preamble

Lecture 3
Switching Mode Optimization for Channel Sounding Using Switched Tx and Rx Arrays

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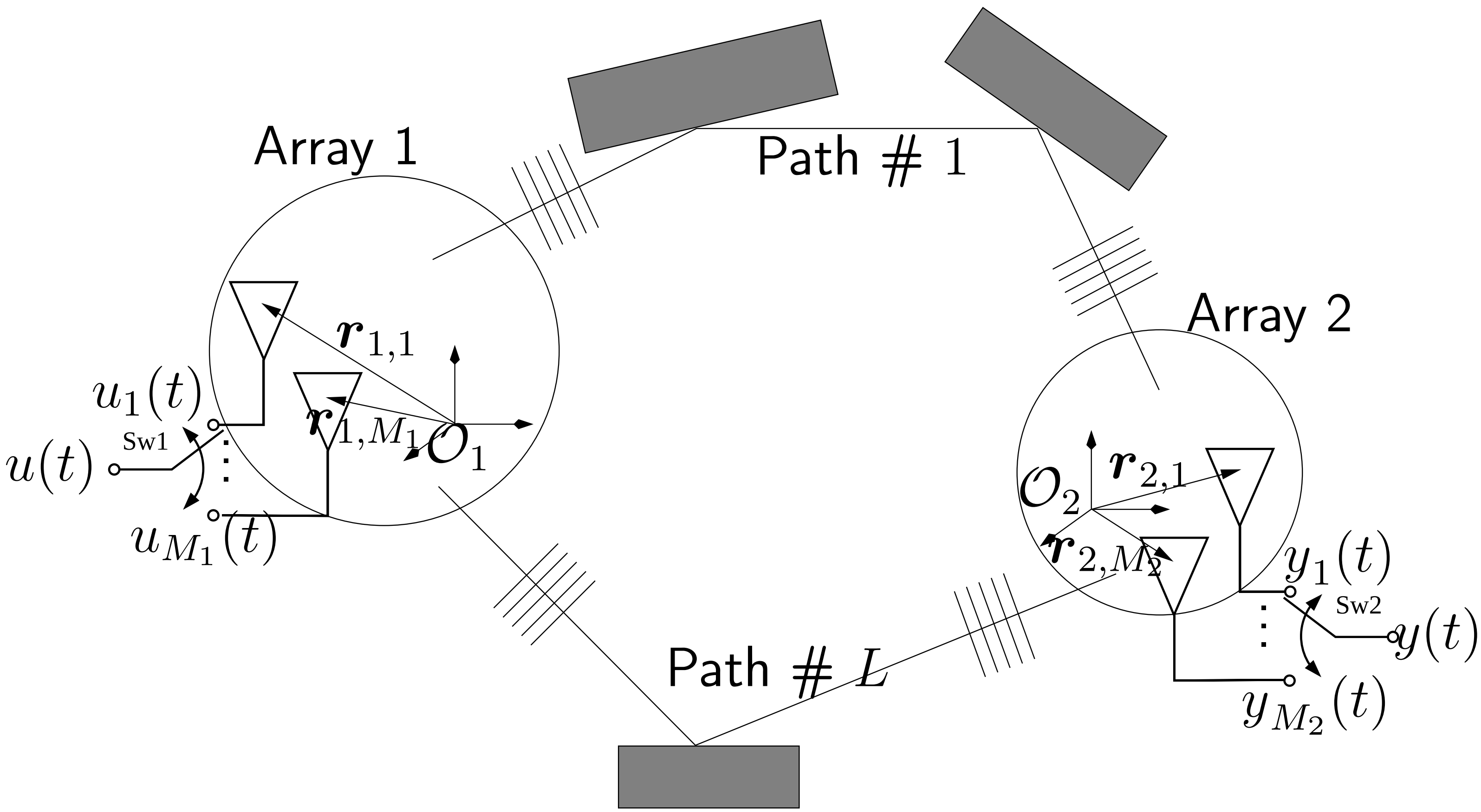
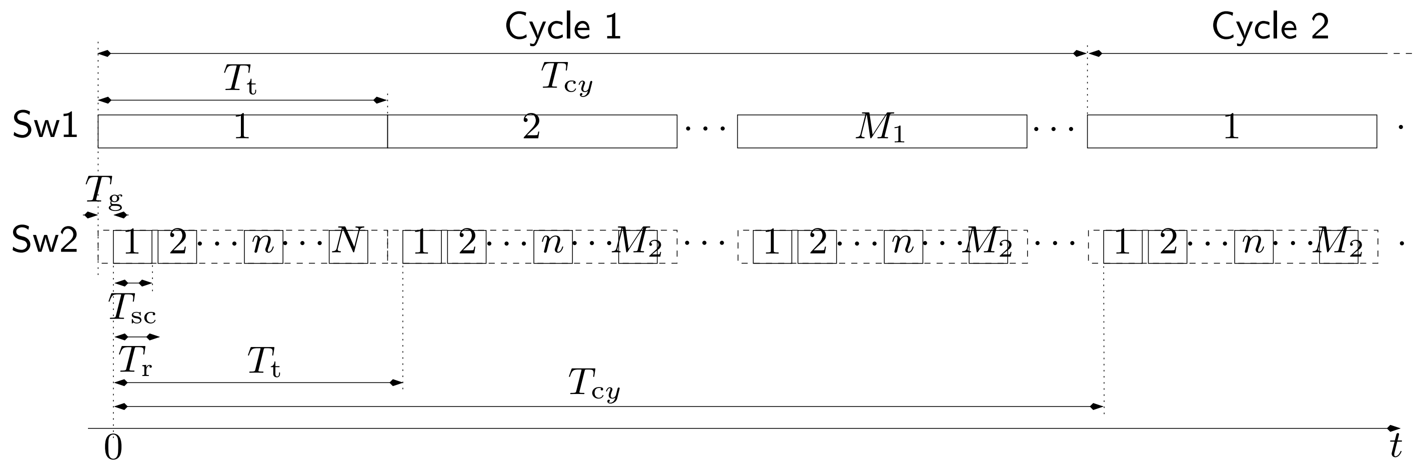
Contents

Channel sounding using switched multiple Tx and Rx antennas



The TDM structure with the traditionally used switching mode.



- $T_{\rm r}$: Time interval between two consecutive scanning periods;
- $T_{\rm cy}$: Time interval between two consecutive measurement cycles;
- $1/T_{\rm cy}$: Measurement cycle rate;
- $1/T_{\rm r}$: Switching rate;
- $[-\frac{1}{2T_{cy}}, +\frac{1}{2T_{cy}}]$: Traditionally used DF estimation range (DFER);
- $[-\frac{1}{2T_r}, \frac{1}{2T_r}]$: Proposed DFER.

$$1/T_{\rm r} = M_2 M_1 R \cdot (1/T_{\rm cy}) \text{ with cycle repetition rate } R \geq 1$$

Signal model

The contribution of the wave propagating through the ℓth path to the output of the Rx array can be written as

$$\begin{aligned}\boldsymbol{s}(t;\boldsymbol{\theta}_\ell) &\doteq [s_1(t;\boldsymbol{\theta}_\ell),\ldots,s_{M_2}(t;\boldsymbol{\theta}_\ell)]^{\mathrm{T}} \\ &= \alpha_\ell \exp\{j2\pi\nu_\ell t\}\boldsymbol{c}_2(\boldsymbol{\Omega}_{2,\ell})\boldsymbol{c}_1(\boldsymbol{\Omega}_{1,\ell})^{\mathrm{T}}\boldsymbol{u}(t-\tau_\ell),\end{aligned}$$

where

- $\boldsymbol{\theta}_\ell \doteq [\boldsymbol{\Omega}_{1,\ell},\boldsymbol{\Omega}_{2,\ell},\tau_\ell,\nu_\ell,\alpha_\ell]$: parameter vector of the ℓth propagation path;
- $\boldsymbol{c}_k(\boldsymbol{\Omega}) = [c_{k,1}(\boldsymbol{\Omega}),\ldots,c_{k,M_k}(\boldsymbol{\Omega})]$: response of Array k in direction $\boldsymbol{\Omega}$, $, k=1,2$;
- $\boldsymbol{\Omega} = \boldsymbol{e}(\phi,\theta) \doteq [\cos(\phi)\sin(\theta),\sin(\phi)\sin(\theta),\cos(\theta)]^{\mathrm{T}} \in \mathbb{S}_2$
- $\boldsymbol{u}(t) \doteq [u_1(t),...,u_{M_1}(t)]^{\mathrm{T}}$: input signal vector.

Signal model

Received signal vector:

$$\boldsymbol{y}(t) = \sum_{\ell=1}^{L} \boldsymbol{s}(t;\boldsymbol{\theta}_\ell) + \boldsymbol{n}(t),$$

where $\boldsymbol{n}(t)$ denotes complex circularly symmetric spatially and temporally white Gaussian noise vector with component spectral height N_0.

Parameter vector:

$$\boldsymbol{\theta} \doteq [\boldsymbol{\theta}_1, ..., \boldsymbol{\theta}_L].$$

Notice that $\dim(\boldsymbol{\theta}) = 8L$.

The SAGE algorithm

Hidden data:

$$\boldsymbol{x}_\ell(t) = \boldsymbol{s}(t;\boldsymbol{\theta}_\ell) + \beta_\ell \boldsymbol{n}_\ell(t), \quad \ell = 1, \ldots, L,$$

where

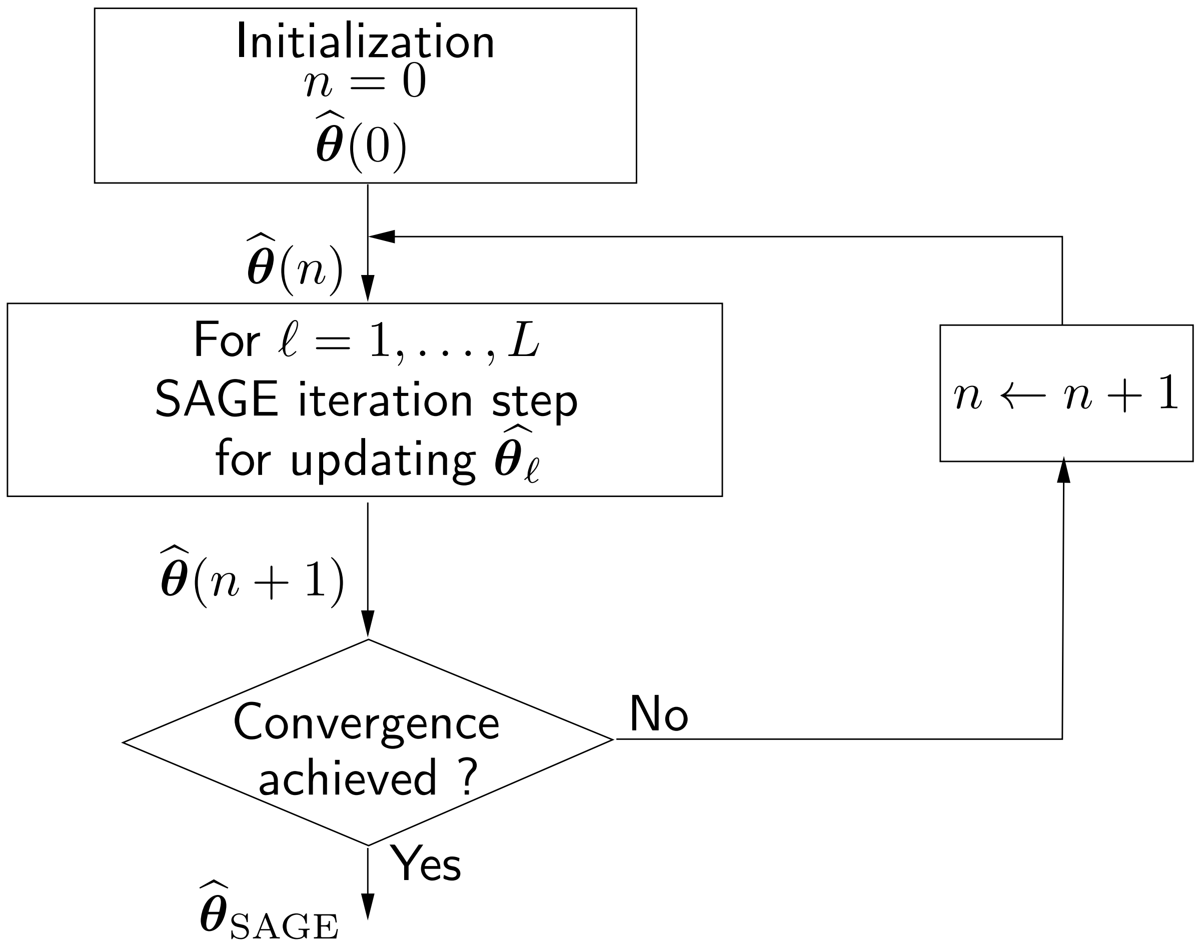
- $\boldsymbol{n}_\ell(t)$, $\ell = 1, \ldots, L$ denotes L independent complex circularly symmetric spatially and temporally white Gaussian noises with component spectral height N_0.
- $\sum_{\ell=1}^{L} |\beta_\ell|^2 = 1$.

Incomplete data:

$$\boldsymbol{y}(t) = \sum_{\ell=1}^{L} \boldsymbol{x}_\ell(t) = \sum_{\ell=1}^{L} \boldsymbol{s}(t;\boldsymbol{\theta}_\ell) + \underbrace{\sum_{\ell=1}^{L} \beta_\ell \boldsymbol{n}_\ell(t)}_{\equiv \boldsymbol{n}(t)}.$$

The SAGE algorithm

Signal flow graph of the SAGE algorithm:



The SAGE algorithm

Objective function:

$$|z(\bar{\boldsymbol{\theta}}_\ell; \hat{x}_\ell)| \doteq |\tilde{\boldsymbol{c}}_2(\boldsymbol{\Omega}_{2,\ell})^{\mathrm{H}} \boldsymbol{X}_\ell(\tau_\ell, \nu_\ell) \tilde{\boldsymbol{c}}_1(\boldsymbol{\Omega}_{1,\ell})^*|,$$

Objective function for joint estimation of the DF and directions:

$$|z(\nu_\ell, \boldsymbol{\Omega}_{1,\ell}, \boldsymbol{\Omega}_{2,\ell}; \hat{x}_\ell)| = |\sum_{i=1}^{I} R_i(\check{\nu}_\ell) S_i(\check{\boldsymbol{\Omega}}_{1,\ell}, \check{\nu}_\ell) T_i(\check{\boldsymbol{\Omega}}_{2,\ell}, \check{\nu}_\ell) + V(\nu_\ell, \boldsymbol{\Omega}_{1,\ell}, \boldsymbol{\Omega}_{2,\ell})|,$$

where

- $\check{\nu}_\ell = \nu'_\ell - \nu_\ell$
- $\check{\boldsymbol{\Omega}}_{1,\ell} = \boldsymbol{\Omega}'_{1,\ell} - \boldsymbol{\Omega}_{1,\ell}$
- $\check{\boldsymbol{\Omega}}_{2,\ell} = \boldsymbol{\Omega}'_{2,\ell} - \boldsymbol{\Omega}_{2,\ell}$, with $(\cdot)'$ denoting the true parameters
- $V(\nu_\ell, \boldsymbol{\Omega}_{1,\ell}, \boldsymbol{\Omega}_{2,\ell})$: Correlated complex circularly Gaussian noise.

Joint estimation of the Doppler frequency and DoA

Example of the ambiguity problem:

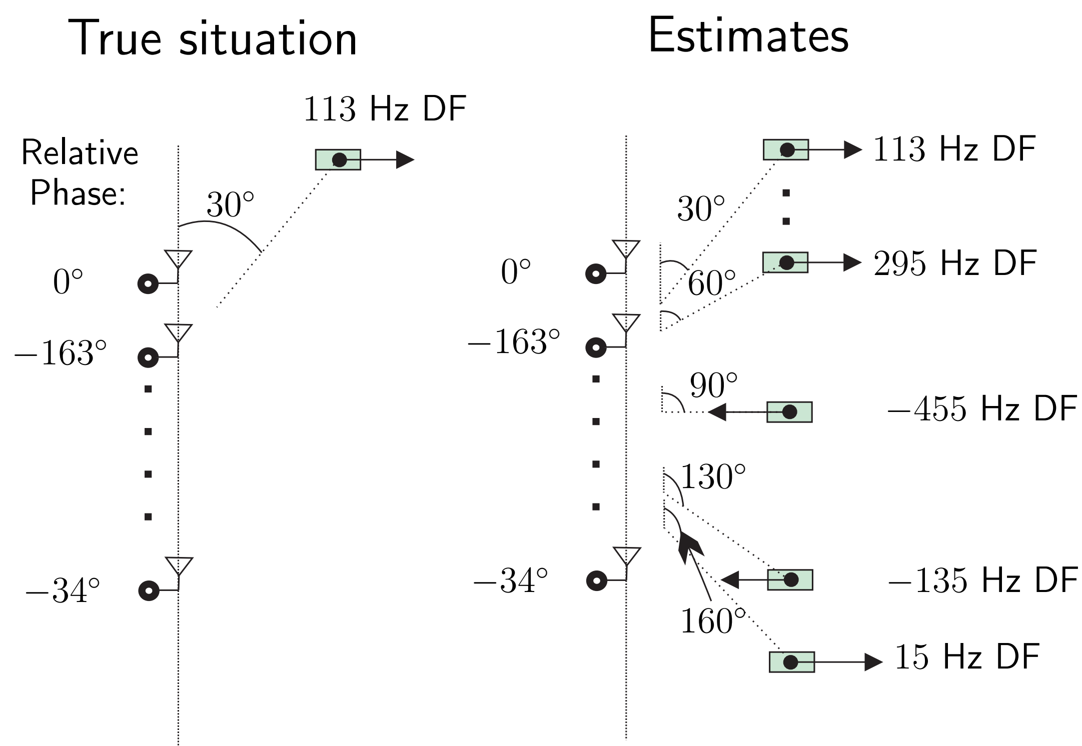
- A SIMO system and linear Rx antenna array.
- ψ : Incident Angle (IA) with respect to the Rx array axis.

Characteristics of the dth path:

- true IA: $\psi'_d = 30^\circ$
- true DF: $\nu'_d = 113$ Hz

Sounder setting:

- $M_2 = 8$
- $T_\mathrm{r} = 1$ ms
- Sequential SM



Joint estimation of the Doppler frequency and DoA

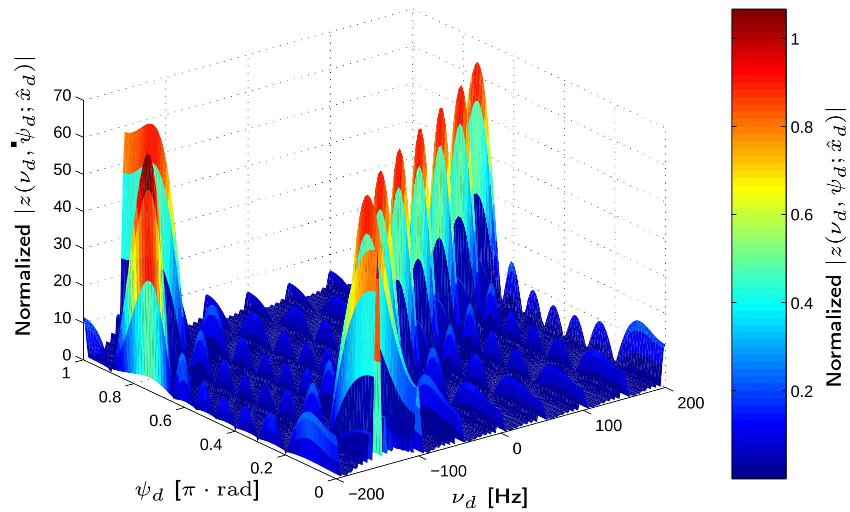
Example of the ambiguity problem:

- Noise-free objective function:

$$z(\nu_d, \psi_d; \hat{x}_d) \propto \frac{\sin(\pi \check{\nu}_d I T_{cy})}{\sin(\pi \check{\nu}_d T_{cy})} \cdot \frac{\sin(M_2 \pi (\check{\nu}_d T_r + \frac{1}{2}\check{\psi}_{2,d}))}{\sin(\pi(\check{\nu}_d T_r + \frac{1}{2}\check{\psi}_{2,d}))},$$

where

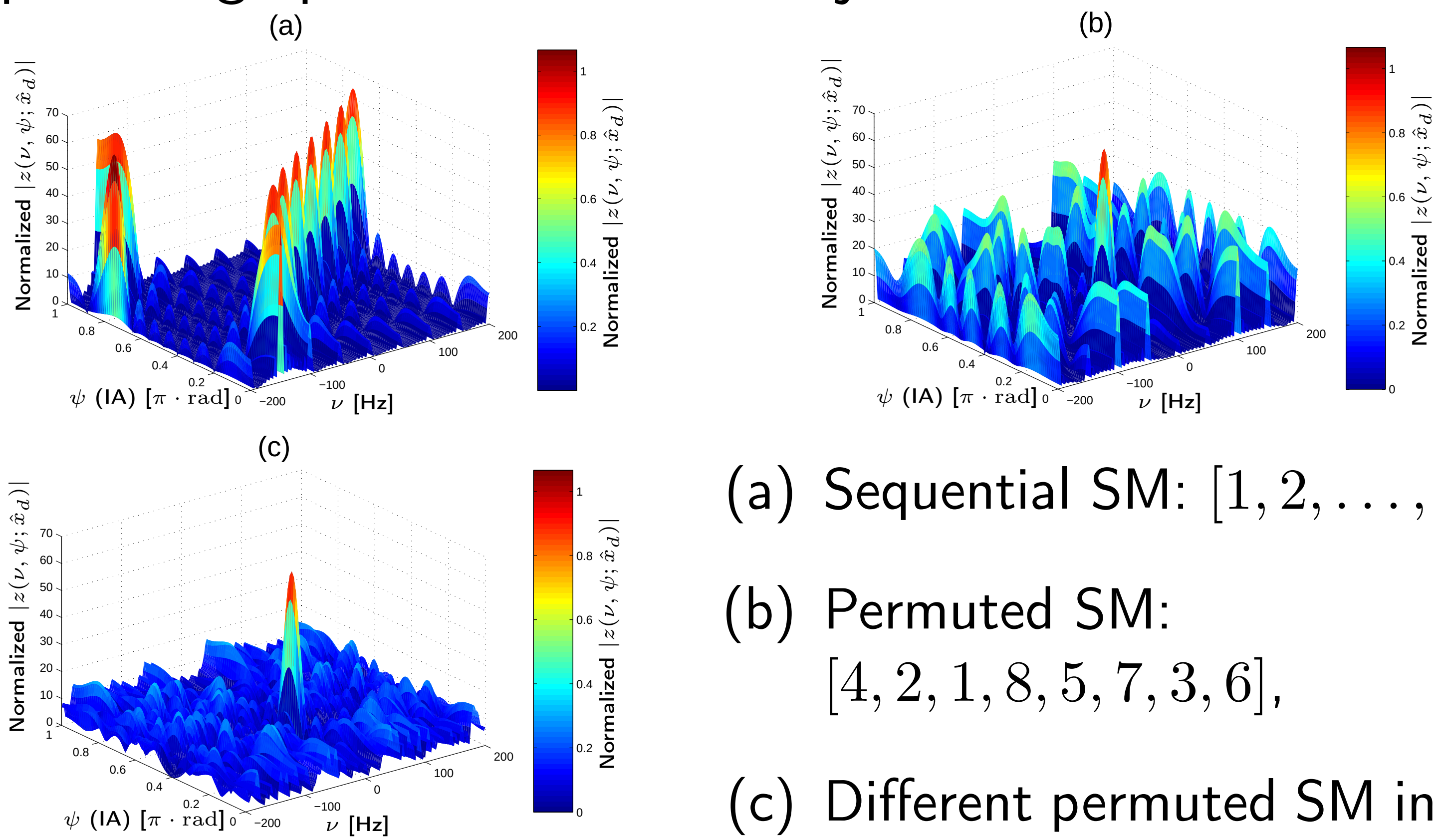
- $\check{\nu}_d = \nu'_d - \nu_d$,
- $\check{\psi}_{2,d} = \cos(\psi'_d) - \cos(\psi_d)$.



Joint estimation of the Doppler frequency and DoA

Switching mode (SM) optimization:

Example: 3D-graphs of normalized objective functions



(a) Sequential SM: $[1, 2, \ldots, 8]$,

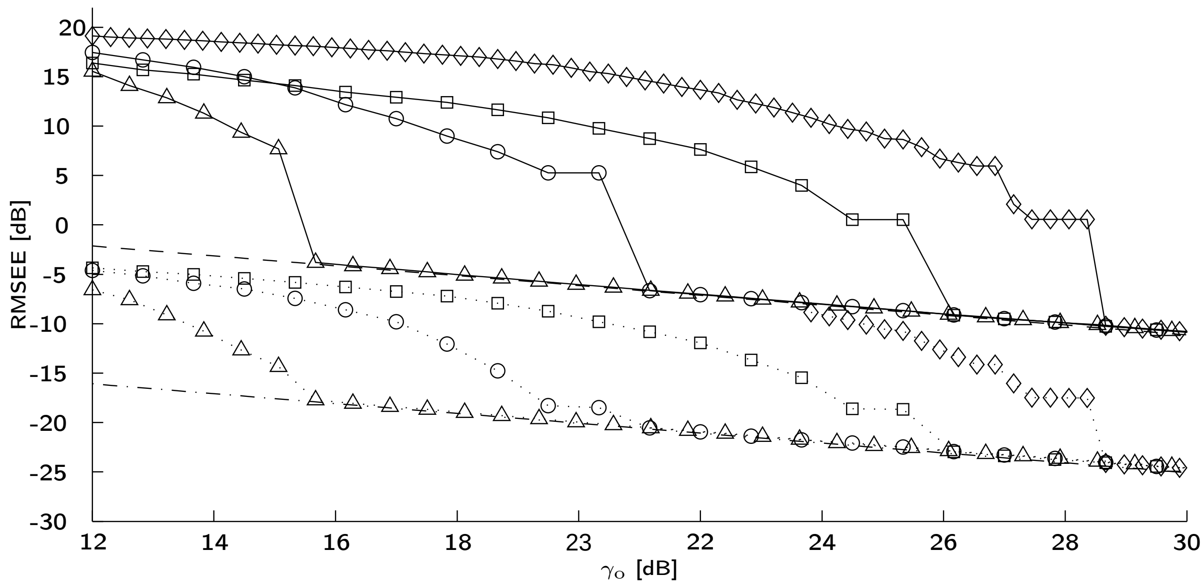
(b) Permuted SM: $[4, 2, 1, 8, 5, 7, 3, 6]$,

(c) Different permuted SM in all cycles.

Joint estimation of the DF and the DoA

Simulations:

RMSEE of DF (solid curves) and direction of arrival (dotted curves) versus SNR for different SMs leading to different NSLs (normalized sidelobe level):



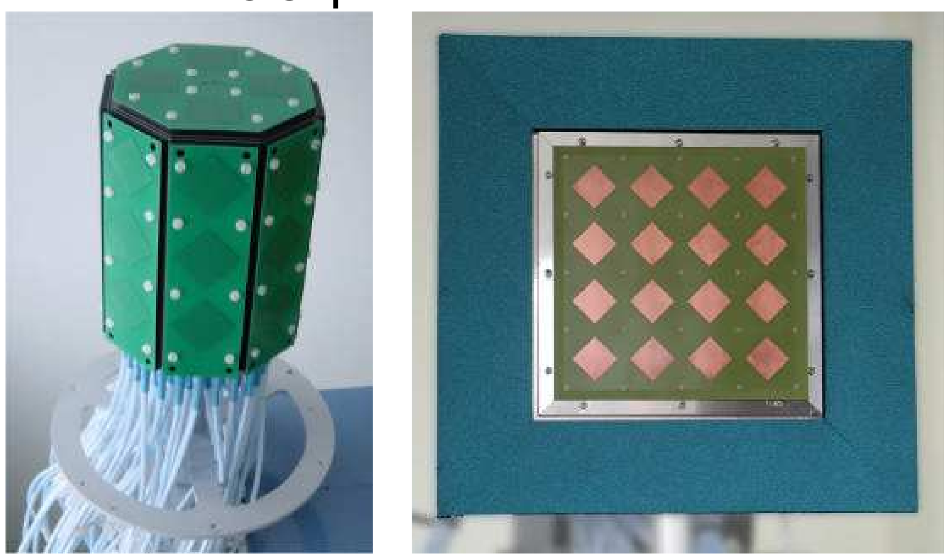
The selected SMs have normalized side-lobe level

$$\mathrm{NSL} = 0.85(\diamond),\ 0.80(\square),\ 0.58(\bigcirc) \text{ and } 0.28(\triangle).$$

Experimental Investigations

Characteristics of the measurement set-up:

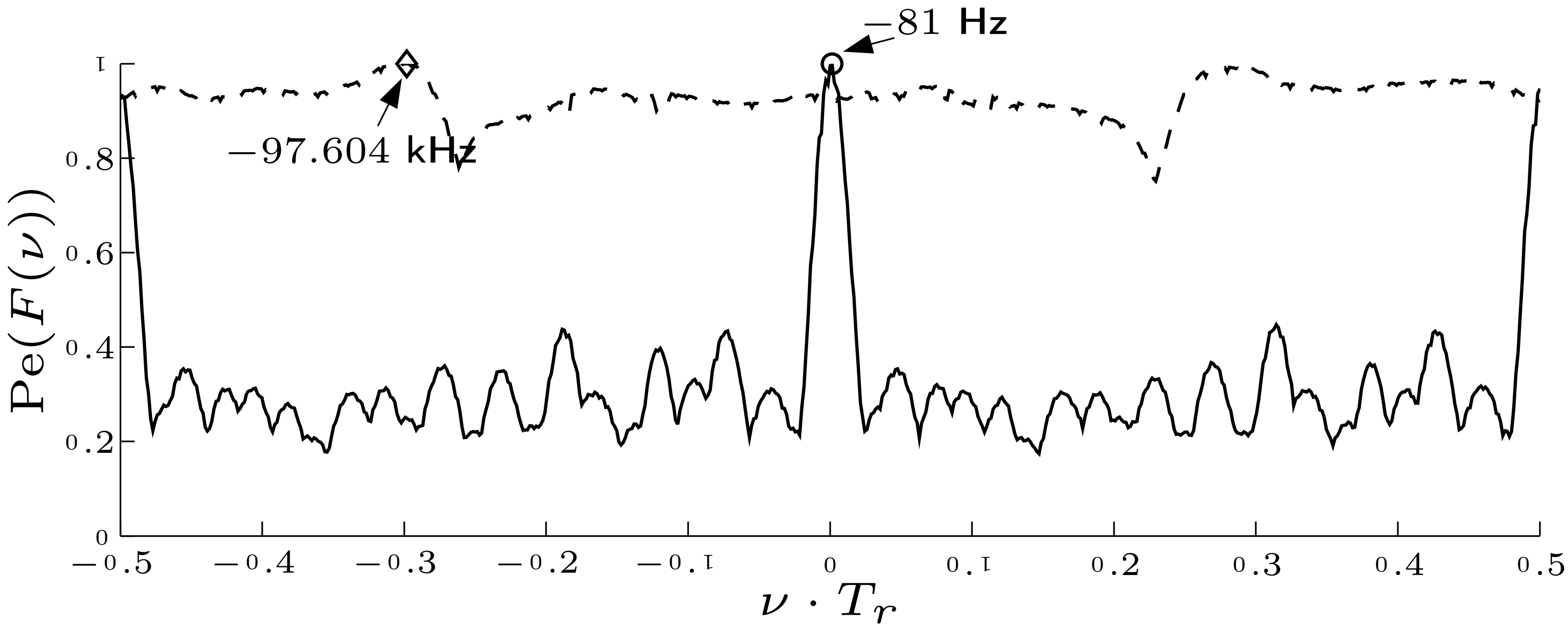
- TDM-MIMO channel sounder, PROPSound
- 3×8 dual-polarized ODA Tx array (M_1=54), (a)
- 4×4 dual-polarized planar Rx array (M_2=32), (b)
- Line-of-sight
- Two scenarios: Patch-wise identity SM and Patch-wise random SM.
- $\nu = -59$ Hz for the LOS path in both cases.



(a) (b)

Experimental Investigations

Joint estimation of the DF and DoA in the initialization step:
Pseudo-envelope $Pe(F(\nu))$ of objective function:



Conclusions

- The SAGE algorithm can estimate the DF with absolute value up to half the switching rate $\frac{1}{T_{\mathrm{r}}}$ rather than half the measurement cycle rate $\frac{1}{T_{\mathrm{cy}}}$ as commonly believed.
- Notice that the extended DFER does not depend on the numbers of array elements, and the range extension is by a factor $M_2 M_1 R$ compared to the traditionally used DFER.
- Modulo-type SMs used with uniform linear and planar arrays lead to an ambiguity in the estimation of the DF and the directions.
- By using suitably selected SMs, the DF and direction estimator can achieve near-optimum performance provided $\gamma_{\mathrm{o}} > \gamma_{\mathrm{o}}^{\mathrm{th}}$.
- The NSL is a sensible figure of merit associated to SMs for selecting "good" SMs.