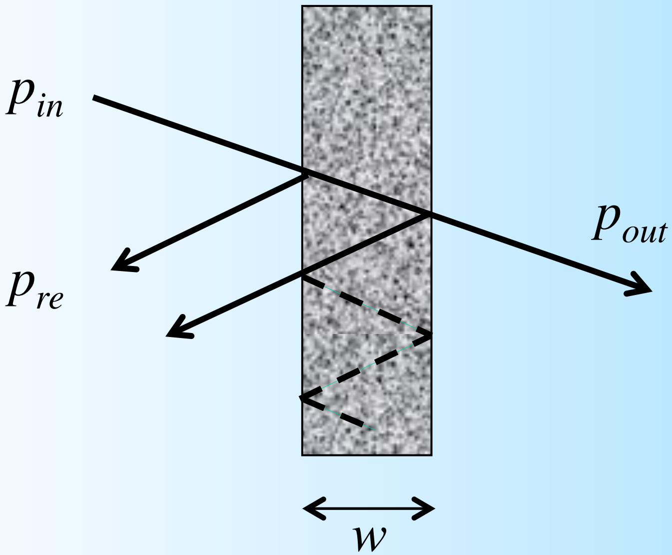
Deterministic radio propagation modeling and ray tracing

1) Introduction to deterministic propagation modelling
2) Geometrical Theory of Propagation I - The ray concept – Reflection and transmission
3) Geometrical Theory of Propagation II - Diffraction, multipath
4) Ray Tracing I
5) Ray Tracing II – Diffuse scattering modelling
6) Deterministic channel modelling I
7) Deterministic channel modelling II – Examples
8) Project - discussion

Transmission through a wall (1/5)

* Hypotheses: - normal or quasi-normal incidence
 - weakly lossy medium



$$|\Gamma| \approx |\Gamma_{TE}| = \left| \frac{\cos\theta_i - \sqrt{\left(\frac{n_2}{n_1}\right)^2 - \sin^2\theta_i}}{\cos\theta_i + \sqrt{\left(\frac{n_2}{n_1}\right)^2 - \sin^2\theta_i}} \right| = \left| \frac{1-\sqrt{\varepsilon_r}}{1+\sqrt{\varepsilon_r}} \right|$$

$$\frac{S_{re}}{S_{in}} = \frac{|E_{re}|^2 / 2\eta}{|E_{in}|^2 / 2\eta} = \frac{|E_{re}|^2}{|E_{in}|^2} \approx |\Gamma|^2$$

(Source: Prof. H.L. Bertoni)

Transmission through a wall (2/5)

In a lossy medium the wavenumber can be written as:

$$k = \omega\sqrt{\mu_0\varepsilon_c} = \omega\sqrt{\mu_0\varepsilon_0\varepsilon_r}$$

The complex relative dielectric constant can be written as:

$$\varepsilon_r = \varepsilon_r' - j\varepsilon_r'' = \frac{\varepsilon}{\varepsilon_0} - j\frac{\sigma}{\omega\varepsilon_0}$$

If the medium is weakly lossy ε”<<ε’ .
A plane wave propagating through the lossy medium has the expression:

$$\mathbf{E} = \mathbf{E_0}e^{-jkr} = \mathbf{E_0}e^{-(\alpha+j\beta)r};$$ with $jk=\alpha+j\beta$

Thus:

$$k = \omega\sqrt{\mu_0\varepsilon_0\varepsilon_r} = \omega\sqrt{\mu_0\varepsilon_0}\sqrt{\varepsilon_r' - j\varepsilon_r''} = \frac{\omega}{c}\sqrt{\varepsilon_r' - j\varepsilon_r''} =$$

$$= \frac{\omega}{c}\sqrt{\varepsilon_r'}\sqrt{1 - j\frac{\varepsilon_r''}{\varepsilon_r'}} \approx \frac{\omega}{c}\sqrt{\varepsilon_r'}\left(1 - j\frac{\varepsilon_r''}{2\varepsilon_r'}\right)$$

Where the series expansion have been truncated at first order

Transmission through a wall (3/5)

Therefore:

$$jk = \alpha + j\beta \approx \frac{\omega}{c}\sqrt{\varepsilon_r'}\left(\frac{\varepsilon_r''}{2\varepsilon_r'} + j\right) \Rightarrow$$

$$\begin{cases} \alpha \approx \dfrac{\omega}{c}\sqrt{\varepsilon_r'}\left(\dfrac{\varepsilon_r''}{2\varepsilon_r'}\right) \\ \beta \approx \dfrac{\omega}{c}\sqrt{\varepsilon_r'} \end{cases}$$

$$\left|E(r)\right| = \left|E(0)\right| \cdot e^{-\alpha r}$$

$$S(r) = S(0) \cdot e^{-2\alpha r}$$

Transmission through a wall (4/5)

The reflection coefficient at normal incidence for the air-medium interface is

$$\Gamma_{0m} = \frac{1-\sqrt{\varepsilon_r}}{1+\sqrt{\varepsilon_r}}$$

The reflection coefficient for the second, medium-air interface is (see the expression of the reflection coefficients for normal incidence)

$$\Gamma_{m0} = \frac{\sqrt{\varepsilon_r}-1}{1+\sqrt{\varepsilon_r}} = -\Gamma_{0m}$$

Now if we consider the first interface we have

$$\frac{S_{refl1}}{S_{inc1}} = \frac{\left|\vec{E}_{refl1}\right|^2}{\left|\vec{E}_{in1}\right|^2} = \left|\Gamma_{0m}\right|^2$$

Transmission through a wall (5/5)

For power conservation we have:

$$S_{inc1} = S_{refl1} + S_{trasm1} \quad \Rightarrow \quad 1 = \frac{S_{refl1}}{S_{inc1}} + \frac{S_{trasm1}}{S_{inc1}} = \left|\Gamma_{0m}\right|^2 + \frac{S_{trasm1}}{S_{inc1}}$$

$$\frac{S_{trasm1}}{S_{inc1}} = 1 - \left|\Gamma_{0m}\right|^2$$

Now the transmitted power at the first interface, properly multiplied by the lossy-medium attenuation factor becomes the incident power at the second interface, therefore we have

$$\frac{S_{refl2}}{S_{inc2}} = \left|\Gamma_{m0}\right|^2 = \left|\Gamma_{0m}\right|^2 = \left|\Gamma\right|^2 \quad ; \quad \frac{S_{transm2}}{S_{inc2}} = \frac{S_{transm2}}{S_{transm1} e^{-2\alpha w}} = \frac{S_{transm2}}{S_{inc1}\left(1 - \left|\Gamma\right|^2\right) e^{-2\alpha w}} = 1 - \left|\Gamma\right|^2$$

Thus:

$$\frac{S_{transm2}}{S_{inc1}} = \frac{S_{out}}{S_{in}} = \left(1 - \left|\Gamma\right|^2\right)^2 e^{-2\alpha w} \Rightarrow L_t = \frac{S_{in}}{S_{out}} = \frac{e^{2\alpha w}}{\left(1 - \left|\Gamma\right|^2\right)^2}$$

Example of Transmission Loss

Brick wall: $\varepsilon_r'=4,\ \varepsilon_r''=0.2,\ w=20$ cm

$$\left|\Gamma\right|^2 = \frac{S_{refl1}}{S_{inc1}} \approx \left|\frac{\sqrt{4}-1}{\sqrt{4}+1}\right|^2 = \frac{1}{9} = 0.11 \quad \text{or} \quad -9.6\text{dB}$$

at 1,800 MHz (λ_o=1/6 m): $\alpha = \dfrac{0.2\pi}{(1/6)\sqrt{4}} = 1.88$

$$\mathrm{L}_t = \frac{S_{in}}{S_{out}} = (1-0.11)^2 e^{2(0.2)(1.88)} = 2.7 \quad \text{or} \quad 4.3\text{dB}$$

(Source: Prof. H.L. Bertoni)

Summary of Reflection and Transmission Loss

Theory

Wall Type	Frequency Band	Ref. loss	Trans. Loss
Brick, exterior	1.8 - 4 GHz	10 dB	10 dB
Concrete block, interior	2.4 GHz		5 dB
Gypsum board, interior	3.4 GHz	4 dB	2 dB

Measured

Exterior frame with metal siding	800 MHz 5 - 6 GHz 5 GHz		4 - 7 dB 9 - 18 dB 36 dB
Brick, exterior	4 - 6 GHz	10 dB	14 dB
Concrete block, interior	2.4 / 5 GHz		5 / 5 - 10 dB
Gypsum board, interior	2.4 / 5 GHz		3 / 5 dB
Wooden floors	5 GHz		9 dB
Concrete floors	900 MHz		13 dB

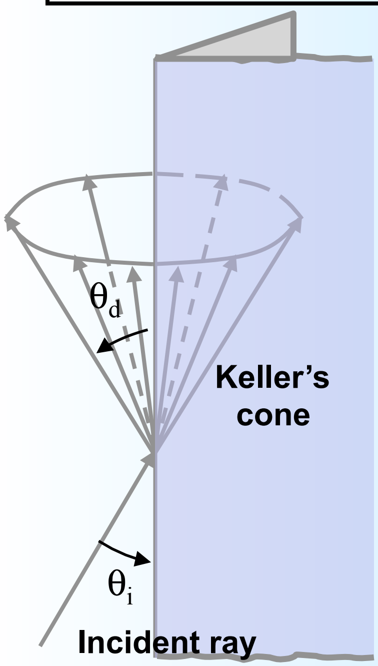
(Source: Prof. H.L. Bertoni)

Geometrical Theory of Diffraction

The extension of GO to the category of diffracted rays was first introduced by J. B. Keller in 1961 and is based on the following assumptions[6] :

I. *A diffracted ray is generated whenever a ray impinges on an edge (or on a vertex)*

II. *For every diffracted ray the Fermat's principle holds*

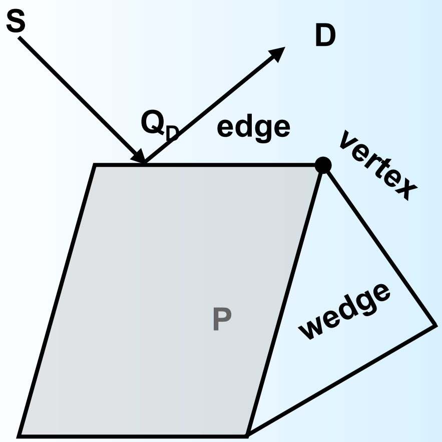


<u>Diffraction law</u>*: *the angles between incident / diffracted ray and the edge satisfy "Snell's law applied to diffraction":

$$n_i \cdot \sin\theta_i = n_d \cdot \sin\theta_d$$

➔ If the rays are in the same material then: $\theta_d = \theta_{i;}$
Therefore diffracted rays ouside the wedge belong to the ***Keller's cone***

The diffracted ray (1/3)



- In urban propagation only straight edges (local field principle) are of interest. Vertex diffraction won't be treated here

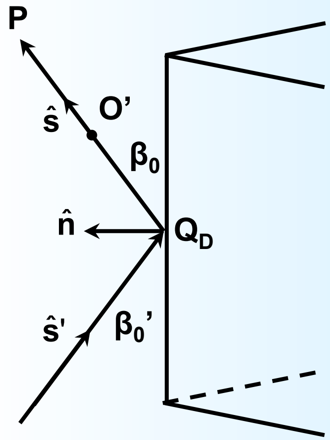
- If the impinging wave is plane (or can be approximated so for the local field principle) then the diffracted wave is cylindrical for perpendicular incidence ($\theta_d=\theta_i=\pi/2$) and conical for oblique incidence (the wavefront is a cone) [7]

- The diffracted wave is so that one caustic coincides with the edge. **Therefore the divergence factor of the diffracted wave/ray is different from that of the incident wave/ray** (see further on)

- The diffracted ray field can be computed by solving Maxwell's equations for a plane , cylindrical or spherical wave incident on a straight conducting edge [7, 8, 9] and somehow subtracting from the solution the incident wave and the reflected wave(s).

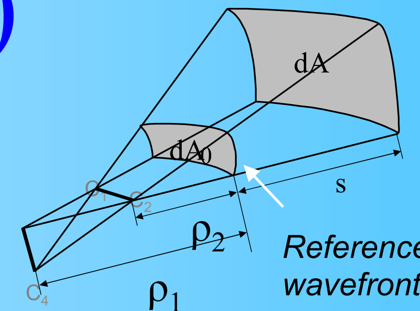
- Then the diffracted field is expanded in a Luneberg-Kline series from which only the first term (high frequency approx.) is kept in order to derive the ***diffraction coefficients***

The diffracted ray (2/3)



The high frequency term has the form:

$$\vec{E}^d(s) = \vec{E}^d(O')\cdot\sqrt{\frac{\rho_1^d\cdot\rho_2^d}{(\rho_1^d+s)\cdot(\rho_2^d+s)}}\cdot e^{-j\beta s}$$



ρ_1^d, ρ_2^d = curvature radii of the diffracted wave.

One caustic coincides with the edge: ρ_2^d corresponds to $O'-Q_D$ where O' is the reference point, origin of the coordinate s.

It is useful to choose $O'=Q_D$ ($\rho_2^d=0$ ➔ simpler expression). However for power conservation reasons $\mathbf{E}^d(O')$ ➔ ∞ for O' ➔ Q_D

Since $\mathbf{E}^d(s)$ cannot change with the reference system, therefore it must be:

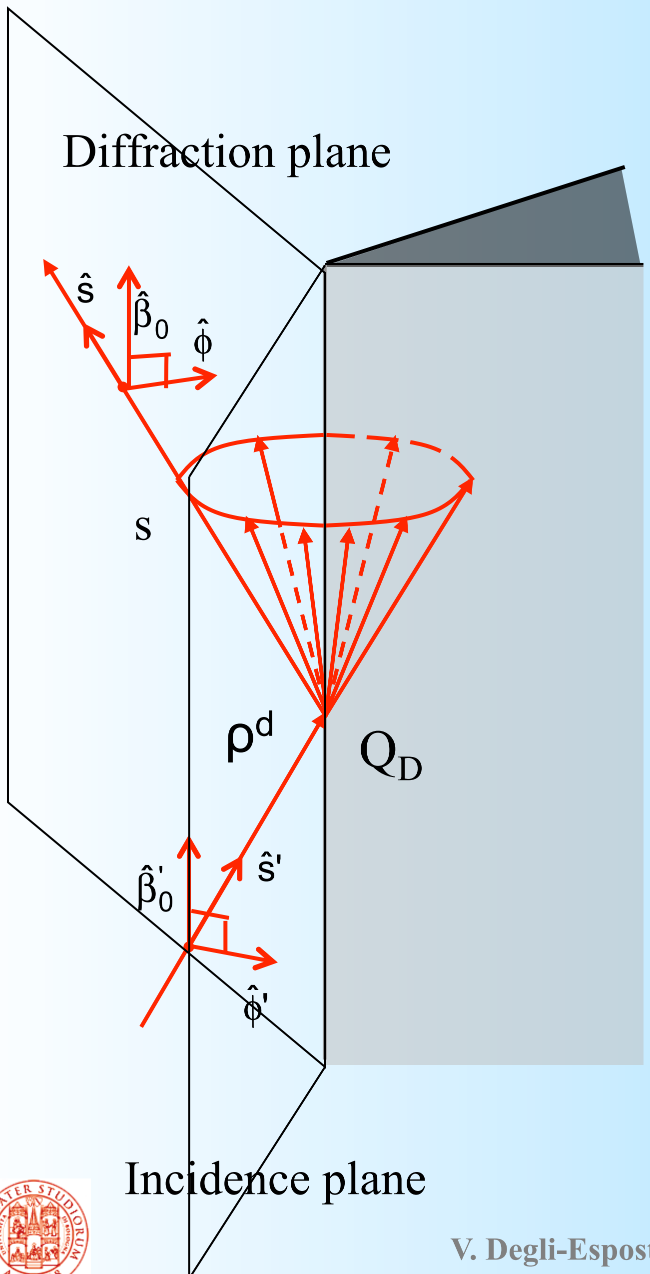
$$\lim_{\substack{O'\to Q_D \\ (\rho_2^d\to 0)}}\left[\vec{E}^d(O')\cdot\sqrt{\rho_2^d}\right] = \textit{finite vector} \equiv \vec{E}^i(Q_D)\cdot\mathbf{D} \quad\Rightarrow\quad \boxed{\vec{E}^d(s) = \vec{E}^i(Q_D)\cdot\mathbf{D}\cdot A(\rho^d, s)\cdot e^{-j\beta s}}$$

with: $A(\rho^d, s) = \sqrt{\dfrac{\rho^d}{(\rho^d+s)\cdot s}}$

D is the ***diffraction matrix,*** which contains the diffraction coefficients

The diffracted ray (3/3)

Attenzione a ro-d e s’



➔ trajectory: Fermat’s principle

➔ Field expression:

$$\begin{bmatrix} E^d_{\beta_0} \\ E^d_{\phi} \end{bmatrix} = \begin{bmatrix} D_s & 0 \\ 0 & D_h \end{bmatrix} \begin{bmatrix} E^i_{\beta_0'}(Q_D) \\ E^i_{\phi'}(Q_D) \end{bmatrix} \cdot A(s,\rho^d) \cdot e^{-j\beta s}$$

divergence factor…

if the proper local reference system is adopted (see figure) then the diffraction matrix reduces to a 2x2 diagonal matrix, otherwise it’s a 3x3 matrix

Φ-polarization is called “hard” (TE), β-polarizationi is called “soft” (TM)

The divergence factor

If $\rho_2^d \rightarrow 0$ as shown, then we get :
$(\rho_1^d \rightarrow \rho^d)$

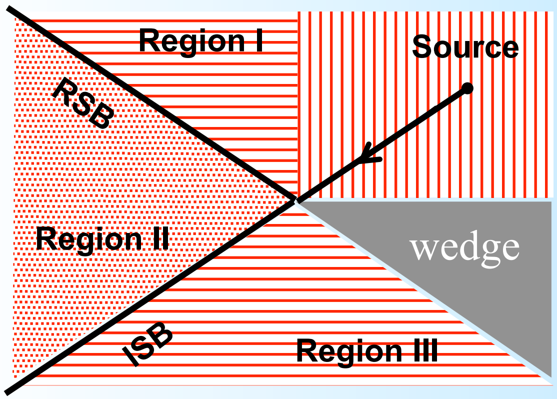
$$A(\rho^d, s) = \sqrt{\frac{\rho^d}{s \cdot (\rho^d + s)}}$$

For a straight edge we have:

$$A(\rho^d, s) = \begin{cases} \dfrac{1}{\sqrt{s}} & \text{for a plane incident wave} \\ \dfrac{1}{\sqrt{s \cdot \sin \beta_o'}} & \text{for a cylindrical incident wave} \\ \sqrt{\dfrac{\rho^d}{s \cdot (\rho^d + s)}} & \text{for a spherical incident wave} \end{cases}$$

- For the computation of the diffraction coefficients we refer in the following to a simple case with a cylindrical incident wave.

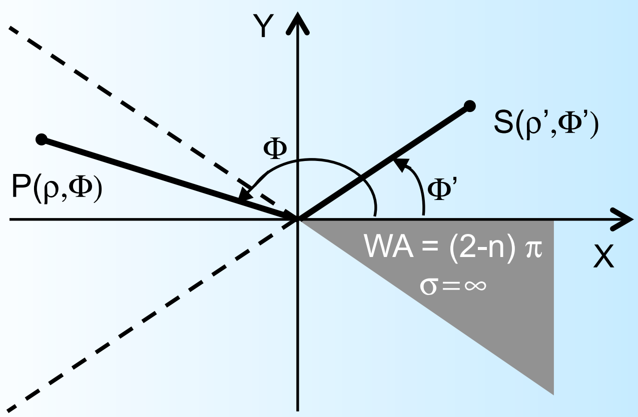
The diffraction coefficients for a canonical 2D problem



ISB : Incidence Shadow Boundary

RSB : Reflection Shadow Boundary

R I : direct + reflected + diffracted
R II : direct + diffracted
R III : diffracted



Hypotheses:

- unlimited perfectly conducting wedge of angular width WA $=(2-n)\pi$ $(0 \leq n < 2)$
- Infinite uniform linear source parallel to the edge with constant current $I_0\,\mathbf{i_z}$

⇩

cylindrical incident wave with normal incidence

The diffraction coefficients

Adopting the method described above the following Keller's diffraction coefficients are obtained (***G****eometrical* ***T****heory of* ***D****iffraction, GTD*) [9]

$$D^S(\phi,\phi',n) = \frac{-e^{-j\pi/4}\cdot\sin\left(\pi/n\right)}{n\sqrt{2\pi\beta}}\cdot\left[\frac{1}{\cos\left(\pi/n\right)-\cos\left(\xi^-/n\right)} - \frac{1}{\cos\left(\pi/n\right)-\cos\left(\xi^+/n\right)}\right]$$

$$D^H(\phi,\phi',n) = \frac{-e^{-j\pi/4}\cdot\sin\left(\pi/n\right)}{n\sqrt{2\pi\beta}}\cdot\left[\frac{1}{\cos\left(\pi/n\right)-\cos\left(\xi^-/n\right)} + \frac{1}{\cos\left(\pi/n\right)-\cos\left(\xi^+/n\right)}\right]$$

$\xi^- = \Phi - \Phi'$

$\xi^+ = \Phi + \Phi'$

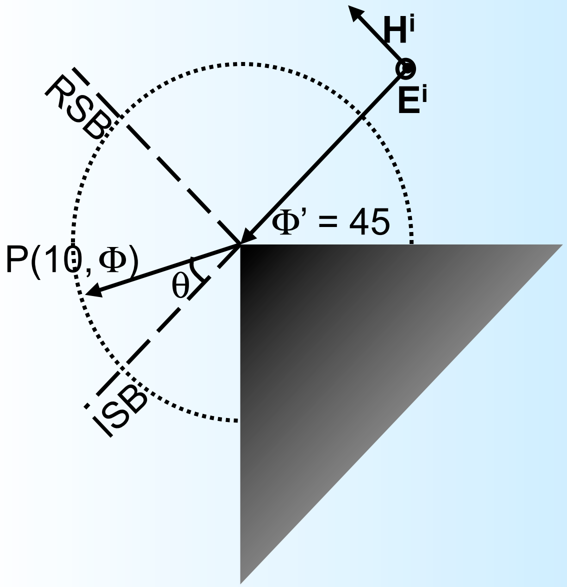
Such coefficients have singularities on the shadow boundaries, i.e. when:

$\xi- = \phi - \phi' = \pi$ (ISB)

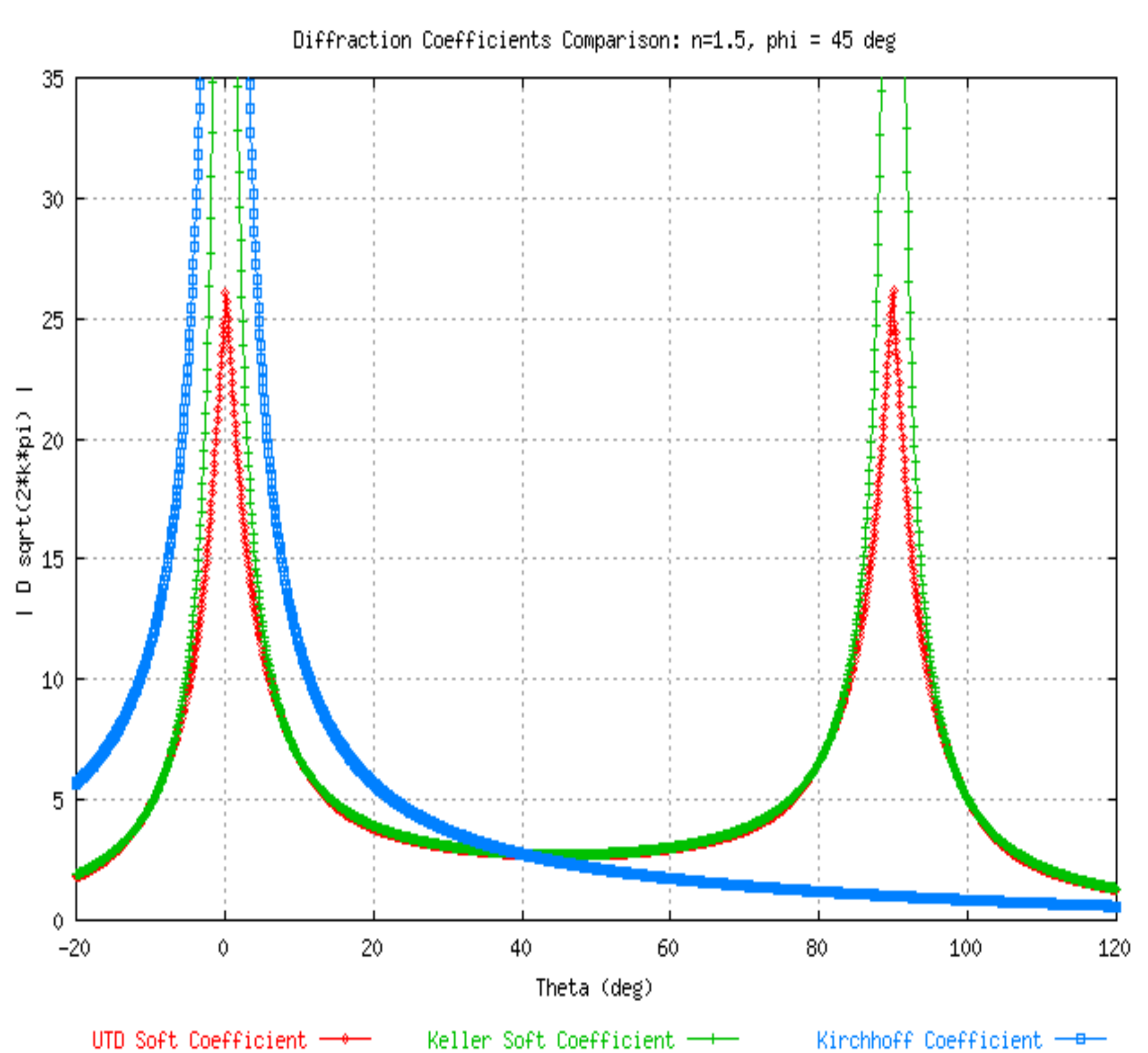
$\xi+ = \phi + \phi' = \pi$ (RSB)

Therefore also other, more complicated coefficients have been derived which do not have such singularity: the UTD (***U****niform* ***T****heory of* ***D****iffraction*) coefficients

Example (1/2)



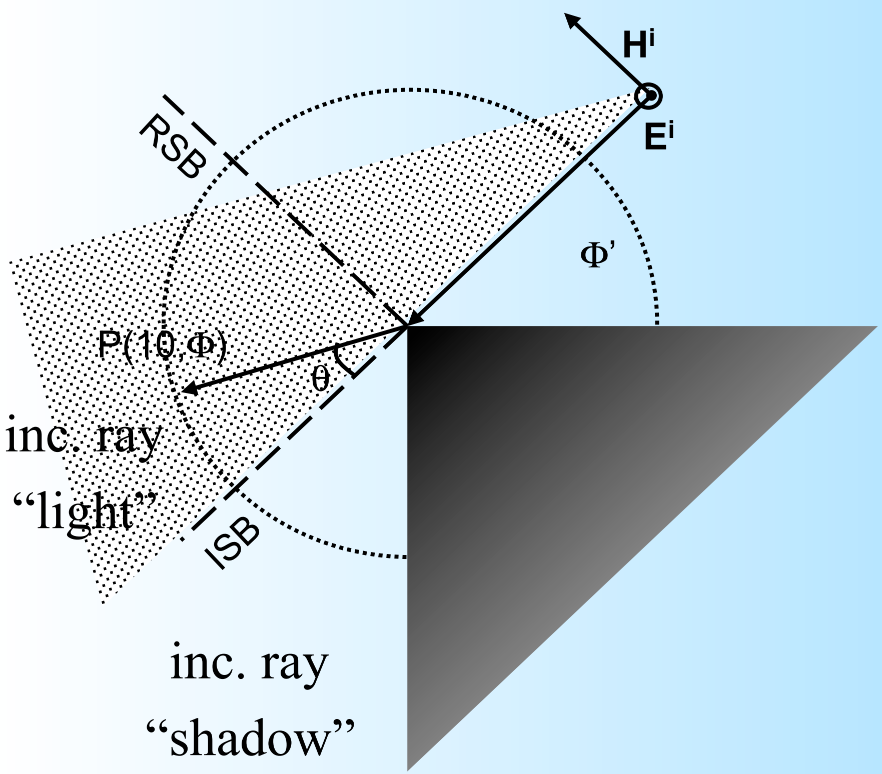
H^i
E^i
RSB
ISB
Φ' = 45
P(10, Φ)
θ

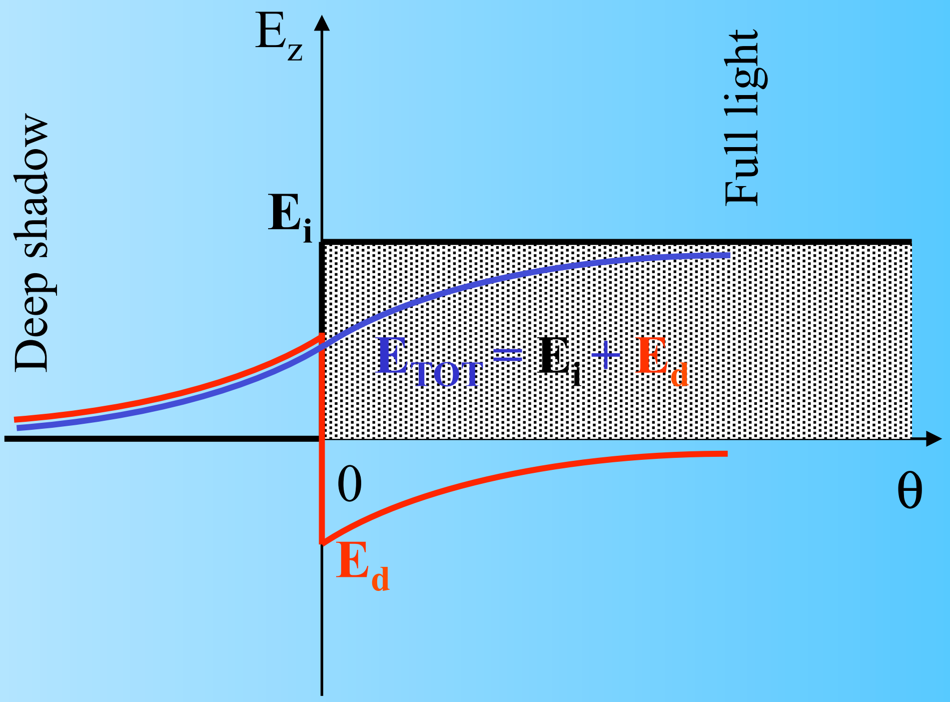


Diffraction Coefficients Comparison: n=1.5, phi = 45 deg
| D sqrt(2*k*pi) |
Theta (deg)
35
30
25
20
15
10
5
0
-20
0
20
40
60
80
100
120
UTD Soft Coefficient
Keller Soft Coefficient
Kirchhoff Coefficient

Example (2/2)

UTD, considering the diffracted ray and the incident ray





Other notes on GTP

- A single ray can undergo multiple interactions. The resulting ray is therefore a polygonal line and the proper interaction coefficients must be applied at each interaction. The proper divergence factor for the overall piece-wise path must then be applied.

- Reflection and transmission do not change the form of the divergence factor of a ray. Diffraction does.

- Diffraction coefficients for oblique incidence and dielectric wedges have also been derived by some authors

- The interaction called “diffuse scattering” is important but is not treated here. It will be treated further on.

Computation Examples: reflection

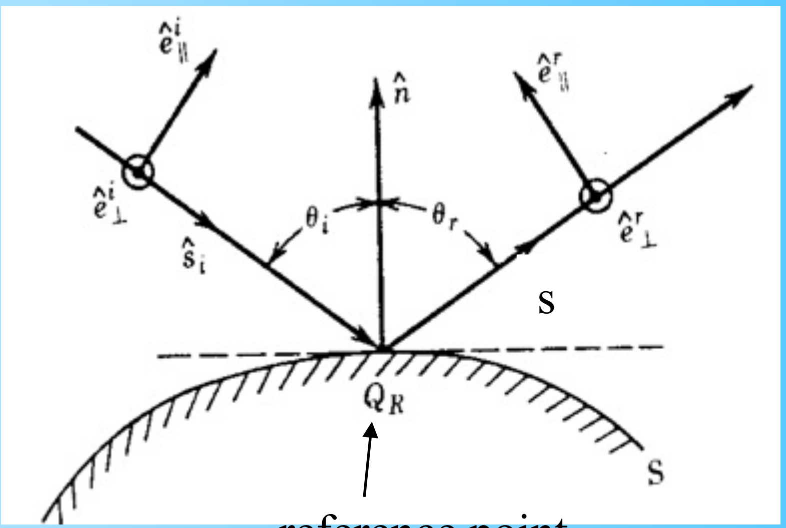
For the generic incident astigmatic wave we can write:

$$\vec{E}_r(s) = \underbrace{\vec{E}(Q_R)}_{\text{field at reference point } (Q_R\text{ , s=0})} \cdot \underbrace{\underline{\underline{\mathbf{R}}}(Q_R,\theta_i)}_{\text{Reflection coefficient (Dyadic)}} \cdot \underbrace{\sqrt{\frac{\rho_1 \cdot \rho_2}{(\rho_1+s)\cdot(\rho_2+s)}}}_{\text{divergence or spreading factor}} \cdot \underbrace{e^{-j\beta s}}_{\text{Phase factor}}$$

The use of the Dyadic Reflection coefficient [8] allows to refer to a fixed reference system

$$\underline{\underline{\mathbf{R}}} = \Gamma_{//}\left(\hat{e}^i_{//}\hat{e}^r_{//}\right) + \Gamma_{\perp}\left(\hat{e}^i_{\perp}\hat{e}^r_{\perp}\right)$$

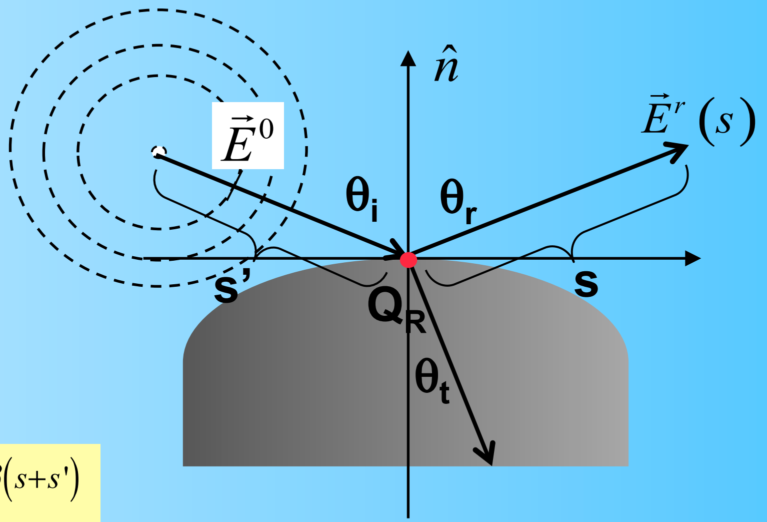
$$\left(\bar{a}\,\bar{b}\right) \triangleq \begin{pmatrix} a_x b_x & a_x b_y & a_x b_z \\ a_y b_x & a_y b_y & a_y b_z \\ a_z b_x & a_z b_y & a_z b_z \end{pmatrix}$$



reference point

Reflection (II)

For a spherical incident wave the expression above becomes ($\rho_1 = \rho_2 = s'$):



$$\vec{E}_r(s) = \vec{E}^0 \frac{e^{-j\beta s'}}{s'} \cdot \underline{\underline{\mathbf{R}}} \cdot \frac{s'}{s+s'} e^{-j\beta s} = \vec{E}^0 \cdot \underline{\underline{\mathbf{R}}} \cdot \frac{e^{-j\beta(s+s')}}{s+s'}$$

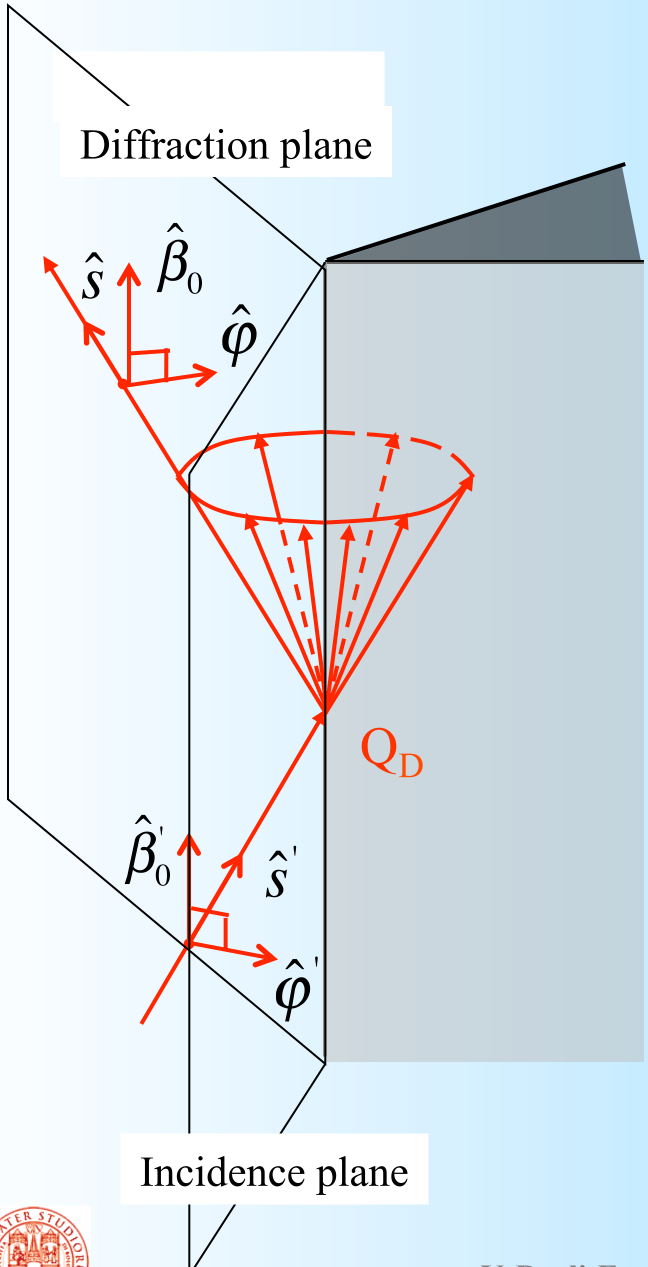
which is equivalent to

$$\begin{bmatrix} \vec{E}_{r\ TE}(s) \\ \vec{E}_{r\ TM}(s) \end{bmatrix} = \begin{bmatrix} \Gamma_{TE} & 0 \\ 0 & \Gamma_{TM} \end{bmatrix} \cdot \begin{bmatrix} \vec{E}^0_{TE} \\ \vec{E}^0_{TM} \end{bmatrix} \cdot \frac{e^{-j\beta s'}}{s'} \frac{s'}{s+s'} e^{-j\beta s}$$

Divergence factor for a spherical wave

Incident field in Q_R

Diffraction



Diffraction coefficients ➔ Diffracted field

$$\begin{bmatrix} E^d_{\beta_0} \\ E^d_{\phi} \end{bmatrix} = \begin{bmatrix} D_s & 0 \\ 0 & D_h \end{bmatrix} \begin{bmatrix} E^i_{\beta_0'}(Q_D) \\ E^i_{\phi'}(Q_D) \end{bmatrix} \cdot A \cdot e^{-j\beta s}$$

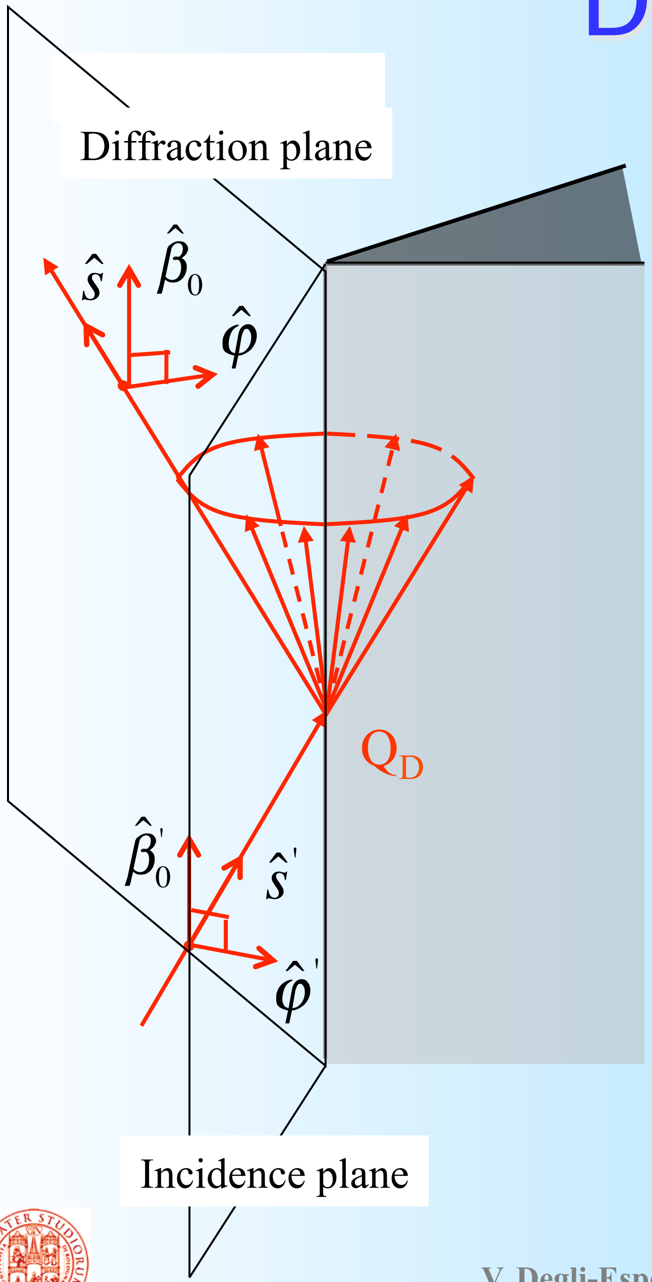
A is the *divergence factor* for the diffracted field. For a spherical incident wave:

$$A(s',s) = \sqrt{\frac{s'}{s \cdot (s'+s)}} \qquad \vec{E}^i(Q_D) = \vec{E}^{0i} \frac{e^{-j\beta s'}}{s'}$$

Therefore we have:

$$\begin{bmatrix} \vec{E}^d_{\beta_0} \\ \vec{E}^d_{\phi} \end{bmatrix} = \begin{bmatrix} D_s & 0 \\ 0 & D_h \end{bmatrix} \begin{bmatrix} \vec{E}^{0i}_{\beta_0'} \\ \vec{E}^{0i}_{\phi'} \end{bmatrix} \cdot \frac{1}{\sqrt{s \cdot s' \cdot (s'+s)}} \cdot e^{-j\beta(s+s')}$$

Diffraction (II)



Using the the Dyadic Diffraction coefficient:

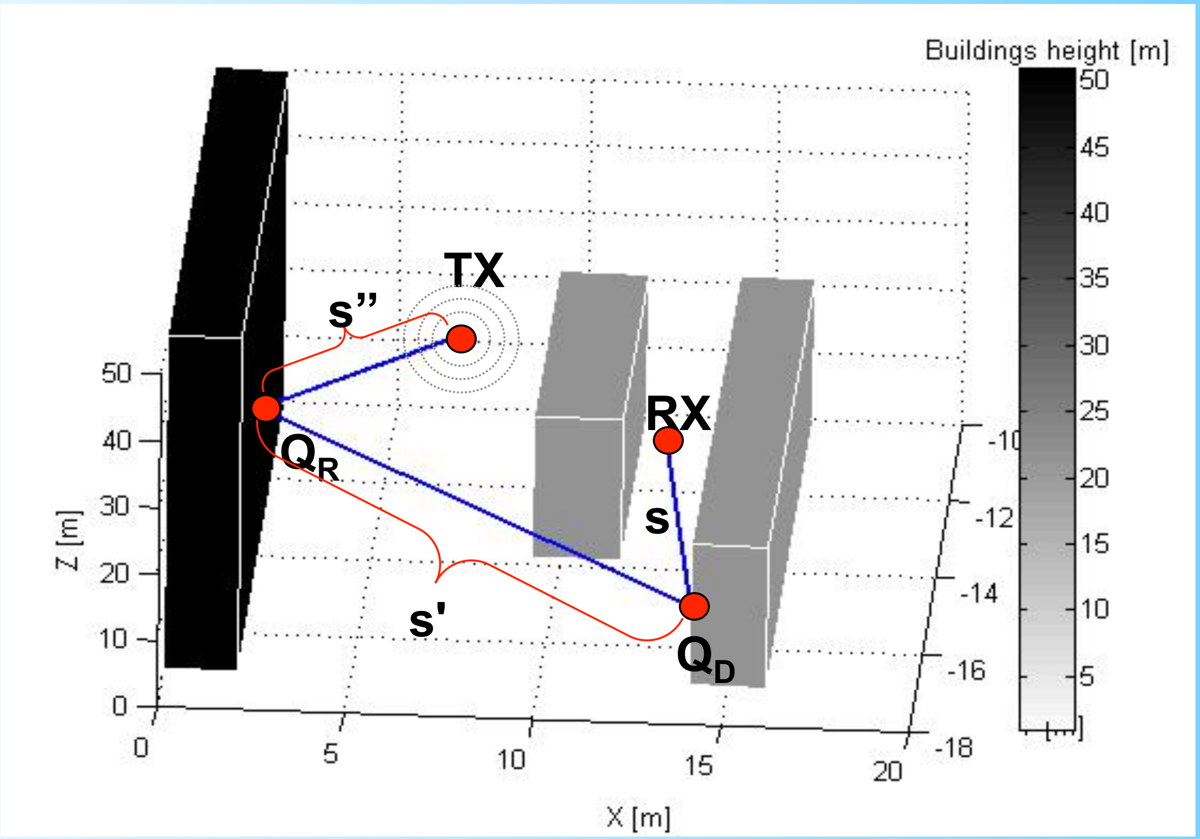
$$\underline{\underline{\mathbf{D}}} = D_s\left(\hat{\beta}_0'\hat{\beta}_0\right) + D_h\left(\hat{\phi}'\hat{\phi}\right)$$

we have

$$\overline{E}^d = \overline{E}^0 \cdot \underline{\underline{\mathbf{D}}} \cdot \frac{1}{\sqrt{s \cdot s' \cdot \left(s' + s\right)}} \cdot e^{-j\beta\left(s+s'\right)}$$

Double interaction (1/2)

Reflection + Vertical Edge Diffraction



Field at the reflection point: $\vec{E}(Q_R) = \vec{E}^0 \dfrac{e^{-j\beta s''}}{s''}$

Double interaction (2/2)

The field at the diffraction point is:

$$\vec{E}(Q_D) = \underbrace{\vec{E}^0 \cdot \frac{e^{-j\beta s''}}{s''}}_{\vec{E}(Q_R)} \cdot \underline{\underline{\mathbf{R}}} \cdot \frac{s''}{s' + s''} e^{-j\beta s'} = \vec{E}^0 \cdot \underline{\underline{\mathbf{R}}} \cdot \frac{e^{-j\beta(s'+s'')}}{s' + s''}$$

Finally, the field at the RX can be computed as:

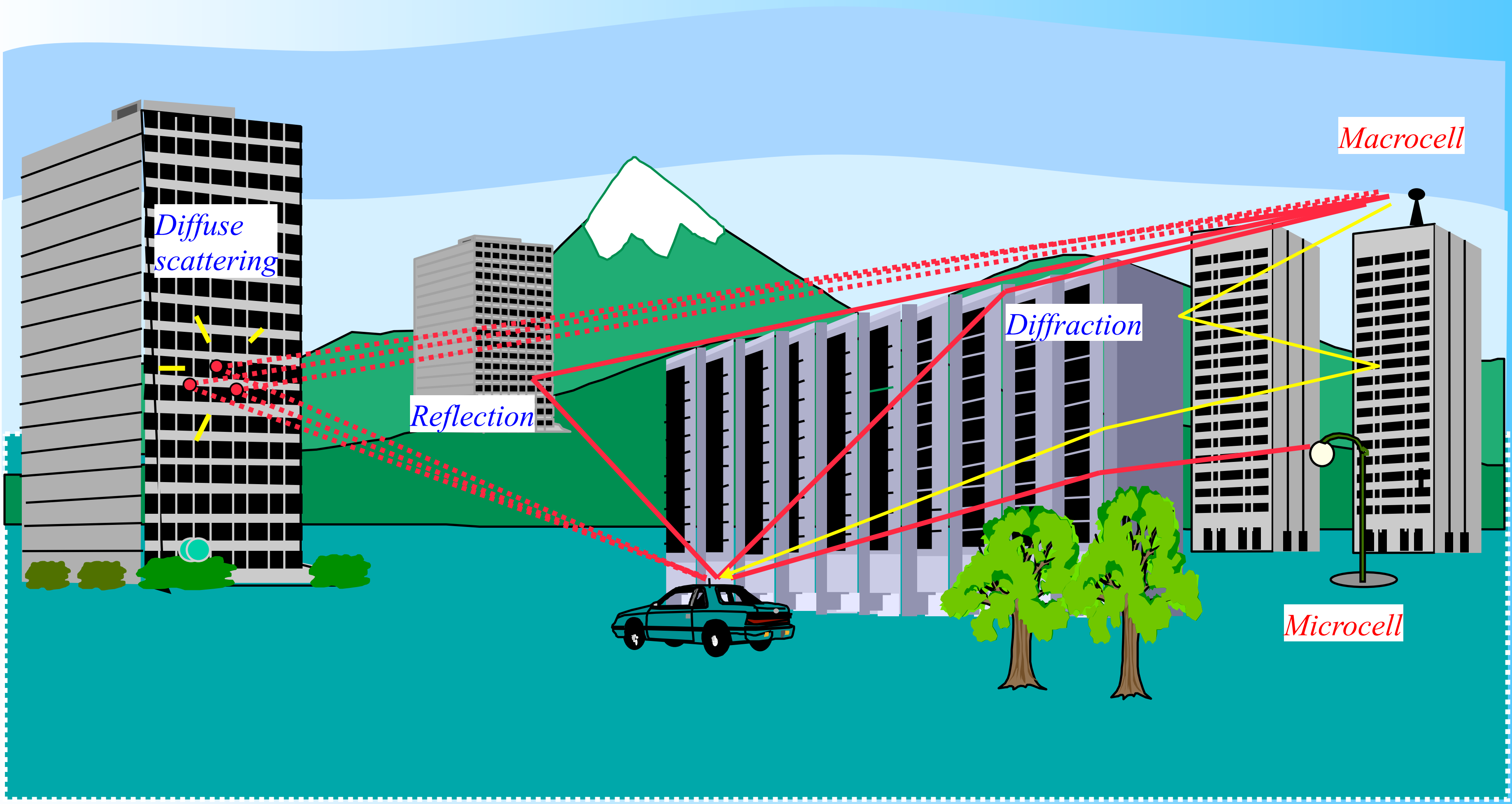
$$\vec{E}(Rx) = \vec{E}(Q_D) \cdot \underline{\underline{\mathbf{D}}} \cdot \sqrt{\frac{(s' + s'')}{s\left[s + (s' + s'')\right]}} \cdot e^{-j\beta s} =$$

$$= \vec{E}^0 \cdot \underline{\underline{\mathbf{R}}} \cdot \underline{\underline{\mathbf{D}}} \cdot \frac{1}{s' + s''} \cdot \sqrt{\frac{(s' + s'')}{s\left[s + (s' + s'')\right]}} \cdot e^{-j\beta(s+s'+s'')} =$$

$$= \vec{E}^0 \cdot \underline{\underline{\mathbf{R}}} \cdot \underline{\underline{\mathbf{D}}} \cdot \frac{1}{\sqrt{s(s' + s'')(s + s' + s'')}} \cdot e^{-j\beta(s+s'+s'')}$$

Superposition of multiple rays (1/2)

(Multipath propagation…)



Superposition of multiple rays (2/2)

The total field at a given position P can be computed through a coherent, vectorial sum of the field of all rays reaching P (difficult to determine though…):

$$\bar{E}(P) = \sum_{i=1}^{N_r} \bar{E}_i(P)$$

Moreover, the delays and angles of departure/arrival of the different ray contributions can be recorded get a multidimensional prediction.
In fact the GTP, determining its trajectory, also yields the following parameters for the i-th ray:

s^i total unfolded length

$t^i = s^i / c$ propagation delay

$\chi^i \equiv (\theta_T^i, \phi_T^i)$ angles of departure

$\psi^i \equiv (\theta_R^i, \phi_R^i)$ angles of arrival