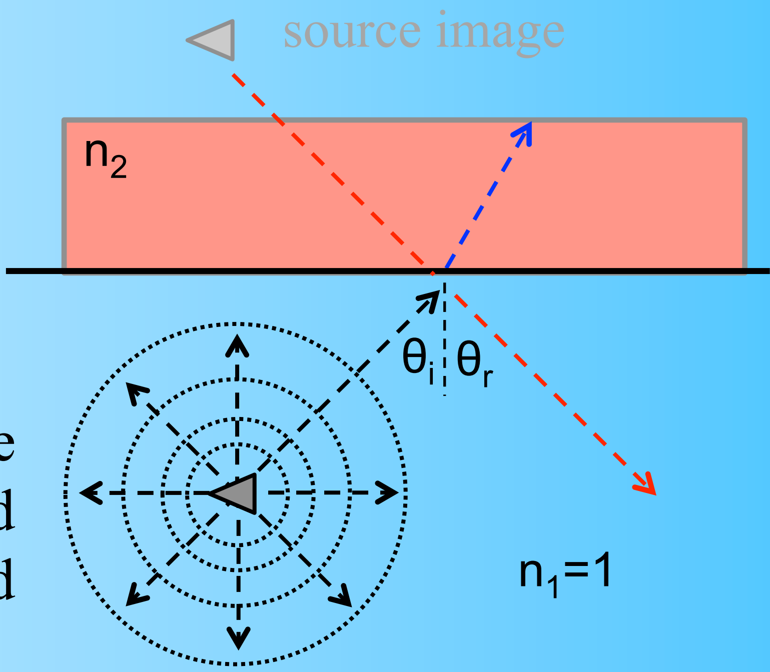
"Fermat's principle"

- The ray trajectory is always so as to minimize path (or optical-path …)



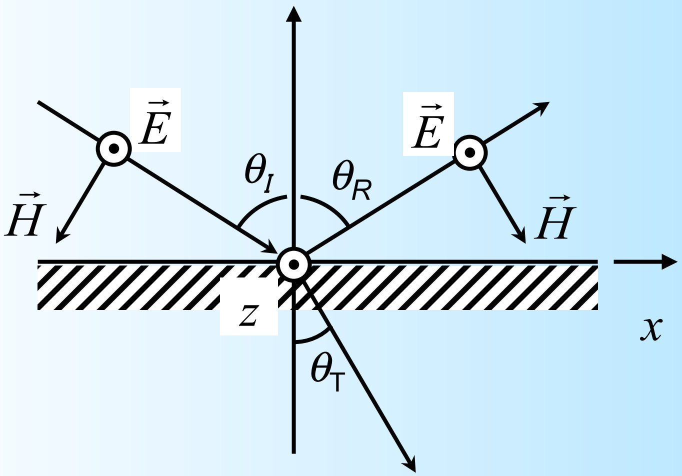
Ray Reflection and Transmission (1/2)

radial rays spring from the transmitting antenna



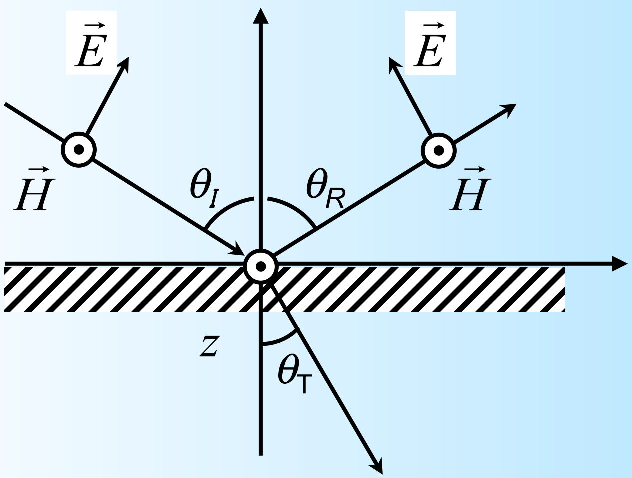
- when a ray impinges on the plane surface the corresponding wave is reflected and transmitted, thus generating **reflected** and **transmitted** rays
- The incident ray trajectory is modified according to the *Snell's laws of reflection (transmission)*. The field amplitude / phase change at the interaction point according to proper Fresnel's *reflection (transmission) coefficients*

Reflection and Transmission Coefficients



- TE polarization

$$\Gamma_{TE} = \frac{\cos\theta_i - \sqrt{\left(\frac{n_2}{n_1}\right)^2 - \sin^2\theta_i}}{\cos\theta_i + \sqrt{\left(\frac{n_2}{n_1}\right)^2 - \sin^2\theta_i}} \; ; \; \tau_{TE} = 1 + \Gamma_{TE}$$

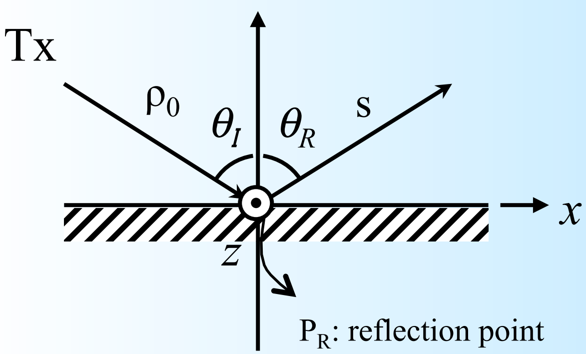


- TM polarization

$$\Gamma_{TM} = \frac{\left(\frac{n_2}{n_1}\right)^2 \cos\theta_i - \sqrt{\left(\frac{n_2}{n_1}\right)^2 - \sin^2\theta_i}}{\left(\frac{n_2}{n_1}\right)^2 \cos\theta_i + \sqrt{\left(\frac{n_2}{n_1}\right)^2 - \sin^2\theta_i}} \; ; \; \tau_{TM} = 1 + \Gamma_{TM}$$

Field formulation

Reflected ray

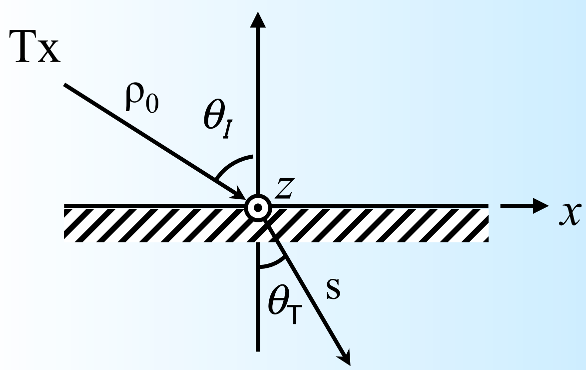


- direction: reflection law or Fermat's principle
- Field expression:

$$\begin{bmatrix} \vec{E}_r^{TE}(s) \\ \vec{E}_r^{TM}(s) \end{bmatrix} = \begin{bmatrix} \Gamma_{TE} & 0 \\ 0 & \Gamma_{TM} \end{bmatrix} \cdot \begin{bmatrix} \vec{E}_i^{TE}(P_R) \\ \vec{E}_i^{TM}(P_R) \end{bmatrix} \cdot \frac{\rho_0}{\rho_0 + s} e^{-j\beta s}$$

Divergence factor

Transmitted ray

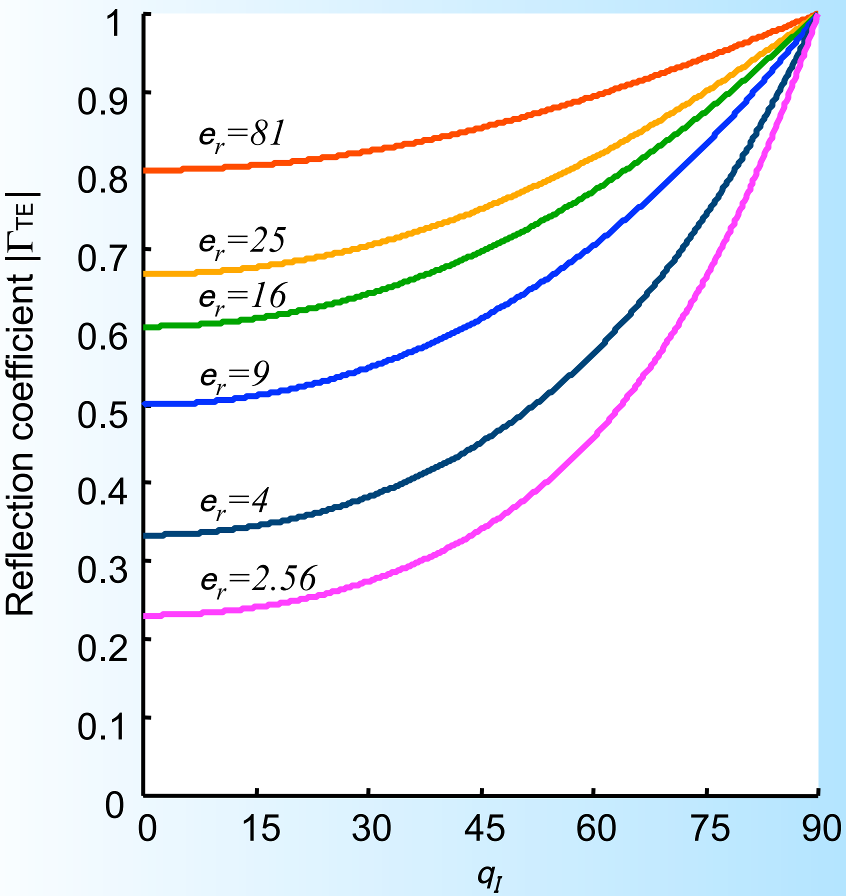


- direction: Snell's law or Fermat's principle
- Field expression:

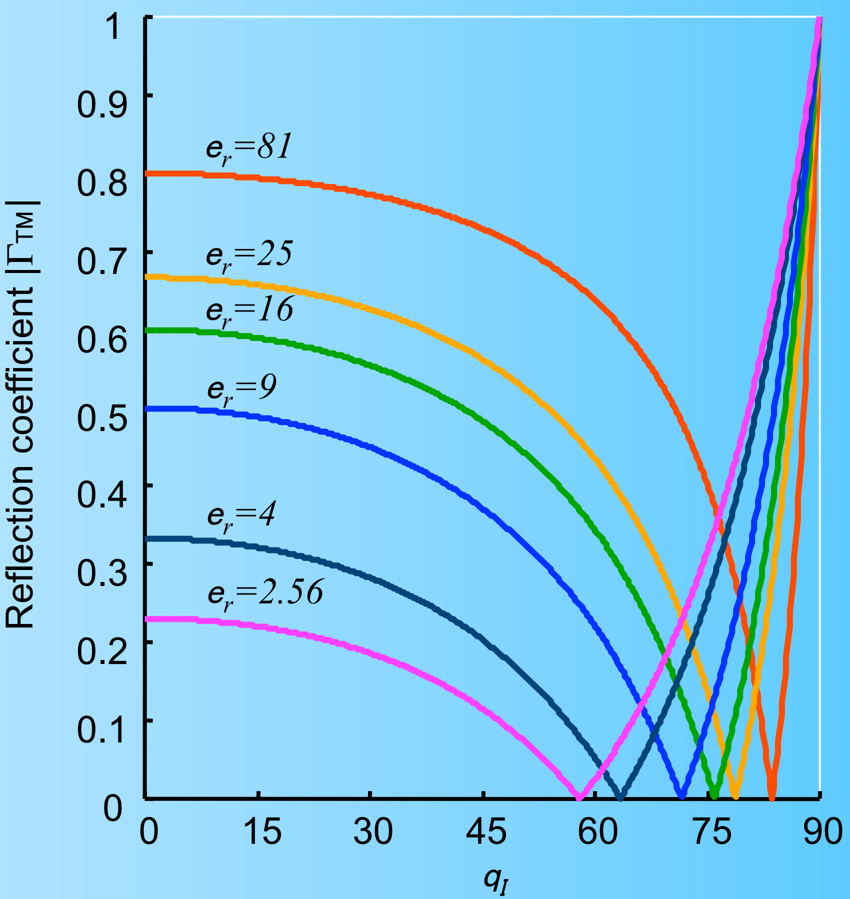
$$\begin{bmatrix} \vec{E}_t^{TE}(s) \\ \vec{E}_t^{TM}(s) \end{bmatrix} = \begin{bmatrix} \tau_{TE} & 0 \\ 0 & \tau_{TM} \end{bmatrix} \cdot \begin{bmatrix} \vec{E}_i^{TE}(P_R) \\ \vec{E}_i^{TM}(P_R) \end{bmatrix} \cdot \frac{\rho_0}{\rho_0 + s} e^{-j\beta' s}$$

Fresnel's coefficients

Example: dielectric materials

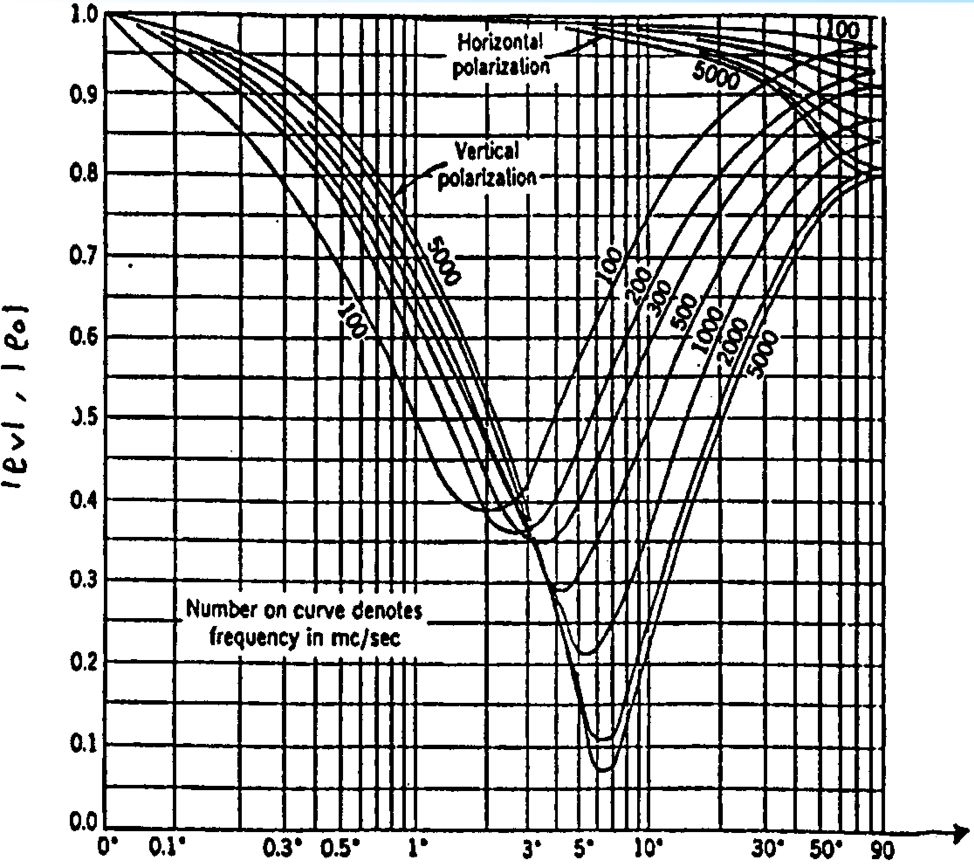


TE Polarization

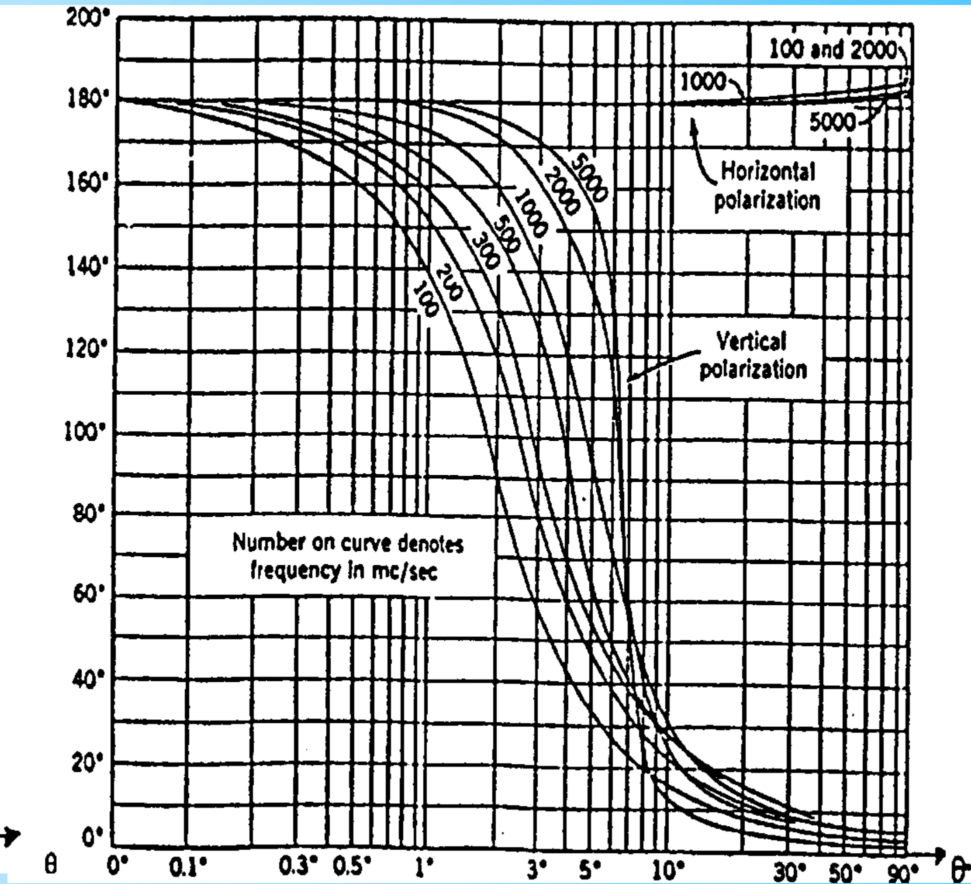


TM Polarization

Example: sea water



Module of Γ_{TE} and Γ_{TM}



Phase of Γ_{TE} and Γ_{TM}

Example: material constants

When conductivity exists, the following constant is often used

$$\varepsilon_{rc} = \varepsilon_r - j\frac{\sigma}{\varepsilon_o \omega} = \varepsilon_r - j\varepsilon''$$

Material*	e_r	s (S/m)	e'' @ 1 GHz
Lime stone wall	7.5	0.03	0.54
Dry marble	8.8		0.22
Brick wall 4.4	0.01	0.18	
Cement	4 - 6		0.3
Clear glass	4 - 6		0.005 - 0.1
Metalized glass	5.0	2.5	45
Lake water	81	0.013	0.23
Sea Water 81	3.3	59	
Dry soil	2.5	--	--
Earth	7 - 30	0.001 - 0.03	0.02 - 0.54

* Common materials are not well defined mixtures and often contain water. "Effective" material properties depend on exact mixture, and on water content. These are approximate numbers taken from several sources.