

Tongji University Examination  
Year 2012 – 2013 I semester (A)

Signature of preparer:

Signature of examiner:

Number of course:102209

Course name: Fundamentals of Communication Systems

Type of examination:Exam

This examination is for: Intermediate exam (\_\_\_), final exam (\_\_\_), reexamine (\_\_\_)

Year\_\_\_\_\_ Specification \_\_\_\_\_ number of student \_\_\_\_\_ Name\_\_\_\_\_ Marks obtained:\_\_\_\_\_

The examination consists of three parts, i.e . Part I, questioning & answering; Part II, Computation; and Part II, Calculation and Analysis. The full marks are 100.

**Part 1, questioning & answering.**

Please answer the following 10 questions. Totally 30 points, each question 3 points.

1. What are the two modes of communications? Briefly describe their differences.

*Solution: Two modes of communications are analog and digital communications. There are many differences between the two modes: such as, 1) the signals are continuous and can take any arbitrary values in the analog communications, while the signals are discrete and take only predefined values in the digital communications; 2) For Digital communications, error correcting techniques can be used. Digital encryption can be used. Different kinds of message can be integrated to transmit in one system. For analog signals, these techniques may not be used.*

2. Which techniques in the following belong to the so-called first/second/third/fourth generation of wireless communications? AMPS, GSM, IS-95, CDMA, Bluetooth, WCDMA, CDMA, CDMA2000, TD-SCDMA, LTE, TD-LTE, WiMAX, WiFi, WLAN, Zigbee?

*Solution:*

- *1G: AMPS;*
- *2G:GSM, IS-95, Bluetooth, CDMA, WLAN, WiFi;*
- *3G: WCDMA, CDMA2000, TD-SCDMA;*
- *4G: LTE, TD-LTE, WiMAX,Zigbee*

3. What is the relationship between the bit rate  $R_b$  and symbol rate  $R_B$ ? What is the relationship between the bit error probability  $P_b$  and the symbol error probability  $P_s$ .

*Solution:  $R_b = R_B \log_2 M$ ,  $P_b = P_s / \log_2 M$*

4. Is the claim "Very small objects cast no shadow" correct? and state the reasoning.

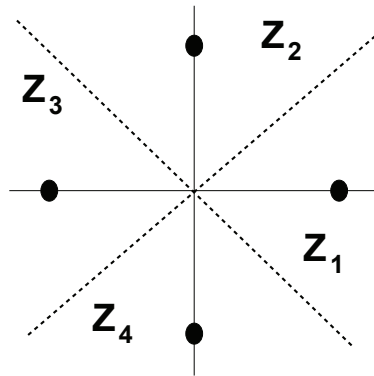
*Solution: This is correct. The shadowing is created by the diffraction. Diffraction happens when the electromagnetic waves hit an edge. The newly generated wave may*

not cover the region nearby the building. However when the object is small, such region is so small that can be negligible. Therefore we say very small objects cast no shadow.

5. What is the difference between the Okumura model and Hata model

*Solution: The Okumura model makes use of empirical tables. The Hata model is more analytical than the Okumura model and can be used to compute the path loss as a function of the distance from the base station to the mobile phone.*

6. Please draw the decision regions for QPSK.



7. To demodulate  $M$ -ary modulated signal, how many matched filters are needed at least? why?

*Solution: We need to have at least  $M$  matched filter, provided the  $M$  symbols are linearly independent. This is because the dimension of the signal space equals  $M$ . Each matched filter has the response identical with the  $m$ th transmitted message.*

8. Compared with phase modulation, frequency modulated signals have constant envelope and occupy larger bandwidth than the original signals. Please
- Shortly describe at least two advantages and two disadvantages of frequency modulation.
  - Select modulation scheme that has more robust performance: MSK, or PSK in a fast fading scenario, and tell the reason.

*Solution: The advantages:*

- *Nonlinear amplifiers can be used with high power efficiency*
- *Modulated signal is less sensitive (more robust) to amplitude distortion introduced by the channel or the hardware*

*The disadvantages:*

- *Higher bandwidth occupancy than amplitude and phase modulations*
- *Requires frequency reference detection*
- *Phase discontinuity may increase signal bandwidth*

*In a fast fading scenario, it is better to use MSK, since the PSK needs the phase reference, which can be time-variant and thus difficult to estimate. Furthermore, the MSK uses different frequencies, which may have less time-variant channel coefficients.*

9. What is the “Non decision-feedback PLL”? One kind of “Non decision-feedback PLL” relies on the distribution of the data. The symbol distribution is often assumed to be Gaussian along each signal dimension. Please shortly explain why this assumption is important.

*Solution: Solution: The structure of a non-decision-directed carrier phase recovery loop depends on the underlying distribution of the data. The constellation of the data would be symmetric.*

10. Please describe at least two common features and two significant differences between the space diversity and the angle/direction diversity.

*Solution: Solution: the common features: 1) Both diversities exploiting the channel diversity in the space domain; 2) both techniques use multiple antennas. Differences: 1) The difference is that for the space diversity, multiple antennas are used which are separated in space, in order for different antennas to observe different channel. For the angle/direction diversity, the antennas may have different directional radiation patterns, in order to cover different directions; 2) for the angle/direction diversity, a single antenna can be used whose position can be adjusted. The space diversity cannot use a single antenna.*

## Part II, calculating and analysis.

Please answer the following 8 questions. Totally 40 points, each question 5 points.

1. In order to avoid  $180^\circ$  phase shift, people use  $\pi/4$ -DQPSK instead of purely DQPSK. The steps of generating the  $\pi/4$ -DQPSK signals are: 1) Information bits are first differentially encoded as DQPSK, and then 2) every other symbol transmission is shifted in phase by  $\pi/4$ . Please use the afore-mentioned steps to find the phase transition of  $\pi/4$ -DQPSK modulated signals for the bit sequence 01 – 11 – 10 – 00 – 11 – 00, where “01” to the left of the sequence are transmitted first. The Gray coding is used for information encoding.

*Solution: The phase transition for DQPSK is  $90^\circ, 180^\circ, 270^\circ, 0^\circ, 180^\circ, 0^\circ$ . Initial phase is  $0^\circ$ . Then the phases of the symbol transmitted are  $90^\circ, 270^\circ, 180^\circ, 180^\circ, 0^\circ, 0^\circ$ . The phase transition for  $\pi/4$ -DQPSK is that  $90^\circ, 315^\circ, 180^\circ, 225^\circ, 0^\circ, 45^\circ$ .*

2. We use  $r_c = 1/T_c$  to denote the fading rate where  $T_c$  is the average fading duration in time, and  $r_s = 1/T_s$  to represent the symbol transmission rate. Please describe under the following scenarios, whether the outage probability and error probability are suitable for describing the system performance:

- $r_c \gg r_s$
- $r_c \approx r_s$
- $r_c \ll r_s$

**Solution:**

- $r_c \gg r_s$ :  $T_c \ll T_s$ , the fast fading scenario. The probability of symbol errors applies to describing the system performance.
- $r_c \approx r_s$ :  $T_c \approx T_s$ . The average error probability is a reasonably good figure of merit for the channel quality.
- $r_c \ll r_s$ :  $T_c \gg T_s$ , the slow fading scenario. The outage and average error probability are often combined.

3. Assuming that a 2–dimensional signal space is used for designing the constellation of 8PSK, please answer the following questions:

- Write the vector representation of the symbols  $\mathbf{s}_i$  for  $i = 1, \dots, 8$
- Give an example of the basis functions that can be possibly used?
- Now we use the direct-sequence spread spectrum (DSSS) technique to modulate the signals. In order to have the processing gain in decibel equal to 20 dB, how many chips should the spreading code have?

**Solution:**

- *Write the vector representation of the symbols  $\mathbf{s}_i$  for  $i = 1, \dots, 8$   
Signal constellation for MPSK ( $A > 0$ )*

$$s_{i1} = A \cos[2\pi(i-1)/8], \quad s_{i2} = A \sin[2\pi(i-1)/8], \quad i = 1, \dots, 8$$

- *Give an example of the basis functions that can be possibly used?*

$$\phi_1(t) = g(t) \cos(2\pi f_c t)$$

$$\phi_2(t) = g(t) \sin(2\pi f_c t)$$

- *Now we use the direct-sequence spread spectrum (DSSS) technique to modulate the signals. In order to have the processing gain in decibel equal to 20 dB, how many chips should the spreading code have?*

$$10 \cdot \log_{10}(N/M) = 20$$

$$N = M \cdot 10^{20/10}$$

$$= 800$$

4. For a mixed BPSK and BPAM modulation, the constellations  $s_1 = 2A$  and  $s_2 = -A$  for  $A > 0$  are transmitted. Please find the decision regions  $Z_1$  and  $Z_2$  corresponding to the constellations  $s_1$  and  $s_2$ , and plot the decision regions.

*Solution: The signal space is one-dimensional, so  $\mathbf{r} = r \in \mathcal{R}$ . The decision region  $Z_1 \subset \mathcal{R}$  is defined by*

$$Z_1 = \{r : \|r - 2A\| < \|r - (-A)\|\} = \{r : r > A/2\}.$$

*Thus,  $Z_1$  contains the positive numbers larger than  $A/2$  on the real line. Similarly*

$$Z_2 = \{r : \|r - 2A\| < \|r - (-A)\|\} = \{r : r < A/2\}.$$

*So  $Z_2$  contains all numbers less than  $A/2$ .*

5. The continuous phase FSK signal has the following form

$$s_i(t) = A \cos \left[ 2\pi f_c t + 2\pi\beta \int_{-\infty}^t u(\tau) d\tau \right]$$

where  $u(t) = \sum_k a_k g(t - kT_s)$  is a 16-PAM signal modulated with the information bit stream. Calculate

- the transmission bandwidth of  $s(t)$  by assuming small  $\beta$  and  $T_s = 1$  ms. The function  $g(t)$  is a Raised Cosine pulse with the roll off factor of 0.5, and  $T = T_s$ .
- the additional/extra bandwidth compared with a linearly modulated signal.

*Solution:  $\Delta f_c = 0.5/T_s = 0.5$  KHz. The pulse shape  $g(t)$  occupies the band  $[-1/(2T), 1/(2T)]$ , i.e.  $1/T = 1/T_s = 1$  KHz. For  $M = 16$ , we have totally  $B_s = M\Delta f_c + 2B_g = 16 \cdot 0.5 + 2 \cdot 1 = 10$  KHz. The extra bandwidth is  $M\Delta f_c = 8$  KHz.*

6. Find the bit error probability  $P_b$  and symbol probability  $P_s$  of QPSK assuming  $\gamma_b = 10$  dB. Compare the exact  $P_b$  with the approximation  $P_b \approx P_s/2$  based on the assumption of Gray coding. Finally, compute  $P_s$  based on the nearest neighbor bound using  $\gamma_s = 2\gamma_b$  and then compare with the exact  $P_s$ .

*Solution:*

- *The QPSK  $P_b$  is the same as the BPSK (since the QPSK is just two of BPSK).  $\gamma_b = 10^{10/10} = 10$*

$$P_b = Q(\sqrt{2\gamma_b}) = Q(\sqrt{20}) = 2.5 \cdot 10^{-10}$$

- *The exact symbol error probability  $P_s$  is*

$$\begin{aligned} P_s &= 1 - [1 - Q(\sqrt{\gamma_s})]^2 \\ &= 1 - [1 - Q(\sqrt{2\gamma_b})]^2 \\ &= 5.0792 \cdot 10^{-10} \end{aligned}$$

- *Nearest neighbor approximation:*

$$P_s \approx 2Q(\sqrt{\gamma_s}) = 5.0793 \cdot 10^{-10}$$

7. Determine the required  $\bar{\gamma}_b$  for BFSK modulation in slow Rayleigh fading such that 99% of the time (or in space),  $P_b(\gamma_b) < 10^{-4}$ .

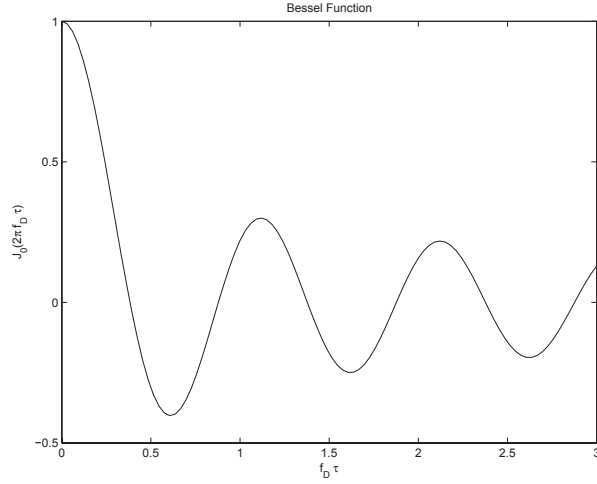
*Solution: Solution: For BPSK modulation in AWGN the target BER is obtained at 8.5 dB, i.e. for*

$$\begin{aligned} P_b(\gamma_b) &= 10^{-4} \\ Q(\sqrt{\gamma_b}) &= 10^{-4} \\ \gamma_b &= 2 \cdot 10^{0.85} \\ &= 11.5\text{dB} \end{aligned}$$

*Since we want  $P_{out} = p(\gamma_b < \gamma_0) = 0.01$  we have*

$$\bar{\gamma}_b = \frac{\gamma_0}{-\ln(1 - P_{out})} = \frac{2 \cdot 10^{0.85}}{-\ln(1 - 0.01)} = 31.49\text{dB}$$

8. What is the time diversity? Let's consider an environment which has the auto-correlation coefficients described by a Bessel function depicted in the following figure, where  $f_D$  and  $\tau$  represent the maximum absolute value of Doppler frequency and time difference respectively. In the case where the Doppler frequency of the channel is distributed within the range  $[-600, 1200]$  Hz, what would be the minimum time difference in order to have the maximum gains for the time diversity (Hint: the first zero-crossing point for the Bessel function is  $f_D\tau = 0.38$ .)



*Solution: The time diversity is to obtain the channels in different time instants in order to have uncorrelated channels. In the case considered,  $f_D = 1200$  Hz, thus the minimum time difference can be  $0.38/f_D = 3.17 \times 10^{-4}$  s.*

### Part III, computing.

Please answer the following 4 questions. Totally 30 points.

1. The two-ray model is used when a single ground reflection dominates the multipath effect. Let's consider a system with  $h_t = 10$  meters,  $h_r = 1.5$  meters,  $d \gg h_t + h_r$ , the system carrier frequency is 900 MHz. The propagation is as depicted in Figure 1. If the reflection on the ground introduces a phase rotation of  $\pi/4$ , please find an approximate of the distance where the received power has the first null.

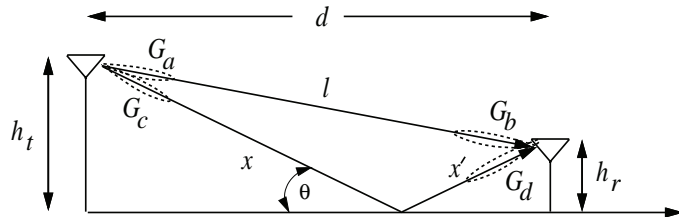


Figure 1: Two-ray model

Solution: Since the reflection has already introduced  $\pi/4$  additional phase, the phase difference  $\Delta\phi$  needs to be  $\pi - \pi/4 = 3\pi/4$ . Thus

$$\begin{aligned}\Delta\phi &\approx \frac{4\pi h_t h_r}{\lambda d} \\ d &\approx \frac{4\pi h_t h_r}{\lambda \Delta\phi} \\ &= \frac{4\pi 10 \cdot 1.5}{3 \times 10^8 / (9 \cdot 10^8) (3/4)\pi} \\ &= 180 \frac{4}{3} \\ &= 240\end{aligned}$$

2. Consider a 9QAM constellation with  $d_{min} = \sqrt{2}$ . What is the additional energy required to send one extra bit (16QAM) while keeping the same bit error probability.

Solution: We are going to use the following approximation:

$$P_b \approx \frac{2(\sqrt{M} - 1)}{\sqrt{M} \log_2 M} Q\left(\sqrt{\frac{3\bar{\gamma}_s}{M - 1}}\right)$$

It can be calculated that for 9QAM,  $\bar{\gamma}_s = \frac{3}{N_0}$ , and for 16QAM,  $\bar{\gamma}_s = \frac{5d^2}{2N_0}$ . Equating  $P_{b,9QAM} = P_{b,16QAM}$  yields

$$\frac{4}{3 \log_2 9} Q\left(\sqrt{\frac{9}{8N_0}}\right) = \frac{3}{8} Q\left(\sqrt{\frac{d^2}{2N_0}}\right) \quad (1)$$

3. Please refer to Figure 2, and consider the following conditions to calculate *i*) the average energy per symbol, and *ii*) the probability of symbol errors  $P_s$  using the nearest neighbor approximation:

Case I  $b = 1$ ,  $N_0 = 0.02$ ,  $a = b$ , the angle theta =  $45^\circ$ ;

Case II  $b = 1, N_0 = 0.02$ ,  $a = b/2$ , the angle theta =  $45^\circ$ .

Solution: Solution: We calculate the  $d_{min}$  for Case I:  $d_{min} = 2 * b * \sin(45/2) = 2 * 1 * 0.3827 = 0.7654$ . Thus assuming the  $M_{min} = 2$  for the 8 outer points, and  $P_s = 2 * Q(0.7654/0.2) = 6.2 * 10^8$ .

For the case II, the outer points has the

$$d_{min} = \sqrt{(b \cos(\theta))^2 + (b \sin(\theta) - a)^2} = \sqrt{b^2 - 2ab \sin(\theta) + a^2}$$

when  $\theta = 45^\circ$ ,  $d_{min} = 0.73b$ . Thus

$$\begin{aligned}P_s &= 2 \cdot Q(0.73b/\sqrt{2N_0}) \cdot 4/8 + 4 \cdot Q(0.5b/\sqrt{2N_0}) \cdot 4/8 \\ &= 2 * 2.4 * 10^{-7} * 0.5 + 4 * 4.1 * 10^{-4} * 0.5 \\ &= 8.1 * 10^{-4}\end{aligned} \quad (2)$$

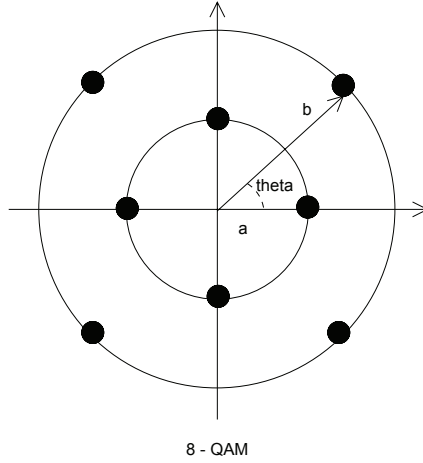


Figure 2: 8QAM

4. Considering a channel with the impulse response of  $h(t) = 0.5\delta(t - \tau_0) + 0.5\delta(t - \tau_1)$  where  $\tau_0 = 5$  ns is the delay of the line-of-sight (LoS) component, and  $\tau_1 = 10$  ns is the delay of a reflected non-line-of-sight component. For a spread spectrum system using maximal linear codes with following two settings, please calculate (1) the required delay offset of the synchronizer in the receiver relative to the line-of-sight component in order to obtain the maximum of received signal power, and (2) the power reduction compared to the case where only the LoS component exist.

- The maximal linear codes with period  $T = 200$  ns and  $N = 100$ ,
- The maximal linear codes with period  $T = 200$  ns and  $N = 40$ ,

*Solution: Solution: For  $\tau = 10\text{ns} = 5T_c$  and  $N = 100$ , the autocorrelation  $\rho_c(\tau)$  will have two peaks. So the maximum power would be  $(1 - 1/N)^2 = (1 - 0.025)^2 = 0.975^2$ . The power reduction equals  $(1 - 0.025)^2 = -0.2199$  dB. For  $\tau = 10\text{ns} = 1T_c$  and  $N = 40$ , the offset would be 5 ns. The power reduction would be  $(2 * 0.495)^2 = -0.0873$  dB.*